MATHEMATICAL MODELLING OF PIEZOELECTRIC DISK TRANSFORMER WITH RING ELECTRODES IN PRIMARY ELECTRICAL CIRCUIT

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Abstract: *Thanks to its unique properties piezoceramics has applications in various fields of engineering and technology. Disk piezoelectric devices are widely used in the elements of information systems. Mathematical model of piezoceramic disk transformer with ring electrode in primary electrical circuit is constructed. An estimate is obtained and calculated and experimental curves of the frequency dependence of the modulus of transformation coefficient of piezoceramic disk transformer with ring electrode in primary electrical circuit are compared in the work. As a result of research of real device's mathematical model a set of geometrical, physical and mechanical and electrical parameters of a real object can be determined which provides realization of technical parameters of piezoelectric functional element specified in technical specifications. The cost of the saved resources is the commercial price of the mathematical model.*

Keywords: PIEZOELECTRIC DISK ELEMENT, PHYSICAL PROCESSES, AMPLITUDE-FREQUENCY RESPONSE

1. Introduction

Thanks to its unique properties piezoceramics has applications in various fields of engineering and technology. The relevance of the use of various functional elements of piezoelectronics in radio electronics, information and power systems is explained by their high reliability and small dimensions, which solves the problem of miniaturization of such systems. Piezoelectric disks with surfaces partially covered electrodes are often used to create various functional piezoelectronic devices. Disk piezoelectric devices are widely used in the elements of information systems. In disk piezoelectric elements with surfaces partially covered by electrodes we can simultaneously excite oscillations of compression-tension and transverse bending vibrations. Manipulating the geometric parameters of electrodes and their location relative to each other, you can have a significant effect on the energy of oscillatory motion particular type of material particles of piezoelectric disk volume. It should be especially noted that this piezoelectric element has compatibility with microsystem technology, so it can be made as microelectromechanical structures (MEMS) [1]. One of the main elements of functional piezoelectronics is piezoelectric transformer (PT). Research has shown that PTs can compete with traditional electromagnetic transformers on both efficiency and power density [2-4]. PTs are therefore an interesting field of research [5]. The favorable attributes of the PT are low weight and size and potentially low cost. One additional important characteristic is the high voltage isolation of the ceramic materials used to build PTs [6]. In addition, a piezoelectric transformer is more suitable for mass production than traditional, coil-based transformers [7].

The ***purpose*** of this paper is to set out the principles of mathematical models construction that are sufficiently adequate to real devices and occurring physical processes using the simplest example of axially symmetric radial oscillations of the piezoelectric disk.

2. Calculation of transformation ratio of piezoelectric transformer with ring electrode in primary electrical circuit

Let us consider a disk piezoelectric transformer (Fig. 1), primary electrical circuit of which consists of electric potential difference generator $U_1 e^{i\omega t}$ (where U_1 is an amplitude value of electric potential difference; $i=\sqrt{-1}$ is an imaginary unit; ω is an angular frequency; t is a time) with output electrical impedance Z_g and ring electrode (position 1 in Fig. 1). The secondary electrical circuit consists of an electrode in the form of a circle (position 2) with connected electronic circuit to it with input electrical impedance Z_n, on which an electric potential difference $U_2 e^{i\omega t}$ is formed. The primary and secondary circuits of piezoelectric transformer do not have a galvanic connection. The energy exchange between the primary and secondary circuits is carried out by means of axisymmetric radial vibrations of the piezoceramics material particles in the volume of thickness polarized disk (position 3 in Fig. 1).

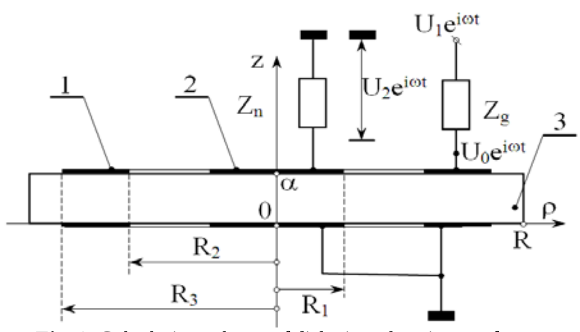

Fig. 1. *Calculation scheme of disk piezoelectric transformer*

It is obvious that the work of function piezoelectronic element, which is schematically shown in Fig. 1, is fully described by transformation ratio $K(\omega,\Pi)=U_2/U_1$ (Π is a set of geometrical and physical and mechanical properties of the piezoelectric transformer), which is a mathematical model of the device under consideration. Scheme of construction of piezoelectric transformer's mathematical model is outlined in [8].

