ON A MATHEMATICAL MODEL OF LAND-USE CHANGE

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Abstract: *The paper is devoted to a mathematical model of land-use change proposed by Dobson et. al. We formulate and investigate some quantitative properties of the corresponding Cauchy problem. We construct a numerical algorithm for approximate solution of the problem and present some of the numerical results. Their meaning is explained and discussed.*

Keywords: LAND-USE, MATHEMATICAL MODEL, CAUCHY PROBLEM, NUMERICAL SIMULATIONS

1. Introduction

In the distant past people had practically no potential to influence the nature. However since the early 19-th century (the beginning of the Industrial Age) and especially since the rapid increase in the number of inhabitants of the Earth the technological potential for changing the natural resources has multiplied many times. The main cause of all problems is undoubtedly the increase in the world's population – more than 7 billion now, which means that it increased around 7-fold since the year 1800. Another several billion newcomers are expected by 2050, even with all the cataclysms and epidemics that can kill millions and depopulate entire regions. These epidemics (such as AIDS, for example) are a reality now in large areas of Africa and Asia, unlike Europe and North America where, despite their absence, population growth is about zero or negative. Six countries are currently "responsible" for over half of the annual human growth on Earth by 80 million people - India, China, Pakistan, Nigeria, Bangladesh and Indonesia [1].

If their economic (industrial) development follows their population growth at the same pace and their standard of living is close to that of the US and the European Union, this would mean almost immediate depletion of most resources such as oil, minerals, arable land and water. The Earth, which is currently suffocate under the burden of its inhabitants, will have to absorb a new amount of carbon dioxide and harmful emissions.

The need of more and more resources (water, food etc.) for the continuously increasing amount of people worldwide leads to deforestation, related with clearance of forests for agriculture, building of cities, the fragmentation of forests, where large forest areas appear to be fragmented on numerous smaller plots located in agricultural lands or developing cities, which definitely affects forests and dependent species [2]. The deforestation leads to serious problems because the forests are very important to the health status of the environment. Forests provide numerous and vital ecosystem services for the environment and the climate. They help, for example, to regulate our climate and keep the river basins in a sustainable state by providing us with clean water. Forests contribute to purifying the air we breathe. The growth of the forest fund often helps capture large amounts of carbon dioxide from the atmosphere. This also helps to maintain and preserve biodiversity, as many species live in and depend on forests. The forests are also an important economic resource not only for the production of wood but also for other raw materials used for medicines and other products. Thus, forests have important functions for people's well-being and rest.

Methods of mathematical modeling are widely used for the study of complex processes, allowing mathematical description and an opportunity for performing numerical experiments. Investigation and modeling of complex phenomena are preceded by phenomenological observations and experimental work. After collecting the experimental data the investigation continues with interpretation and prediction of the behaviour of the system by identifying areas of independent variables, selection of the state parameters and definition of the parameters of the system under study. Thus one comes to the formulation of a model or to the description of the unknown and known variables and the relationships between them.

An interesting example of mathematical modeling is the work of Thomas Malthus "Essay on the principle of population" (1798) [3]. There Malthus mentioned the conflict between the growing population and the limited capacity of the environment, which has to satisfy the continuously increasing needs of natural resources. Due to Malthus, in certain moment of population development, its aspiration for growth should transform into "fight for survival". The theory of Malthus had strong impact on Charles Darwin and his book "On the origin of species" [4] devoted to the theory of survival of the most adaptive individuals. In the early 20-s of the 20-th century Pearl [5], Lotka [6] and Volterra [7] separately developed mathematical models for studying populations. These models provoked conducting a series of experimental studies on predator-prey interactions, competitive relationships between species and the regulation of populations.

The aim of the present paper is to study a model proposed in [8]. It describes the temporary change of land-use in particular situations. We formulate the model and study some of its properties. Further we perform numerical experiments and give some of their results.

2. Mathematical model of land-use dynamics

Here we present a model of land-use dynamics proposed by Dobson et al. [8]. The usefulness of such models follows from the high rate at which natural habitats can be converted to other uses. For example, pristine or almost pristine habitats may be colonized by humans and transformed into agricultural land. Moreover, the agricultural land can be converted into degraded land or into housing developments, cities etc. [8]. In some situations conversion to agricultural land can be partially reversible. For example: the conversion of forests into farmlands can be followed by regeneration of forests over substantial areas after abandoning of the farms (transformations observed in some countries [8]).

The model proposed by Dobson et al. [8] describes the dynamics of the size of human population supported by agriculture, denoted by W, the amount of land under agriculture, denoted by Y, the amount of degraded land (in recovery), denoted by V, and the amount of pristine or recovered land (undisturbed forest), denoted by X.

The model is the following system of four ordinary differential equations:

$$(1)\quad \frac{dX}{dt} = sV - dWX$$

$$(2)\quad \frac{dY}{dt} = dXW + bV - aY$$

$$(3)\quad \frac{dV}{dt} = aY - (b+s)V$$

$$\frac{dW}{dt} = rW\frac{Y - hW}{Y} \qquad (4)$$

The meaning of the variables and of the parameters of the model is the following. The variable X denotes the area of pristine forest habitat, which can be converted to agriculture land (area Y). Agriculture land transforms into unused land (denoted by V) after a time period 1/a. It is also assumed that the unused land can be restored to agriculture land after an interval of time 1/b. The variable W denotes the number of people that use the land. The parameter r denotes the growth rate of human population. Its dynamics is described by logistic expression with carrying capacity proportional to the land amount h needed to support one human. The parameter d denotes the rate of transformation of forests into agriculture land, while the parameter s denotes the rate of recovery of degraded land to forest.

