APPLYING QUEUE THEORY AT STUDY OF REFUSALS OF REQUESTS RECEIVED IN UNIVERSAL AUTOMOTIVE SERVICE

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Abstract: The refusal of requests received in a universal automotive service workshop in the city of Rousse was investigated. The present work analyses the average monthly requests from the workshop customers. The number of the actual repairs was also determined. The work in the service have been seen as a mass service system with a non-stationary mid-month incoming stream with queries. The basic values of the system parameters were calculated under non-stationary conditions and Mat Lab application was created. After the model has been validated, a service conversion option was proposed to reduce the refusals. The proposed approach can serve as a methodology for analysing and optimizing of the activity of other universal automotive service.

Keywords: queue theory, refusals of requests, modelling, universal automotive service, correction, operating mode, work organization

1. Introduction

In the maintenance of transport equipment the main objective is the technical condition of the fleet to be always correct, using minimal resources. This can be achieved with different strategies and methodologies. The most modern trends used in the maintenance of vehicles is the development of different methods and methodologies for predicting the technical condition change [13].

Outsourcing the maintenance of vehicles is the most widely used method. The most common reasons for using this are:
- freeing resources for core business activities;
- improving productivity and quality;
- saving of funds;
- use of external competencies;
- transfer of the risks [10].

Using outsourcing has a number of drawbacks. The first is related to the employment of the service itself, which is the time for repair of the damaged vehicle (the time for order / order processing). The second disadvantage is related to the quality of the performed repairs / service.

The full query processing time includes the queue time and the time for processing the query itself. Queue is directly dependent on the time required for the query to be processed, i.e., the shorter the processing time for the query is, the queue is the smaller as well. According to the online business catalog “bussines.bg” there are 30 vehicle workshops in the town of Rousse [3]. The average number of working posts in these workshops is 7, which means that the working posts for servicing and repairing cars in Rousse are 210. In 2009, 90,871 cars were registered in city of Rousse [4]. From what has been written here follows that there will be a chronic shortage of working posts in the city. A queue of damaged cars will be produced in the workshops. For many workshops, getting big queues leads to denial of potential customers and financial losses. The purpose of this report is to provide a practical solution to this problem by researching a specific workshop.

The goal will be achieved by setting and solving the following tasks:
- collecting and processing statistical information from the services of a universal automotive workshop based on previous periods;
- defining the type of mass-service system;
- input modelling;
- solving the system of differential equations;
- defining the main features of the system;
- preparing proposals for adjusting the regime or organization of work in the workshop.

2. Exposition

In order to describe the operation mode of the car service, considered as a mass service, it is necessary to know the characteristics of the incoming flow of cars considered as a stochastic process, the service intensity, the maximum length of the tail and the number of service units [6, 11].

For the inbound flow of freight we can make the following assumptions:
- ordinary flow - The probability of two or more cars occurring for an elementary time interval is infinitely small compared to the probability of occurrence of only one car. The normality feature means that the cars come as single, not in group of two,three and so on at the same time.
- flow without consequences - the number of cars arriving in the system for time interval Δt does not depend on how many vehicles have already arrived, i.e. does not depend on the history of the studied phenomenon (the flow without action afterwards (Poisson flow).
- stationarity / non-stationarity of the flow [8, 13] - for sufficiently long periods of time - 1 month, 6 months, 1 year, etc. it is possible to assume the steady-state of the incoming stream, that is to say, with certain conventions, the probability of occurrence of a certain number of cars in a given, sufficiently long interval depends only on the length of that interval. Generally, in arbitrary periods, the λ stream is non-stationary λ = λ (t). This non-stationarity is clearly distinguishable over a period of one business year (about 300 working days).

