THE COMPUTATION OF ROTOR'S MOTION IN CYLINDRICAL CHAMBER FILLED WITH VISCOUS GAS

РАСЧЕТ ДВИЖЕНИЯ РОТОРА В ЦИЛИНДРИЧЕСКОЙ КАМЕРЕ, ЗАПОЛНЕННОЙ ВЯЗКИМ ГАЗОМ

Prof., Dr. Dementev O.
Chelyabinsk State University, Chelyabinsk, Russia
e-mail: dement@csu.ru

Abstract: The problem rotor's movement in a stationary circular cylindrical chamber having finite length and filled with viscous gas is solved by the method of direct numerical integration of the set of equations describing pressure distribution in a thin layer of viscous gas and the motion of a rotating statically disbalansed cylinder. The rotor moving in the gravitational field is influenced by the impressed forces which vary periodically in time. Unsteady pressure equation is approximated by the symmetric stable finite-difference scheme of the second order accuracy. Stability criteria of a rotating rigid unstable cylinder (a rotor) motion subject to problem parameters are studied. The inner cylinder is influenced by outer forces which vary periodically in time. Trajectories of the rotor stationary motion for various velocities of rotation, disbalance values, amplitudes and frequencies of outer forces are calculated. Conditions of contact free motion of the cylinder, rotating in the chamber, are determined.

Keywords: Influence of form, stability, gas, rotation.

1. Let us consider two horizontal coaxial circular cylinders with length *L* and radius R_1 and R_2. The space between the cylinders is filled with viscous gas. The center of mass of the inner rigid solid cylinder (rotor) situated outside its rotational axis (static instability). The rotor moves round its symmetry axis with constant angular speed ω. Outer cylinder (chamber) is immovable. A gap between the cylinders is significantly less than their radii that is why to determine pressure distribution in a thin layer of viscous gas one can use Reynolds equation. In cylindrical coordinate system (r, φ, z), axis *z* of which is oriented along the axis of outer cylinder, the equation for pressure *p* is the following [1,2].

(1) $$\frac{1}{12\mu}\frac{\partial}{\partial z}\left(h^3\rho\frac{\partial p}{\partial z}\right)+\frac{1}{12\mu R_1^2}\left(h^3\rho\frac{\partial p}{\partial\varphi}\right) = \frac{\partial}{\partial t}(\rho h)+\frac{\omega}{2}\frac{\partial}{\partial\varphi}(\rho h),$$

where ρ is gas density; $h=h(\varphi)$ – local thickness of a gap between cylindrical surfaces $(R_1\le r\le R_1+h)$; μ – coefficient of dynamic viscosity of gas.
Boundary conditions:

(2) given $z=\pm L/2 \quad p=p_0$

(p_0 – pressure in the medium around the layer). Having found the field of pressure in the gas layer from the problem (1), (2), let us find the force applied by the gas to the rotating cylinder of length *L*:

(3) $$F_x=-2\int_0^{L/2}\int_0^{2\pi}pR_1\cos\varphi dzd\varphi,$$
$$F_y=-2\int_0^{L/2}\int_0^{2\pi}pR_1\sin\varphi dzd\varphi$$

While calculating reaction of a gas layer we took into account only pressure forces. They are much more than frictional forces [2] with the accuracy which was used while deriving the equation (1). Motion of a cylinder rotating in the gravitational field under the action of outer forces which periodically change in time is described by the equations (in the coordinate system, connected with the center of the immovable chamber).

(4) $$m\ddot{x}=F_x+m\delta\omega^2\cos\omega t +mg\left(1+a_1\cos\omega_1 t\right),$$
$$m\ddot{y}=F_y+m\delta\omega^2\sin\omega t+mga_2\sin\omega_1 t.$$

Here *m* – rotor mass; δ – value of shift of mass center from rotation axis; *g* – acceleration due to gravity; a_1, a_2 – amplitudes of outer periodic effects (i.e., a case when metal cylinder moves in alternating electromagnetic field); ω_1 – frequency of outer effects. At the beginning rotation axis coincides with a chamber axis.

