MODELLING OF BIMORPH PIEZOELECTRIC ELEMENTS FOR INFORMATIONAL SYSTEMS

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Abstract: *The relevance of the use of various functional elements of piezoelectronics in informational and measuring systems is explained, first of all, by their high reliability, as well as small dimensions and weight, which greatly facilitates the solution of the problem of miniaturization of such systems. Currently, there are no reliable and valid methods of constructing of mathematical models of piezoelectric transducers, which could be used as a theoretical basis for characteristics and parameters calculating of this class of functional elements of modern piezoelectronics. The purpose of this article is to solve the problem of the excitation of transverse bending oscillations in bimorph piezoelectric element. Construction and features of mathematical description of bimorph piezoelectric element, the principle of which is based on the use of axisymmetric transverse bending oscillations, are considered. The solution of the problem of transverse bending oscillations excitation in bimorph piezoelectric element by the difference of electric potentials is obtained.*

Keywords: PIEZOELECTRIC DISK, BIMORPH ELEMENT, PHYSICAL PROCESSES, MATHEMATICAL DESCRIPTION.

1. Introduction

According to the results of the analysis of the world market of micro-devices based on the piezoelectric effect (piezoactuators, jet nozzles, microphones, pressure, vibration sensors, etc.), the consulting company "Yole Développement" (France) found that, for today, this market has almost reached the mark of $ 85 million, and by 2023 it will be about $ 310 million (while the Compound Annual Growth Rate will be 30.3%) [1]. It should be noted that the main accent that is being made by this company in the analysis of the piezoelectric device market is the research of devices with piezoelectric ceramics PZT (lead zirconate titanate). The unconditional advantages of using PZT piezoelectric ceramics are: low cost of raw materials and technology for the manufacture of piezoceramic elements, a relatively simple manufacturing technology, the majority of technological operations are unified with operations of microelectronics technologies, high inertness to the influence of climatic factors of the environment (including for aggressive and dangerous environments temperature, pressure, acidity, radioactivity, etc.), highly stable indicators of electromechanical coefficient, electrical and mechanical strength, ultra-high sensitivity to external mechanical influences and others [2].

The relevance of the use of various functional elements of piezoelectronics in in-formational and measuring systems, radio electronics is explained by their high reliability and small dimensions, which solves the problem of miniaturization of such systems. Piezoelectric disks with surfaces partially covered by electrodes are often used to create various functional piezoelectronic devices. Manipulating the geometric parameters of electrodes and their location relative to each other, you can have a significant effect on the energy of oscillatory motion particular type of material particles of piezoelectric disk volume. It should be especially noted that this piezoelectric element has compatibility with microsystem technology, so it can be made as microelectromechanical structures (MEMS) [3]. In this regard, the technologies and devices that use the direct and / or reverse piezoelectric effect in the principles of their work are promising. Bimorph piezoceramic elements, in which elastic oscillations of transverse bending are excited, are used to solve a fairly wide range of scientific and technical problems.

A significant scientific contribution to the development of the theoretical, mathematical and methodological and experimental basis for the development of new devices with piezoelectric ceramics was made by many scientists. For solving problems related to the improvement of manufacturing technologies for piezoceramic elements have been devoted, for example, works [4, 5].

The aim of the paper is solving the problem of the excitation of oscillations of transverse bending in bimorph piezoelectric element.

2. Materials and methods

Let consider the construction, which is shown in Fig. 1, a.

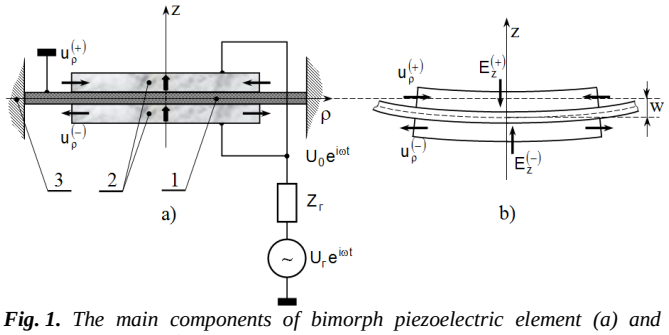


Fig. 1. *The main components of bimorph piezoelectric element (a) and transverse bending formation circuit (b)*