For service intensity data by the service owner, it is known that the service time of a vehicle is a relatively constant quantity and is about half a working day (4 hours) based on a plan, μ = const. Regarding the number of service channels (servers), if necessary it can reach up to 3 (1≤n≤3). Two installers are needed to ensure continuous work for 8 hours on one channel. Again, according to the owner's data with more than m = 12 waiting in the queue, he refuses the order of the day, or the client renounces himself. To test the system's operation, it is necessary to find the probability that the system will have a number of cars at the time t when the n server [7, 2]

\[ P_k(t) = \sum_{n=m}^{\infty} P_n(t), k = 0, n + m, t \in [1, T], 1 \leq n \leq 3, \quad (1) \]

where for one period T is taken one full working month T = 1 of 12 months. The beginning of the first working month t = 1 coincides with the astronomical beginning of the year, and the end of the last working month t = T = 12, with the end of the astronomical year.

For the queue theory model, the following can be summarized: a non-stationary stream of requests with density λ (t), supplied to a mass service system with n serving channels. Request service time is a random variable with an indicative distribution and parameter μ = const. A car arriving at a busy time stands in the waiting line and "patiently" waits for service, unless there is more than m = 12 in the service queue, a limited waiting system and a limited number of cars in the queue. The main indicator will be the intensity of returned requests, and at what time of year these peaks are the highest, as well as the average number of underserved queries in those peaks.
From everything told above so far, it can be said that the system is of type (M / M / s) in non-stationary mode. To describe a system of this type, the following differential equations of Kolmogorov (Erlang-Kolmogorov) [mitrofanova] is in effect [1, 7]:

\[
\frac{dp_k}{dt} = -\lambda P_k(t) + \mu P_{k+1}(t)
\]

\[
\frac{dp_k}{dt} = \lambda P_{k-1}(t) - (\lambda + k\mu)P_k(t) + \mu(k + 1)P_{k+1}(t)
\]

\[
\frac{dp_{k+1}}{dt} = \lambda P_{k-1}(t) - (\lambda + \mu)P_k(t) + \mu P_{k+2}(t)
\]

\[
\frac{dp_s}{dt} = \lambda P_{s-1}(t) - (\lambda + s\mu)P_s(t) + \mu P_{s+1}(t)
\]

\[n \text{ is the number of channels and the maximum queue length when all servers are occupied. In some cases (with endless waiting) the differential equation system is open and for the numerical decision it is necessary to take the extra algebraic condition } \sum_{i=0}^{n} P_i(t) = 1 \text{ for normality. In the more general case, the maximum length of the queue is large. The computational features associated with a system (2) following the possible introduction of an algebraic equation are as follows:}
\]

- a large-scale system (generally)
- the system is differential-algebraic
- the system is of the "rigid system" type.

This can be summarized as followed: a system of rigid, differential-algebraic equations in some large-scale slashes. Numerical methods have been developed to overcome these difficulties. A Matlab program was developed to solve a system (2), using the built-in "solver" ode15s, implementing the Gir method. When entering \( \lambda(t), \mu, n, m \), the application returns a numerical solution to \( P_k(t) \). Artificially, the precision is greater than the default for the solver from 10\(^{-6}\) to 10\(^{-9}\) to absolute error from 10\(^{-3}\) to 10\(^{-5}\) to relative error [3, 12].

Input stream \( \lambda(t) \), generally every day, is different, and some seasonality is also highlighted. The following table 1 gives statistics for the applications received over a period of 3 years between 2015 and 2017.

<table>
<thead>
<tr>
<th>Month\Year</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>139</td>
<td>145</td>
<td>144</td>
</tr>
<tr>
<td>2</td>
<td>102</td>
<td>106</td>
<td>108</td>
</tr>
<tr>
<td>3</td>
<td>149</td>
<td>145</td>
<td>148</td>
</tr>
<tr>
<td>4</td>
<td>137</td>
<td>152</td>
<td>148</td>
</tr>
<tr>
<td>5</td>
<td>155</td>
<td>162</td>
<td>160</td>
</tr>
<tr>
<td>6</td>
<td>165</td>
<td>168</td>
<td>172</td>
</tr>
<tr>
<td>7</td>
<td>187</td>
<td>188</td>
<td>184</td>
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<td>8</td>
<td>190</td>
<td>186</td>
<td>184</td>
</tr>
<tr>
<td>9</td>
<td>169</td>
<td>175</td>
<td>176</td>
</tr>
<tr>
<td>10</td>
<td>130</td>
<td>122</td>
<td>128</td>
</tr>
<tr>
<td>11</td>
<td>130</td>
<td>128</td>
<td>132</td>
</tr>
<tr>
<td>12</td>
<td>119</td>
<td>118</td>
<td>120</td>
</tr>
</tbody>
</table>