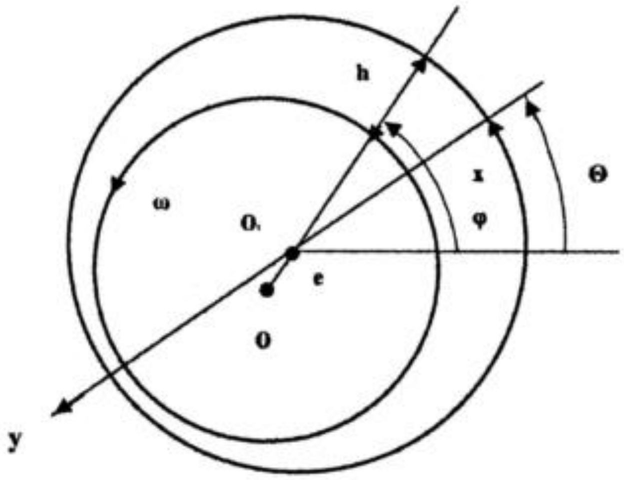

Fig. 1

2. Let us solve the problem of a rotor motion by the method of direct numerical integration of the set of equations describing a cylinder motion and pressure distribution in a gas layer [3, 4]. Let us rewrite the equation (1) in the form of the law of energy conservation of the volume $R_1\Delta\varphi\Delta z h_j$; including the node *i*, *j* of the lattice in cylindrical coordinate system [5] ($\Delta\varphi$ – angular step dimension and Δz – is the step in a coordinate):

(5) $$R_1\Delta\varphi\Delta z\frac{\partial}{\partial t}\left(h_j\rho_{i,j}\right)-\frac{h_j^3\Delta\varphi R_1}{12\mu\Delta z}\left[\rho_{i+\frac{1}{2},j}\left(-p_{i,j}+p_{i+1,j}\right)- -\rho_{i-\frac{1}{2},j}\left(p_{i,j}+p_{i-1,j}\right)\right]\left[\rho_{i,j+\frac{1}{2}}h^3_{j+\frac{1}{2}}\left(p_{i,j+1}-p_{i,j}\right)- -\rho_{i,j-\frac{1}{2}}h^3_{j-\frac{1}{2}}\left(p_{i,j}-p_{i,j-1}\right)\right]+ +\frac{\omega R_1\Delta z}{2}\left[\rho_{i,j+\frac{1}{2}}h_{j+\frac{1}{2}}-\rho_{i,j-\frac{1}{2}}h_{j-\frac{1}{2}}\right]=0$$
$$(i=0,1,2,\dots,I-1,\qquad j=1,2,\dots,J).$$

here

$$\Delta\varphi = \frac{2\pi}{J};\ \Delta z = \frac{L}{2I};\ \varphi_j = j\Delta\varphi;\ h_j = = C - x\cos\varphi_j - y\sin\varphi_j;$$

C – mean value of radial gap; I, J – quantity lattice nodes in axial and peripheral direction. Relation between pressure and gas density is defined by ratio $\frac{p}{\rho} = const.$

Boundary condition (2) in the form of finite difference are the following:

(6) $$p_{I,j} = p_0, p_{-1,j} = p_{1,j}, p_{i,0} = p_{0,J}, p_{i,J+1} = p_{i,1}$$

$$(i = 1,2,\dots,I,\ j = 1,2,\dots,J).$$

Here $\Delta\varphi = 2\pi / J$; $\Delta z = L/2I$; $\varphi_j = j\Delta\varphi$; I, J – quantity of lattice nodes in axial and peripheral direction. Relation between pressure and gas density is defined by the ratio $p/\rho = \text{const}$.

While writing ratios we used conditions of periodicity

$$p(z,\varphi) = p(z,\varphi + 2\pi)$$

and smoothness $\left(\frac{\partial p}{\partial z}\right)_{z=0} = 0.$

Let us consider equations (5) in dimensionless form, using the following units of measurements: a distance accross the layer – C (mean value of radial gap), a distance along the layer – R_1, time – $1/\omega$, pressure – p_0:

(7)
$$2\Lambda\Delta\varphi\Delta z\frac{\partial}{\partial t}\left(h_j\rho_{i,j}\right) - h_j^3\frac{\Delta\varphi}{\Delta z}\left[\rho_{i+1/2,j}\left(p_{i+1,j} - p_{i,j}\right) - \rho_{i-1/2,j}\left(p_{i,j} - p_{i-1,j}\right)\right]_j$$
$$= \frac{\Delta z}{\Delta\varphi}[\rho_{i,j+1/2}h^3_{j+1/2}\left(p_{i,j+1} - p_{i,j}\right)$$
$$-h^3_{j-1/2}\rho_{i,j-1/2}\left(p_{i,j} - p_{i,j-1}\right)]$$
$$+\Lambda\Delta z\left(\rho_{i,j+1/2}h_{j+1/2} - \rho_{i,j-1/2}h_{j-1/2}\right);$$
$$\left(h_j = 1 - x\cos\varphi_j - y\sin\varphi_j\right).$$

Using the method of varying direction [6] let us reduce the set problem of calculation of pressure field in a layer to a sequence of one-dimensional problems where we assign coefficients, non-linear terms and mass outlay of gas in axial direction to time moment $t_{k+1/2}$ (k – number of a time step) and other values – to t_k. Let us approximate the derivative $\partial p/\partial t$ by the difference formula of the second order centered with respect to $t_{k+1/4}$. We solve the derived nonlinear difference system of equations along the lattice lines considering index j as a parameter by the sweep method. Initial approach $p^0_{i,j}$ at every time step is defined with the help of linear extrapolation

$$\left(p^{k+1/2}_{i,j}\right)^0 = 1,5p^k_{i,j} - 0,5p^{k-1}_{i,j} \quad (k = 1,2,\dots),$$

where at the first time step $\left(p^{1/2}_{i,j}\right)^0 = p^0_{i,j}$.

At the next stage of the method of varying directions in equations (7) let us refer linear components of mass outlays in peripheral directions to t_{k+1} and coefficients and other members of the equations to $t_{k+1/2}$, the derivative $\partial p/\partial t$ center with respect to $t_{k+3/4}$. Then, following the method of the cyclic sweep method [7] find the pressure in all lattice nodes and the force effecting a rotor from a gas layer in the time moment t_{k+1}.

3. Let us consider the motion of a rotating unstable rotor in the field of gravity without periodic effects $\left(a_1 = a_2 = 0\right)$. A rotor, being at the initial period of time in the center of a chamber, begins to move. Orbits of steady-state motion of the rotor are nearly circular ($\delta' = \delta/C = 0,3$; the velocity of rotation *n=50 rev/s; n=350 rev/s*).

Let us turn to studying motion of a rotor in the presence of set external periodic disturbances of the finite amplitude. Let us set disturbances only in vertical direction, i.e., we assume that $a_1 = 1$, $a_2 = 0$ and $\omega_1 = \omega$ (frequencies of the rotor motion and of external disturbances coincide). Calculations of trajectories of the rotor motion are carried out for the speed values of its rotation n in the interval from 50 to 3000 revolutions per second at the change of relative disbalance δ' from 0,1 to 1. Steady-state orbits of the rotor motion are close to vertical elliptic ones. It was found that at small $\left(n < n_1\right)$ and big $\left(n > n_2\right)$ velocities of rotation the motion of the cylinder was not stable: stationary closed trajectories of the axis of revolution were not formed and the rotor touched the chamber (Fig. 2). There is a limited zone of rotor velocities $\left[n_1,\ n_2\right]$ (Fig. 3) for a constant value of disbalance where it is possible for a rotor to move avoiding a contact with a chamber. Left boundary of a zone of steady-state orbits formation practically does not depend on relative disbalance and right boundary moves to the domain of big velocities of revolution with decreased disbalance. Small ellipticity of the chamber (the function of a gap in this case had the form of ([8]) $h_j = 1,066 - 0,133\cos^2\varphi_j - x\cos\varphi_j - y\sin\varphi_j$) caused extention of formation zone for steady-state elliptic trajectories by means of increasing of the biggest critical velocity of revolution n_2. When velocity increases by the factor of 10 (from 50 to 1500 rev/s) the maximal amplitude of the orbit a increases by the factor of 33.