Position 1 in Fig. 1, *a* denotes a metal disk, the thickness of which $2h$ is substantially less than its diameter $2R$, i.e., strong inequality holds $h/R<<1$. Along the contour $\rho = R$ the metal disk is rigidly fixed on an absolutely fixed support (position 3). Wherein, the elements of the cylindrical surface ($\rho = R$; $0 \le \phi \le 2\pi$; $-h \le z \le h$) (ρ, ϕ, z are coordinate axes of the cylindrical coordinate system) do not have the ability to move along the coordinate axes ρ, ϕ, z. Two identical electroded piezoceramic disks are attached to the surfaces $z = \pm h$ of a metal disk by conductive glue (position 2 in Fig. 1, *a*). Investigation of current-conducting electrodes of piezoelectric elements modified by low-energy ribbon-shaped electron stream is presented in [6]. Electroded surfaces of piezoceramic disks, which are in electrical contact with a metal plate through a thin layer of conductive adhesive, always have the same, zero (Fig. 1, *a*) electric potential. Piezoceramic disks are not glued to the metal plate in an arbitrary manner, but in such a way that the direction of electrical polarization of these disks, which is shown in Fig. 1, *a*, bold arrows were the same. We will assume that these directions coincide with the positive direction of the coordinate axis z (Fig. 1, *a*). In this case, the matrix of the material constants of the upper and lower disks have the same construction and, most importantly, the elements of the matrix of the piezoelectric modules of the upper and lower disks have the same sign.

If on the top and bottom electroded surface of the piezoceramic disks are applied the electric potential difference as shown in Fig. 1, *a*, then the physical state of each of the disks will be determined by the following rations:

$$\sigma_{ij}^{(\pm)} = c_{ijk\ell}^{E}\varepsilon_{k\ell}^{(\pm)} - e_{kij}E_k^{(\pm)}, \quad (1)$$

$$D_m^{(\pm)} = e_{mij}\varepsilon_{ij}^{(\pm)} + \chi_{mn}^{\varepsilon}E_n^{(\pm)}, \quad (2)$$

where the plus sign determines the physical state of the upper piezoceramic disk, and the minus sign determines the lower; $c_{ijk\ell}^{E}$,

e_{kij}, χ^{ε}_{mn} are material constants – elements of the matrix of elastic modulus, piezoelectric modules and dielectric constants (see ratios; $\sigma^{(\pm)}_{ij}$, $\varepsilon^{(\pm)}_{k\ell}$, $E^{(\pm)}_{k}$ and $D^{(\pm)}_{m}$ are amplitude values that vary in time according to the law $e^{i\omega t}$ of the components of the elastic stress and strain tensors and the electric field strength and electrical induction vectors, respectively.

Suppose that at an arbitrarily fixed moment of time on the electroded surfaces $z=\pm(\alpha+h)$, where α is thickness of piezoceramic elements of equal size, of disks there is a positive electrical potential U_0. It is obvious that the axial components $E^{(\pm)}_{z}$ of the electric field intensity vector in the upper and lower disks have opposite directions, i.e. $E^{(\pm)}_{z}=\mp E_z$ (Fig. 1, *b*). It follows that the stress-strain states of the upper and lower disks at any time have opposite signs. If the disks are glued to the metal plate strictly coaxially, then the stress-strain state of the piezoceramic disks and, as a result, all shown in Fig. *1*, *a*, design has axial symmetry. This means that the shear stresses $\sigma^{(\pm)}_{\phi\beta}$ ($\beta=\rho,z$) and strains $\varepsilon^{(\pm)}_{\phi\beta}$ are zero.

In the low frequency range, when the scale of the spatial inhomogeneity of the stress-strain state far exceeds the thickness α of the piezoceramic disks, the mode of axisymmetric planar oscillations is realized in these disks. Wherein, the direction of the radial displacements $u^{(\pm)}_{\rho}$ of the material particles of the piezoceramic disks will have opposite directions (Fig. 1). This leads to the appearance of bending moments M_ρ, which force the entire structure to perform axisymmetric transverse bending vibrations. The source of these oscillations is the generator of the difference of electric potentials. Naturally, the adhesive bond introduces some slippage of the components of the structure relative to each other. At the same time, it can be argued that with high quality gluing together with a thin layer of high-molecular glue, these slippages can be neglected in a wide frequency range, up to the frequency of the third electromechanical resonance.

Shown in fig. 1, *a* construction it is accepted to call a bimorph piezoelectric element [2].

The bimorph piezoceramic element is completely naturally divided into two parts – active zone ($0\le\rho\le R_0$),where R_0 is piezoceramic disk radius, and passive zone – metal ring $R_0\le\rho\le R$.

In order to construct the equation of harmonic oscillations of the material particles of the active zone of a bimorph element, let us consider the integral characteristics of the stressed state of this region.

First, we write the expressions for calculating the deformations in the active zone. Since it implements an axisymmetric transverse bending, axisymmetric deformations $\varepsilon^{(\pm)}_{\rho\rho}$ and $\varepsilon^{(\pm)}_{\phi\phi}$ should be defined as follows

$$\varepsilon^{(\pm)}_{\rho\rho}=\varepsilon_{\rho\rho}(z)=-z\frac{\partial^2 w_0}{\partial\rho^2},\ \varepsilon^{(\pm)}_{\phi\phi}=\varepsilon_{\phi\phi}(z)=-\frac{z}{\rho}\frac{\partial w_0}{\partial\rho}, \qquad (3)$$

where w_0 is the deflection of the active zone of the bimorph piezoelectric element depending on the values of the radial coordinate ρ.