To model \( \lambda(t) \), it is advisable to select a relatively elementary function who have a periodicity. This is appropriate given the seasonal fluctuations. To approximate the average values for the period 2015-2017, two methods are used - least squares method and minimax method. The use of the minimax method is motivated by the fact that the biggest error is minimal.

The model should be as simple as possible but reflects the most characteristic behavior of the real stream. For a model, the following trigonometric line is selected, non-linear to the quoted ratios:

\[
\lambda(t) = a_0 + a_1 \cos(wt) + b_1 \sin(wt)
\]

The coefficients \( a_0, a_1, b_1, w \), calculated by the least squares method are:

\( a_0 = 154.734311, a_1 = -2.134153, b_1 = -31.383019, w = 0.660038 \)

The coefficient of determination is \( R^2=0.8132 \) (statistically significant).

The coefficients \( a_0, a_1, b_1, w \), estimated by the minimax method are:

\( a_0 = 154.760396, a_1 = -12.684669, b_1 = -30.630357, w = 0.647557 \)

Figures 1 and 2 show the graphs of \( \lambda(t) \) with coefficients calculated by least squares method and minimax method.

**Fig. 1** The function \( \lambda(t) \) with coefficients found by least squares method

**Fig. 2** The function \( \lambda(t) \) with coefficients found by the minimax method

For the service intensity of one channel - \( \mu \), taking into account the service time of one car from one channel (we consider the working month for 25 days), we receive a \( \mu = 50 \) cars a month from a single channel for a month.

The starting state of the system \( P_0(t_0) \) is unknown. It is known that these types of processes are persistent and after a long period of time they enter into regular mode of operation. Therefore, an initial state may be taken arbitrarily. The integration of the system needs to be done not for a period of time but for a sufficient number of periods. In this way, probability functions \( P_k(t) \) begin to bend to their regular values. After repeated integration with different end times it was found that only after 5-6 periods the \( P_k(t) \) functions enter the regular mode (for two adjacent periods, remain the same). Accuracy is also increased here, with integration being done over 20 periods, with the difference of all \( P_k(t) \) in the last and penultimate periods being less than 10\(^{-8}\) for each t. Taking into account the proximity of \( \lambda(t) \) calculated by the coefficients of (4) and (5), all calculations are further made at \( \lambda(t) \) by coefficients of (5) (minimax method).

The following graphs reflect the results of the system decision (2) at the following values:

\( \lambda(t) \) calculated with coefficients of (5) \( \mu = 50, n = 3 \) (3 running servers), \( m = 12 \) (up to 12 in the queue).

Figure 3 illustrates the probability of having exactly \( k \in [0;3] \) cars in the system. It is noteworthy that the most likely values for a
small or zero number of cars are between 5 and 9 months (May-September).

![Graph of the probability of having exactly the k∈[0;3] vehicle in the system](image1)

**Fig. 3** Graph of the probability of having exactly the k∈[0;3] vehicle in the system

![Graph of the likelihood of having an exact k∈[4; 7] vehicle in the system](image2)

**Fig. 4** Graph of the likelihood of having an exact k∈[4; 7] vehicle in the system

![Graph of the likelihood of having an exact k∈[8; 11] vehicle in the system](image3)

**Fig. 5** Graph of the likelihood of having an exact k∈[8; 11] vehicle in the system

In Fig. 6 shows that shortly after the fourth to the middle of the ninth month the probabilities of a big queue grow at high speed, with a peak coming just before the seventh month.