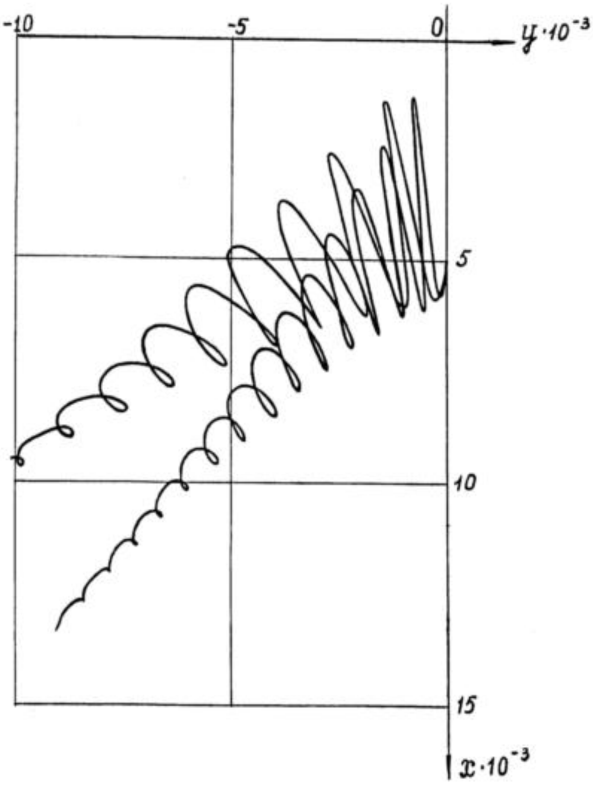

Fig. 2

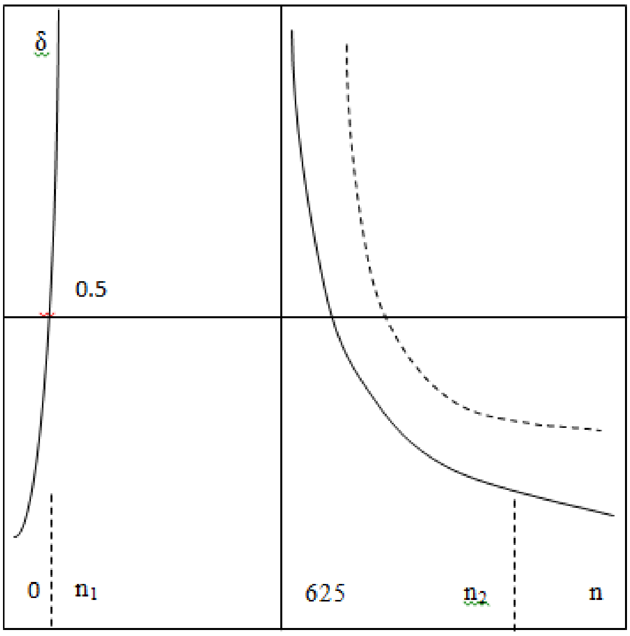

Fig.3

Increasing of the amplitude of horizontal diturbances a_2 up to 10 at low-frequecy disturbances ($\omega_1 = 1s^{-1}$) actually does not distort steady-state circular trajectories of the rotor motion and does not cause their distruction. This conclusion confirms calculations carried out for velocities of cylinder rotation in the interval from 100 to 500 rev/s. At the amplitude of external effects equal to $a_2 = 100$ ($a_1 = 0, \omega_1 = 1s^{-1}, n = 150$ rev/s) a cylinder gets in touch with a chamber very quickly ($\delta' = 0.3$; a_1=0, a_2=1, $\omega_1 = \omega = 2\pi 150$ s^{-1}; a_1=0, a_2=0.1, ω_1=1 s^{-1}, $n = 150$ and 100 rev/s). Change of disbalance from 0,1 to 0,85 in the presence of vertical disturbances with the frequency equal a half of the rotor revolution frequency ($\omega = 200\pi$ s^{-1}, $a_2 = 0$, $a_1 = 1$) does not influence significantly a formation and orientation of steady – state trajectories of the rotor. Influence of vertical disturbances of finite amplitude in comparison with the case of absence of disturbances causes increase of the amplitude of the steady-state trajectory by the factor of 8-10, shift of the orbit centre from the first quarter to the third quarter of the plane (x, y) and transformation of a circular orbit into an elliptic one.

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