In the absence of shear stresses $\sigma^{(\pm)}_{\phi\beta}$ the generalized Hooke's law (1) is represented by the following rations

$$\sigma^{(\pm)}_{\rho\rho}=-z\left(c^E_{11}\frac{\partial^2 w_0}{\partial\rho^2}+\frac{c^E_{12}}{\rho}\frac{\partial w_0}{\partial\rho}\right)+c^E_{12}\varepsilon^{(\pm)}_{zz}-e_{31}E^{(\pm)}_{z}, \qquad (4)$$

$$\sigma^{(\pm)}_{\phi\phi}=-z\left(c^E_{12}\frac{\partial^2 w_0}{\partial\rho^2}+\frac{c^E_{11}}{\rho}\frac{\partial w_0}{\partial\rho}\right)+c^E_{12}\varepsilon^{(\pm)}_{zz}-e_{31}E^{(\pm)}_{z}, \qquad (5)$$

$$\sigma^{(\pm)}_{zz}=-zc^E_{12}\left(\frac{\partial^2 w_0}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial w_0}{\partial\rho}\right)+c^E_{33}\varepsilon^{(\pm)}_{zz}-e_{33}E^{(\pm)}_{z}, \qquad (6)$$

$$\sigma^{(\pm)}_{\rho z}=\sigma^{(\pm)}_{z\rho}=2c^E_{55}\varepsilon^{(\pm)}_{\rho z}-e_{15}E^{(\pm)}_{\rho}, \qquad (7)$$

where $E^{(\pm)}_{\rho}$ is the radial component of the electric field strength vector in the upper (plus sign) or lower (minus sign) piezoceramic disk.

The electrical state of the piezoceramic disks is determined by the electric induction vector $\vec{D}^{(\pm)}$, which is defined by two components $D^{(\pm)}_{\rho}$ and $D^{(\pm)}_{z}$. The circumferential component $D^{(\pm)}_{\phi}\equiv 0$ due to the axial symmetry of the physical state of the piezoceramic disk. In accordance with the general formulation (2) of the law of electric polarization of a dielectric with piezoelectric properties, we can write the following rations:

$$D^{(\pm)}_{\rho}=2e_{15}\varepsilon^{(\pm)}_{\rho z}+\chi^{\varepsilon}_{11}E^{(\pm)}_{\rho}, \qquad (8)$$

$$D^{(\pm)}_{z}=-e_{31}z\left(\frac{\partial^2 w_0}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial w_0}{\partial\rho}\right)+e_{33}\varepsilon^{(\pm)}_{zz}+\chi^{\varepsilon}_{33}E^{(\pm)}_{z}. \qquad (9)$$

When writing rations (4) – (6) and (9), the same in magnitude material constants were designated, as is customary in the mechanics of a deformable solid, by the same symbols.

As shown earlier, for thin piezoceramic disks in the low-frequency region, approximate estimates are valid $\left(D^{(\pm)}_{\rho}, E^{(\pm)}_{\rho}\right)\cong 0\,\forall(\rho z)\in V$ and $\left(\sigma^{(\pm)}_{\rho z}, \varepsilon^{(\pm)}_{\rho z}\right)\cong 0\,\forall(\rho z)\in V$, where V is disk volume.

If the bimorph piezoelectric element oscillates in a vacuum or, which is practically the same, in air, then the reaction of the environment $\sigma^{*}_{zz}=0$, and the ration (6) in the low-frequency region can be written in the following form

$$-zc^E_{12}\left(\frac{\partial^2 w_0}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial w_0}{\partial\rho}\right)+c^E_{33}\varepsilon^{(\pm)}_{zz}-e_{33}E^{(\pm)}_{z}=0,$$

where it comes, that

$$\varepsilon^{(\pm)}_{zz}=\frac{e_{33}}{c^E_{33}}E^{(\pm)}_{z}+z\frac{c^E_{12}}{c^E_{33}}\left(\frac{\partial^2 w_0}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial w_0}{\partial\rho}\right). \qquad (10)$$

Eliminating the axial deformation $\varepsilon^{(\pm)}_{zz}$ from expressions (4), (5) and (9), with using the ration (10), we obtain the following formulas:

$$\sigma^{(\pm)}_{\rho\rho}=-z\left(c_{11}\frac{\partial^2 w_0}{\partial\rho^2}+\frac{c_{12}}{\rho}\frac{\partial w_0}{\partial\rho}\right)-e^{*}_{31}E^{(\pm)}_{z}, \qquad (11)$$

$$\sigma^{(\pm)}_{\phi\phi}=-z\left(c_{12}\frac{\partial^2 w_0}{\partial\rho^2}+\frac{c_{11}}{\rho}\frac{\partial w_0}{\partial\rho}\right)-e^{*}_{31}E^{(\pm)}_{z}, \qquad (12)$$

$$D^{(\pm)}_{z}=-e^{*}_{31}z\left(\frac{\partial^2 w_0}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial w_0}{\partial\rho}\right)+\chi^{\sigma}_{33}E^{(\pm)}_{z}, \qquad (13)$$

where $c_{11}=c^E_{11}-\left(c^E_{12}\right)^2/c^E_{33}$, $c_{12}=c^E_{12}-\left(c^E_{12}\right)^2/c^E_{33}$, $e^{*}_{31}=e_{31}-e_{33}c^E_{12}/c^E_{33}$, $\chi^{\sigma}_{33}=\chi^{\varepsilon}_{33}+e^2_{33}/c^E_{33}$ are elastic modulus, piezoelectric modulus and dielectric constant for constant mode (equality to zero) axial stress $\sigma^{(\pm)}_{zz}$, i.e. for planar harmonic oscillations mode of the polarized in thickness thin piezoceramic disk.