Methodology for calculating the transformation ratio of piezoelectric transformer with ring electrode in primary electrical circuit is shown in [9]:

$$K(\omega,\Pi)=\frac{U_2}{U_1}=\frac{K_2(\Omega,\Pi)}{1-i\omega C_0^{\sigma} Z_g K_3(\Omega,\Pi)}, \qquad (1)$$

where

$$K_2(\Omega,\Pi)=\frac{2 f_l(\omega) K_{31}^2 A_{12}\left[J_1(\Omega R_1/R)/(\Omega R_1/R)\right]}{1-2 f_l(\omega) K_{31}^2 A_{11}\left[J_1(\Omega R_1/R)/(\Omega R_1/R)\right]};$$

$K_{31}^2=\left(e_{31}^*\right)^2/\left(c_{11}\chi_{33}^{\sigma}\right)$ is a squared electromechanical coupling coefficient for the mode of radial oscillations of thickness polarized piezoceramic disk material particles; e_{31}^* and χ_{33}^{σ} are material constants and dielectric permittivity for planar stress-strain state of the polarized across the thickness piezoceramic element; c_{11} is a modulus of elasticity for the mode of axially symmetric radial oscillations of the piezoceramic disk material particles; $f_l(\omega)$ is a switching on function or load characteristic of the output ring electrode of the piezoelectric transformer; $f_l(\omega)=\dfrac{i\Omega^*}{1-i\Omega^*}$, $\Omega^*=\omega\tau_n$ is a dimensionless quantity; $\tau_n=C_1^{\sigma} Z_n$ is a time constant of secondary electrical circuit; $C_1^{\sigma}=\pi R_1^2 \chi_{33}^{\sigma}/\alpha$ is a static electrical capacitance in secondary electrical circuit;

$$K_3(\Omega,\Pi) = \frac{2K_{31}^2}{1-\beta^2}\left\{\left[K_2(\Omega,\Pi)A_{41} + A_{42}\right]J(\Omega) + \right.$$
$$\left. + \left[K_2(\Omega,\Pi)A_{51} + A_{52}\right]N(\Omega)\right\} - 1\,;$$
$$J(\Omega)=\left[J_1(\Omega R_3/R)-\beta J_1(\beta\Omega R_3/R)\right]\big/(\Omega R_3/R)\,;$$
$$N(\Omega)=\left[N_1(\Omega R_3/R)-\beta N_1(\beta\Omega R_3/R)\right]\big/(\Omega R_3/R)\,;$$

$\beta = R_2/R_3$ is a geometrical parameter of the ring; $J_1(\Omega)$ and $N_1(\Omega)$ are Bessel and Neumann functions of the first order; Ω is a dimensionless wave number; coefficients A_{ij} are defined in [9].

All calculations are performed for piezoceramic disk with radius $R = 33\cdot10^{-3}$ m and thickness $\alpha = 3\cdot10^{-3}$ m, made of thickness polarized PZT type piezoceramics with following parameters: $\rho_0 = 7400$ kg/m3; $c_{11}^E = 112$ GPa; $c_{12}^E = 62$ GPa; $c_{33}^E = 100$ GPa; $e_{33} = 20$ C/m2; $e_{31} = -9$ C/m2; $\chi_{33}^\varepsilon = 1800\chi_0$; $\chi_0 = 8{,}85\cdot10^{-12}$ F/m is a dielectric constant; $Q_M = 100$ is a quality factor of piezoceramics.

In Fig. 2 it is shown the calculated (solid line) and the experimentally obtained (dashed line) curves of the frequency dependence of the modulus of piezoceramic ring-dot disk transformer's transformation coefficient. Naturally, the dimensions of the disk transformer in the calculation and experiment are chosen to be the same, i.e., the radius $R = 33\cdot10^{-3}$ m, the thickness $\alpha = 3\cdot10^{-3}$ m and $R_1/R = 12/25$, $R_2/R = 15/25$, $R_3/R = 0{,}999$. The values of the modulus of transformation coefficient of the piezoceramic disk transformer are plotted along the ordinate axis, and the frequency f (dimensionless value $\Omega = \omega\tau_0$, where $\tau_0 = R\big/\sqrt{c_{11}/\rho_0}$ is a time constant of piezoceramic disk) on the abscissa axis. The frequency $f = 15206$ Hz corresponds to the value $\Omega = 1$.