The parameters of the model are assumed to be non-negative. The problem has to be supplemented by corresponding initial conditions.

We study the Cauchy problem related to the model (1) – (4). The following theorems can be easily proved.

Theorem 1.

If the Cauchy problem (1)-(4) with non-negative initial conditions possess a continuously differentiable solution, then this solution is non-negative for every time $t \geq 0$.

Theorem 2.

The system (1) – (4) with non-negative initial conditions possess a unique continuously differentiable solution for every $t \in [0, T]$,

where T is an arbitrary positive constant.

3. Numerical simulations

We solved the model numerically by using the code ode15s from the Matlab ODE suite. Ode15s is a multistep solver using numerical differential formulae [9].

We used the following values of parameters:

$$s = 1, b = 0.2, a = 0.05, p = 0.2, r = 0.5, h = 0.9$$

and initial values:

$$X(0) = 10000, Y(0) = 0.1, V(0) = 0.1, W(0) = 30.$$

We have analyzed the role of parameter d for the dynamics of the system.

The result of the numerical solutions are presented in Fig. 1 – Fig. 4. In Fig. 1 we present the system dynamics for $d = 0.00005$, in Fig. 2 – for $d = 0.0001$, in Fig. 3 – for $d = 0.001$ and in Fig. 4 – for $d = 0.05$.

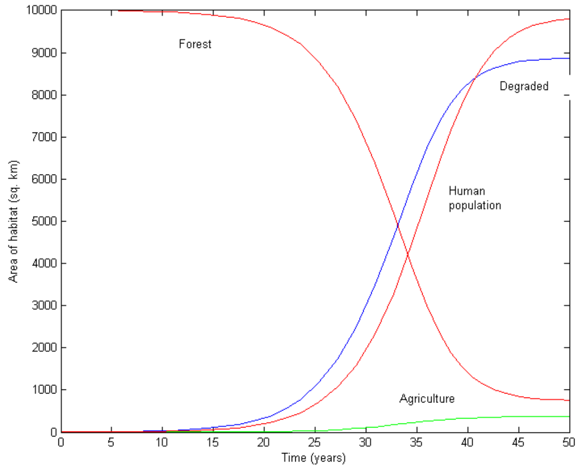

Fig. 1. Dynamics of the system for $d = 0.00005$

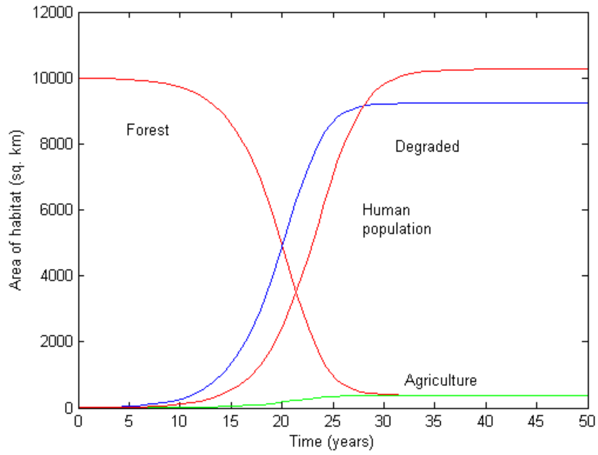

Fig. 2. Dynamics of the system for $d = 0.0001$

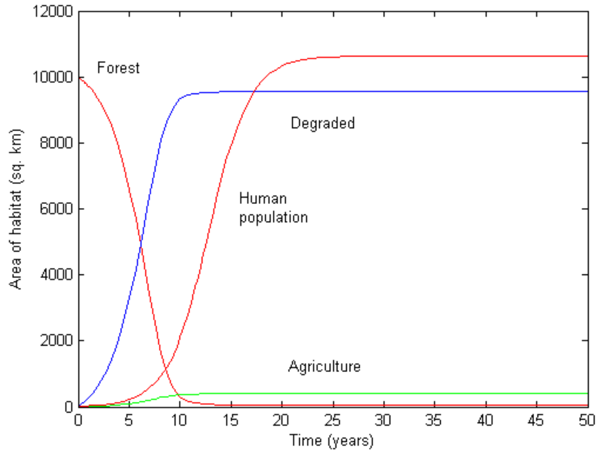

Fig. 3. Dynamics of the system for $d = 0.001$

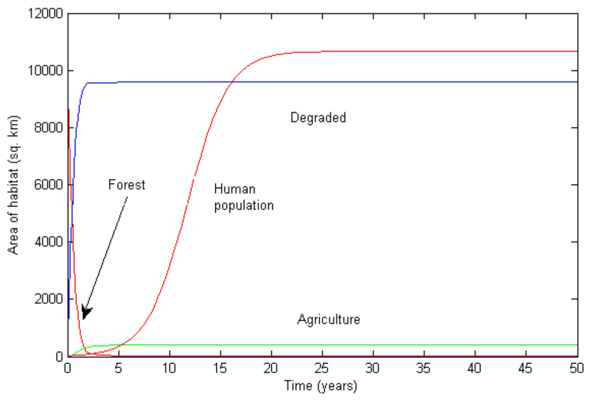

Fig. 4. Dynamics of the system for $d = 0.05$

The numerical solution shows that when the initial patch of pristine forest is invaded by 30 people starting to use land for agriculture, whose area increases, the area of pristine forest declines, while the area of unused land increases. When the value of parameter d is very small (e.g. Fig. 1, Fig. 2), the speed of these transformations is very small and it needs more time in comparison with situations with higher values of this parameters (e.g. Fig. 3, Fig. 4).

CONCLUSIONS

The presented model can be useful for prediction of the temporary dynamics of the various types of land and their change. Our future work will address the problem of estimation of parameters values by using specific field data.

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