From condition $div\vec{D}^{(\pm)}=0$ (conditions of absence of the electrical conductivity) it follows that in this situation conditions $\partial D^{(\pm)}_{z}/\partial z=0$ must be satisfied. This means, that certain expressions (13) of axial components $D^{(+)}_{z}$ and of the electric induction vector, do not depend on values of coordinate z. This fact can be used to determine the electric field strength $E^{(\pm)}_{z}$. Since the adduce is possible $E^{(\pm)}_{z}=-\partial\Phi^{(\pm)}/\partial z$ [7], where $\Phi^{(\pm)}$ is a scalar electric potential in the volume of the upper and lower piezoelectric disk, the following entries will be fair

$$D_z^{(-)} = -e_{31}^* z\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) - \chi_{33}^\sigma \frac{\partial \Phi^{(-)}}{\partial z},$$

$$D_z^{(+)} = -e_{31}^* z\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) - \chi_{33}^\sigma \frac{\partial \Phi^{(+)}}{\partial z}. \quad (14)$$

Let integrate the ration (14) by coordinate z in ranging from $-(\alpha+h)$ to $-h$, and the ration (14) – in ranging from h to $\alpha+h$. Taking into account that $\partial D_z^{(\pm)}/\partial z = 0$, integration results can be written as follows:

$$\alpha D_z^{(-)} = \alpha(h+\alpha/2)e_{31}^*\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) + \chi_{33}^\sigma U_0,$$

$$\alpha D_z^{(+)} = -\alpha(h+\alpha/2)e_{31}^*\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) - \chi_{33}^\sigma U_0,$$

where U_0 is an amplitude value of the electric potential on the electroded surfaces of piezoceramic disks. From recent records, it follows that

$$D_z^{(\pm)} = \mp(h+\alpha/2)e_{31}^*\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) \mp \chi_{33}^\sigma \frac{U_0}{\alpha}. \quad (15)$$

Definitions (13) and (15) are physically equivalent definitions of the same quantity – the axial component $D_z^{(\pm)}$ of electric induction vector. Equating the right sides of the definitions (13) and (15), we obtain the formula for calculating the strength $E_z^{(\pm)}$:

$$E_z^{(\pm)} = z\frac{e_{31}^*}{\chi_{33}^\sigma}\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) \mp (h+\alpha/2)\frac{e_{31}^*}{\chi_{33}^\sigma}\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) \mp \frac{U_0}{\alpha}. \quad (16)$$

Substituting expression (16) into the definitions (11) and (12) of normal stresses, we obtain the following calculation formulas:

$$\sigma_{\rho\rho}^{(\pm)} = -zc_{11}^D\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{k_D}{\rho}\frac{\partial w_0}{\partial \rho}\right) \pm (h+\alpha/2)\frac{\left(e_{31}^*\right)^2}{\chi_{33}^\sigma}\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) \pm e_{31}^*\frac{U_0}{\alpha}, \quad (17)$$

$$\sigma_{\phi\phi}^{(\pm)} = -zc_{11}^D\left(k_D\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) \pm (h+\alpha/2)\frac{\left(e_{31}^*\right)^2}{\chi_{33}^\sigma}\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) \pm e_{31}^*\frac{U_0}{\alpha}, \quad (18)$$

where $c_{11}^D = c_{11} + \left(e_{31}^*\right)^2/\chi_{33}^\sigma$, $c_{12}^D = c_{12} + \left(e_{31}^*\right)^2/\chi_{33}^\sigma$ are elastic modulus that take into account the connectivity (consonant action) of elastic forces and Coulomb forces in the volume of a deformable piezoceramics; $k_D = c_{12}^D/c_{11}^D$.

In the metal layer $-h \le z \le h$ (Fig. 1, *a*) of the active zone bimorph piezoelectric element normal stresses $\sigma_{\rho\rho}^{(м)}$ and $\sigma_{\phi\phi}^{(м)}$ are calculated by the following formulas:

$$\sigma_{\rho\rho}^{(м)} = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{\nu}{\rho}\frac{\partial w_0}{\partial \rho}\right), \sigma_{\phi\phi}^{(м)} = -\frac{Ez}{1-\nu^2}\left(\nu\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right), \quad (19)$$

where E and ν are Young's modulus and Poisson's ratio of metal disk material.