The probability of rejection is given by \( P_{n+m}(t) \), i.e. all channels and places in the queue are busy and the arrived request will be denied (in Figure 6 the curve with purple color). It is also essential to know the density of declined queries. They are given with:

\[
P_n(t) = \lambda(t)P_{n+m}(t) \tag{6}
\]

**Fig. 6** A graph of the probability of having an exact k∈[12; 15] vehicle in the system with \( \lambda(t) \), calculated by the coefficients of (5)

**Fig. 7** Graph of the density of failures

The average volume of returns for the entire period is about 163.24. Also interesting are the volumes between 4-6 months, 6-8 months, 8-10 months, and in the busiest period between 5 and 9 months. After numerical solving of the integral (7), the results are shown in Table 2

<table>
<thead>
<tr>
<th>Months</th>
<th>Average number of returns</th>
<th>% from all returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-6</td>
<td>44.26</td>
<td>27.11</td>
</tr>
<tr>
<td>6-8</td>
<td>74.83</td>
<td>45.84</td>
</tr>
<tr>
<td>8-10</td>
<td>30.92</td>
<td>18.94</td>
</tr>
<tr>
<td>5-9</td>
<td>125.51</td>
<td>76.89</td>
</tr>
</tbody>
</table>

Table 2 shows that almost 77% of the returned applications are for a period of 4 months - from the beginning of May to the beginning of September.

Research shows that with constant work of 3 channels (6 people) there is a marked unevenness in the main indicators of the system. The ineffective mode of operation of the system leads to the need to take adequate measures to optimize it. According to the data from the owner of the garage, the hire of an additional third worker on each channel i.e. a 50% increase in the workforce would not lead to a linear decrease in service time by 50%, and a working day (4 hours) for repair, the time would fall to about 3 hours. Hiring more people from three people per channel does not lead to a decrease in service time.

There is a variant in which three additional people are appointed for a period of 4 months from the beginning of May until the beginning of September. The service speed of \( \mu = 50 \) will increase for this period at \( \mu = 66,667 \), with the other indicators not changing.
In Fig. 8 is a graph of the density of failures when hiring a supplementary labour for a period of 4 months. It appears that in the troubled period, returns have substantially decreased. There are also two large peaks, around the beginning of the year and after the beginning of the fourth month. They can be neutralized in a similar way, but it is not always possible to hire a workforce for a short time.

![Graph of the density of failures in μ different during the season](image)

In the proposed option to improve system performance the returned orders are 57.32, which is 2.85 times less than the original volume.

The study described in this report is also applicable to other repairers because of the similarity of organization of work in them, which is similar to the inputs of the system.

3. Conclusion

1. The theoretical results obtained are intended to clarify the behavior of the system at any time of the day for periods of approximately equal non-stationary intensity (accuracy to constant) of arrival and service.

2. A way is proposed to approximate the input flow density by the most general average statistics.

3. A methodology has been proposed for modeling and testing of the characteristics of a universal car service with similar non-stationary behavior as well as universal car service with a number of service units.

4. When system characteristics are unsatisfactory, the theoretical results will determine what adjustments in service intensity and / or number of channels are to be made for better performance. This would save time and problems of the “trial and error” type.

5. For the study, given the average behavior of intensity, the “average tail length” characteristic is sufficiently synchronized with the theoretical predictions (Table 1 and Figure 8). With sufficient statistics and measurements (a fairly accurate approximate intensity) for other universal car repair shops, it will provide theoretical characteristics that are close enough to the real ones.

6. Improvements can be made at no extra cost (for example opening new positions). This can be done by organizing a job involving more workers. This organization of working time will have the necessary effect in minimizing and adjusting the waiting time and the number of failures. This would result in a much more efficient way of working for the system itself. The waiting time will be approximately the same regardless of the arrival of the cars. The total average waiting time will also be reduced.

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