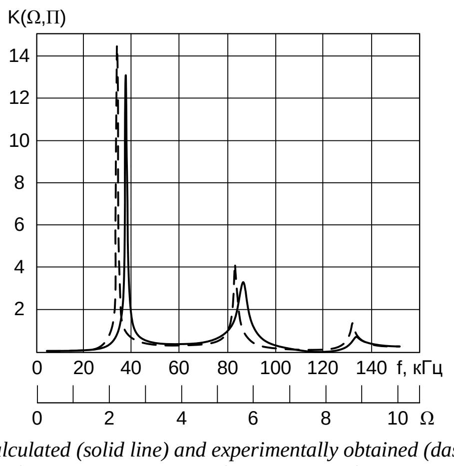

Fig. 2. *Calculated (solid line) and experimentally obtained (dashed line) curves of the frequency dependence of the modulus of piezoceramic ring-dot disk transformer's transformation coefficient*

When building the model, it was assumed that the thickness of the electrodes located on the surfaces of the disk is very small in comparison with the thickness of the disk α. In other words, the thickness of the electrodes, which, as a rule, does not exceed 15 μm, was not taken into account for constructing a mathematical model of piezoelectric transformer based on piezoceramic thin disk ($\alpha/R \ll 1$). It should also be noted that mathematical model (1) was built for ring-dot piezoelectric transformer (see Fig. 1) with surfaces partially covered by electrodes (area 1, $\rho \in [0, R_1]$, and area 3, where $\rho \in [R_2, R_3]$) and in the areas where there are no electrodes (area 2, where $\rho \in [R_1, R_2]$, and area 4, where $\rho \in [R_3, R]$).

As expected, the absolute values of the frequencies of resonances in calculation and experiment differ from each other. So, following the calculation, the frequencies of the first second and third electromechanical resonances are respectively equal to $f_{r1} = 37193\ Hz$, $f_{r2} = 88194\ Hz$ and $f_{r3} = 135330\ Hz$; the frequency ratio $\zeta = f_{r2}/f_{r1} = 2.371$.

The experimental values of the same quantities are, respectively, $f_{r1} = 34491\ Hz$, $f_{r2} = 83728\ Hz$, $f_{r3} = 132325\ Hz$ and $\zeta = f_{r2}/f_{r1} = 2.428$. If the experimental data are assumed to be true, the error in determining the frequency ratio is $\Delta\zeta = 2.3\%$. The obtained results are explained very simply. The numerical values of the frequencies of resonances s are determined by the dimensions and physicomechanical parameters of the material of disk element. The ratio of the resonances frequencies of the same disk is determined practically only by its dimensions. For this reason, a very satisfactory match between the theoretically and experimentally determined resonance frequency ratios is observed. The discrepancy between the absolute values of the resonance frequencies is explained by the discrepancy between the physicomechanical parameters of the piezoceramics, which were incorporated into the calculation and which are inherent in the experimentally investigated object. Comparing the curves, we can conclude that the quality factor of the material of the experimentally investigated sample is at least 1.2 times larger than included in the quality factor calculation.

3. Discussions and Conclusions

It can be asserted that the character of the variation of both curves, shown in Fig. 2, in a fairly wide frequency range coincides with accuracy to details. This means that the qualitative content of the expression (1) is adequate to the processes that occur in real object. In other words, expression (1) is a mathematical model of piezoelectric ring-dot transformer with ring electrode in primary electrical circuit and sufficiently adequate to the real object and the processes occurring in it. The latter allows us to assume that the mathematical description of the stress-strain state of the disk transformer also corresponds quite well to the real state of things.

Main results of this work can be formulated as follows: mathematical model of piezoelectric trans-former with ring electrode in the primary electrical circuit is constructed; calculated and experimentally obtained curves of frequency dependence of modulus of piezoceramic ring-dot disk transformer's transformation coefficient are estimated and compared.

4. References

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