Following the foregoing technology for calculating bending moments, we can write that the linear density M_ρ of bending moments, that are created by normal stresses $\sigma_{\rho\rho}^{(\pm)}$ and $\sigma_{\rho\rho}^{(м)}$, $\sigma_{\rho\rho}^{(м)}$, is defined by the following formula

$$M_\rho = \int_{-(a+h)}^{(a+h)} z\sigma_{\rho\rho}^{(j)}dz = \int_{-(a+h)}^{-h} z\sigma_{\rho\rho}^{(-)}dz + \int_{-h}^{h} z\sigma_{\rho\rho}^{(м)}dz + \int_{h}^{(a+h)} z\sigma_{\rho\rho}^{(+)}dz = -D_1\left(\frac{\partial^2 w_0}{\partial \rho^2} + \frac{\eta}{\rho}\frac{\partial w_0}{\partial \rho}\right) + M_0, \quad (20)$$

where $D_1 = D_{пэ} - D_0 + D_м$; $D_2 = k_D D_{пэ} - D_0 + \nu D_м$; $\eta = D_2/D_1$; $D_{пэ} = 2c_{11}^D\left[(\alpha+h)^3 - h^3\right]/3$ is flexural rigidity of a piezoceramic disk as part of the active zone of a bimorph element; $D_0 = 2\alpha(h+\alpha/2)\left(e_{31}^*\right)/\chi_{33}^\sigma$ is a component of flexural rigidity due to the connectivity of elastic and electric fields in the volume of a piezoceramic disk; $D_м = 2Eh^3/\left[3(1-\nu^2)\right]$ is a flexural rigidity of the metal disk $M_0 = 2(h+\alpha/2)e_{31}^*U_0$ is a flexural moment, which is created by the source of flexural oscillations, i.e. the generator of the difference of electric potentials.

The linear density of the flexural moments M_ϕ is determined in the same way

$$M_\phi = \int_{-(a+h)}^{(a+h)} z\sigma_{\phi\phi}^{(j)}dz = \int_{-(a+h)}^{-h} z\sigma_{\phi\phi}^{(-)}dz + \int_{-h}^{h} z\sigma_{\phi\phi}^{(м)}dz + \int_{h}^{(a+h)} z\sigma_{\phi\phi}^{(+)}dz = -D_1\left(\eta\frac{\partial^2 w_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w_0}{\partial \rho}\right) + M_0. \quad (21)$$

Substituting the determines (20) and (21) into the formula of linear density Q of the transverse forces in the active zone of the bimorph piezoelectric element, we get the following calculation

$$Q = D_1\left(\frac{\partial^3 w_0}{\partial \rho^3} + \frac{1}{\rho}\frac{\partial^2 w_0}{\partial \rho^2} - \frac{1}{\rho^2}\frac{\partial w_0}{\partial \rho}\right). \quad (22)$$

From the conditions of dynamic equilibrium of an element of the volume of the active zone of a bimorph piezoelectric element (see the derivation of formula (17)) follows the equation of harmonic oscillations of the material particles of the active zone

$$\nabla^4 w_0 - \lambda_0^4 w_0 = 0, \quad (23)$$

where $\lambda_0 = \sqrt[4]{\left(2h\rho_м + 2\alpha\rho_{pe}\right)\omega^2/D_1}$ is the wave number of harmonic oscillations of the transverse bending of the active zone of a bimorph piezoelectric element; $\rho_м$ and ρ_{pe} are densities of metal plate and piezoceramics. In deriving equation (23), it was taken into account that $\sigma_z^* \equiv 0$.

Deflection or axial movement w_1 of the material particles of the passive zone, i.e., the metal ring $R_0 \le \rho \le R$, of the bimorph piezoelectric element are solutions of the following equation

$$\nabla^4 w_1 - \lambda_1^4 w_1 = 0, \quad (24)$$

where $\lambda_1 = \sqrt[4]{2h\rho_м\omega^2/D_м}$ is wave number of harmonic oscillations of material particles of a metal ring.

Let consider the features of the joint solution of differential equations (23) and (24).

Since the active zone of the bimorph piezoelectric element contains a point $\rho = 0$, so the general solution of equation (23) is the function $w_0(\rho)$, which is defined as follows

$$w_0(\rho) = A_1 J_0(\lambda_0\rho) + A_2 I_0(\lambda_0\rho), \quad (25)$$

where A_1 and A_2 are constants to be defined; $J_0(\lambda_0\rho)$ and $I_0(\lambda_0\rho)$ are Bessel function and modified zero order Bessel function [8].

The general solution of equation (24) is

$$w_1(\rho) = A_3 J_0(\lambda_1\rho) + A_4 N_0(\lambda_1\rho) + A_5 I_0(\lambda_1\rho) + A_6 K_0(\lambda_1\rho), \quad (26)$$

where $A_3,...,A_6$ are constants to be defined; $N_0(\lambda_0\rho)$ and $K_0(\lambda_0\rho)$ are Neumann function and zero order Macdonald function [8].

On the conditional boundary $\rho = R_0$ between the active and passive zones of a bimorph piezoelectric element, common solutions $w_0(\rho)$ and $w_1(\rho)$ should ensure continuity of the stress-strain state of the bimorph element. This is achieved by stitching on this border the kinematic and dynamic characteristics of the stress-strain state of the active and passive zones. The stitching conditions are written as follows:

$$w_0(\rho)\big|_{\rho=R_0} = w_1(\rho)\big|_{\rho=R_0}, \quad \left.\frac{\partial w_0(\rho)}{\partial \rho}\right|_{\rho=R_0} = \left.\frac{\partial w_1(\rho)}{\partial \rho}\right|_{\rho=R_0}, \quad (27)$$

$$M_\rho(\rho)\big|_{\rho=R_0} = M_\rho^{(m)}(\rho)\big|_{\rho=R_0}, \quad Q(\rho)\big|_{\rho=R_0} = Q^{(m)}(\rho)\big|_{\rho=R_0}, \quad (28)$$

where $M_\rho^{(m)}(\rho)$ and $Q^{(m)}(\rho)$ are linear densities of bending moments and shear forces in the metal ring of the passive zone of the bimorph piezoceramic element. The numerical values of these quantities are calculated by the formulas

$$M_\rho^{(m)}(\rho) = -D_m\left(\frac{\partial^2 w_1}{\partial \rho^2} + \frac{\nu}{\rho}\frac{\partial w_1}{\partial \rho}\right), Q^{(m)}(\rho) = D_m\left(\frac{\partial^3 w_1}{\partial \rho^3} + \frac{1}{\rho}\frac{\partial^2 w_1}{\partial \rho^2} - \frac{1}{\rho^2}\frac{\partial w_1}{\partial \rho}\right). \quad (29)$$

After substituting determines (25) and (26) into the matching conditions for solutions (27) – (28), we obtain a non-uniform system of four algebraic equations containing six unknown constants $A_1,...,A_6$. Missing two equations deliver loop fixing conditions $\rho = R$. In practice, three methods of fastening are most simply implemented, namely, rigid (this method is described in the comments for Fig. 1, *a*), hinge and free fastening.

When rigidly fixed, the position of material particles, which in their entirety form a cylindrical surface $\rho = R$, is strictly fixed, which can be formally described by the following rations

$$w_1(\rho)\big|_{\rho=R} = 0, \quad \frac{\partial w_1(\rho)}{\partial \rho}\bigg|_{\rho=R} = 0. \quad (30)$$

With the hinge fixing, the material particles of the cylindrical surface $\rho = R$ cannot move up or down, but the section itself as a whole can freely rotate relative to the axis that passes through the neutral plane $z = 0$ of the metal plate. The above is formalized as follows

$$w_1(\rho)\big|_{\rho=R} = 0, \quad M_\rho^{(м)}(\rho)\big|_{\rho=R} = 0. \quad (31)$$

With free fixing the contour $\rho = R$ is attached to practically weightless elastic supports or threads that do not resist the movement of elements of a cylindrical surface $\rho = R$. The surface $\rho = R$ can freely move up or down and rotate. Since there is no force reaction on the side of elastic supports, the following conditions must obviously be met

$$M_\rho^{(m)}(\rho)\big|_{\rho=R} = 0, \quad Q^{(m)}(\rho)\big|_{\rho=R} = 0. \quad (32)$$

Conditions (27) – (28) and conditions for fixing the contour $\rho = R$ form a heterogeneous system of six algebraic equations, which contains exactly six constants $A_1,...,A_6$. Obviously, such a system of equations is resolved with respect to the desired constants in the only way. This system of equations has the following form

$$m_{jk}A_k = \delta_{j3}\frac{M_0}{D_1\lambda_0^2}, \quad j,k = 1,...,6, \quad (33)$$

where coefficients m_{jk} for rigid, hinge and free fixing are defined in [37]; δ_{j3} is Kronecker symbol equal to one when $j = 3$ and equal to zero for all $j \neq 3$. The right part of the third equation in the system of equations (33) can be represented as follows: $M_0/\left(D_1\lambda_0^2\right) = W_0U_0$, where $W_0 = 2\left(h + \alpha/2\right)/\left(D_1\lambda_0^2\right)$ is an absolute sensitivity of the active zone of a bimorph piezoelectric element (the dimension is meter divided by volt).

The solution of the system of equations (33) can be represented as follows

$$A_k = (-1)^{k+3}W_0U_0\frac{\Delta_{3k}}{\Delta_0}, \quad k = 1,...,6, \quad (34)$$

where Δ_{3k} is an algebraic complement with unknown coefficient; A_k is a determinant of matrix sized 5×5, which is obtained from the matrix of coefficients m_{jk} system of equations (33) by crossing out the third line and k -th column; Δ_0 is a determinant of matrix 6×6 which is obtained from the coefficients m_{jk} of system of equations (33).

3. Experiments and results

If for a while it is assumed that there are no energy losses in the materials of the elements of the bimorph piezoelectric element, then the frequency-dependent change in the determinants Δ_0, that correspond to the rigid ($\Delta_0^{(h)}$), hinged ($\Delta_0^{(sh)}$) and free ($\Delta_0^{(f)}$) fixing of the contour $\rho = R$ of the metal disk, determined by alternating functions whose graphs are shown in Fig. 2. In the process of calculating the numerical values of the determinants were used the following material constants: metal (steel) plate: Young's modulus $E = 200\ GPa$, Poisson's ratio $\nu = 0,28$, density $\rho_m = 7800\ kg/m^3$, plate half thickness $h = 5\cdot10^{-4}\ m$, radius $R = 5\cdot10^{-2}\ m$; piezoceramic (PZT type ceramics) disks: elastic modulus $c_{11}^E = 110\ GPa$, $c_{12}^E = 62\ GPa$, $c_{33}^E = 100\ GPa$, piezoelectric modulus $e_{31} = -9\ C/m^2$, $e_{33} = 18\ C/m^2$, dielectric permittivity $\chi_{33}^\varepsilon = 1300\chi_0$, dielectric constant $\chi_0 = 8,85\cdot10^{-12}\ F/m$, density $\rho_{pe} = 7400\ kg/m^3$, disk thickness $\alpha = 10^{-3}\ m$, disk radius $R_0 = 1,5\cdot10^{-2}\ m$. Obviously, by manipulating the dimensions of the piezoceramic disks (thickness and radius), one can control the numerical values of the resonant frequencies of the bimorph piezoelectric element. The numerical values of the dimensionless frequencies $\Omega = \lambda_1 R$ of the first three resonances were determined for various types of contour fixing $\rho = R$ of metal disk. The symbols r_0 and α_0 indicate the dimensionless radius and thickness of the piezoceramic disks, where $r_0 = R_0/R$ and $\alpha_0 = \alpha/h$. In the process of calculations, the material constants of the components of the bimorph piezoelectric element were used, which are indicated above in the commentary to Fig. 2. Wherein, to the dimensionless frequency $\Omega = 10$ is corresponds the cyclic frequency $f = 9694\ Hz$.

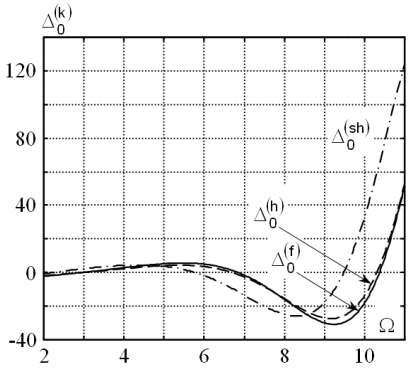


Fig. 1. *Frequency dependent change in the determinant of the system of equations (33)*

Calculations showed that the first three dimensionless resonance frequencies of a bimorph element with rigid contour fixing $\rho = R$ have the highest values compared with the frequencies of resonances that are observed with free and hinged fastening of the edge of a metal plate. This can be explained by the fact that the integral rigidity of the structure of a bimorph element with a rigid clamping of the contour $\rho = R$ of the metal plate is the highest compared with the total (integral) rigidity of the whole structure with other methods of fastening. The frequency of the first resonance with rigid and hinged fastening with increasing parameter r_0 firstly decreases and then begins to increase. This is because, for small values of r_0 the rigidity of the bimorph element increases at a slower rate compared with the increase in the mass of the oscillating element. Upon reaching a certain value of the parameter r_0 its further increase is accompanied by a leading increase in the rigidity of the bimorph piezoelectric element. As the thickness of the piezoceramic disk increases (parameter α_0) these dependences become more pronounced. For the second and third resonances, two (second resonance) and three (third resonance) intervals are observed for the numerical values of the parameter r_0, where there is a decrease and increase in the numerical values of the dimensionless frequencies of resonances. This, most likely, can be explained by inertial effects on the surface areas of the bimorph

element with antiphase motions of material particles.

To complete the mathematical description of a bimorph piezoelectric element, it is necessary to determine the potential U_0 on the electroded surfaces of piezoceramic disks (Fig. 1), i.e. to connect the deflections w of neutral plane with a difference of electric potentials U_z at the output of the electrical signal generator. To do this, it is necessary to determine the electrical impedance of the electromechanical oscillatory system, the design diagram of which is shown in Fig. 1.

From the definition (15) of the axial component $D_z^{(\pm)}$ of the electric induction vector, it follows that the electric charges $Q^{(\pm)}$ on the electroded surfaces of the upper and lower piezoceramic disk are equal in absolute value, but have opposite signs. The values of these charges are calculated by the following formula

$$Q^{(\pm)} = 2\pi \int_0^{R_0} \rho D_z^{(\pm)} d\rho = \mp 2\pi (h+\alpha/2) e_{31}^* R_0 \left.\frac{\partial w_0(\rho)}{\partial \rho}\right|_{\rho=R_0} \mp C_0^\sigma U_0, \quad (35)$$

where $C_0^\sigma = \pi R_0^2 \chi_{33}^\sigma / \alpha$ is a dynamic electric capacitance of a piezoceramic disk as part of bimorph piezoelectric element.

Taking into account the definition (25) of the deflection $w_0(\rho)$ of the neutral layer in the active zone of the bimorph element and the general solution (34) of the system of algebraic equations (33), we can write the following expression

$$\left.\frac{\partial w_0(\rho)}{\partial \rho}\right|_{\rho=R_0} = -\lambda_0 W_0 U_0 \left[\frac{\Delta_{31}}{\Delta_0} J_1(\lambda_0 R_0) + \frac{\Delta_{32}}{\Delta_0} I_1(\lambda_0 R_0)\right],$$

where λ_0 is a wave number of bending oscillations in the active zone of a bimorph element; $W_0 = 2(h+\alpha/2) e_{31}^* / (\lambda_0^2 D_1)$ is an absolute sensitivity of the active zone.

Substituting the last expression into formula (35), we get the following result

$$Q^{(\pm)} = \pm C_0^\sigma U_0 \left[\Psi_0(\omega, \Pi) - 1\right], \quad (36)$$

where $\Psi_0(\omega, \Pi)$ is a function that depends on the frequency and set of parameters (symbol Π) of the components of the active zone of the bimorph piezoelectric element. The numerical values of the function $\Psi_0(\omega, \Pi)$ are calculated by the formula

$$\Psi_0(\omega, \Pi) = 2K_{31}^2 \frac{c_{11}^E (h+\alpha/2)^2 \alpha}{\Delta_0 D_1} \times \left[\Delta_{31} \frac{2J_1(\lambda_0 R_0)}{\lambda_0 R_0} + \Delta_{32} \frac{2I_1(\lambda_0 R_0)}{\lambda_0 R_0}\right],$$

where $K_{31}^2 = (e_{31}^*)^2 / (c_{11}^E \chi_{33}^\sigma)$ is squared electromechanical coupling coefficient of piezoelectric ceramics in the mode of planar oscillations polarized through the thickness of a thin disk.

Let rewrite expression (36) as $Q^{(\pm)} = C_0^\sigma U^{(\pm)} \left[\Psi_0(\omega, \Pi) - 1\right]$, where $U^{(\pm)} = \pm U_0$.

From the last expression it follows that the amplitudes of the electric currents that are flowing through the conductors to the upper and lower piezoceramic disk have the following values

$$I^{(\pm)} = -i\omega C_0^\sigma U^{(\pm)} \left[\Psi_0(\omega, \Pi) - 1\right].$$

The electrical impedance $Z_{pe}^{(\pm)}$ of the upper and lower disks in the composition of the bimorph piezoelectric element is determined from Ohm's law for a part of the electrical circuit as follows

$$Z_{pe}^{(\pm)} = \frac{U^{(\pm)}}{I^{(\pm)}} = -\frac{1}{i\omega C_0^\sigma \left[\Psi_0(\omega, \Pi) - 1\right]} = Z_{pe}. \quad (37)$$

It is clear that equal electrical impedances $Z_{pe}^{(\pm)}$ are connected in parallel. For this reason, the electrical impedance of the bimorph element is $Z_{BE} = Z_{pe}^{(\pm)} / 2 = Z_{pe} / 2$.

The electric potential U_0 with the presence of the final output resistance Z_g in the source of the difference of electric potentials is determined as usual

$$U_0 = \frac{U_g Z_{BE}}{Z_{BE} + Z_g} = \frac{U_g Z_{pe}}{Z_{pe} + 2Z_g}.$$

Substituting relation (37) into the last expression, we get the following calculation formula

$$U_0 = \frac{U_g}{1 - 2i\omega C_0^\sigma Z_g \left[\Psi_0(\omega, \Pi) - 1\right]}. \quad (38)$$

Substituting the expression (38) into the general solution (34), we obtain the final form of the expression for calculating the numerical values of the coefficients A_k:

$$A_k = \frac{(-1)^{k+3} W_0 U_g \Delta_{3k}}{\left\{1 - 2i\omega C_0^\sigma Z_g \left[\Psi_0(\omega, \Pi) - 1\right]\right\} \Delta_0}. \quad (39)$$

Expression (39) completes the solution of the problem of the excitation of oscillations of transverse bending in a bimorph piezoelectric element by the difference of electric potentials, which is produced by a real generator with an output electrical impedance Z_g.

4. Conclusions

The main result of this article can be fixed as follows: design and features of the mathematical description of a bimorph piezoelectric element, the principle of which is based on the use of axisymmetric oscillations of transverse bending are considered; the solution of the problem of the excitation of transverse bending vibrations in a bimorph piezoelectric element for informational systems by the difference of electric potentials, which is produced by a real generator with an output electrical impedance, is obtained.

5. References

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