LET US TEACH MATHEMATICS EFFECTIVELY, LET US TEACH MATHEMATICS TO APPLY

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Abstract: *The problem of students' understanding and mastering mathematics is currently one of the most emphasized topics of expert public discussion. Mathematics is a specific subject. Having mathematical knowledge does not mean only to know definitions, sentences, algorithms or formulas. It is only the first step. The key competencies are to understanding of their mutual context, and ability to apply mathematics for solution of the problems of the life. If we want to teach mathematics more effectively, we need to use the new methods, forms and means of teaching as: the attractive applications, the adequate visualizations, contextual teaching, and observation based learning. A pedagogical experiment, whose results are stated in this article, was realised during the academic year 2015/2016 on the University of Zilina. After using the new alternative forms of teaching the subject Mathematics 1 we carried out a pedagogical experiment. The aim of this experiment was to find out whether this implementation of new methods would have a positive influence on the student's attitudes to mathematics.*

Keywords: TEACHING OF MATH, ALTERNATIVE FORMS OF TEACHING, GEOGEBRA, PEDAGOGICAL EXPERIMENT

1. Introduction

Current school reform introduced to current way of teaching math many changes - changes in content, methods, forms and organization of math education. Nowadays the most important goal of teaching is the shaping a student that understands math and is able to apply appropriate math judgments in practice.

Aforementioned abilities can be improved, if we include practical and applicable math tasks in the education process. Solving problem from areas of Natural, technical, economical or social sciences motivates students and shows them that math can be a useful tool in various aspects of life. Key responsibilities of a math teacher today are: teach math in an attractive, appropriate and interesting way. We found the correct way of teaching math when the student is interested, we develop their imagination, ingenuity, creativity and when we answer their questions following particular life experiences and needs.

Experience shows that math can be applied only by those students that truly understand it. Informal knowledge can be acquired by the student if he is active and encouraged to be creative during the education process [1]. We currently presume that it is information technologies that allow us to positively adduct student's activity during lessons. They give a teacher the opportunity - if the lesson is correctly structured – to construct his own knowledge.

Our article outlines the process of an pedagogical experiment with the goal of improving understanding math's and to increase motivation of students by presenting them with apt practical tasks.

2. Prerequisites and means for solving the problem

When creating lesson scenarios, we were inspired by Kolb's model of learning [3].

Kolb's Experiential Learning Cycle is a universal method of designing an effective didactic process. Kolb's model, among others, is based on experience and practical application of knowledge and skills. Learning by experience does not begin by getting familiarized with the sequenced set of principles, concepts and rules, as in the traditional way of learning, but it starts with specific experiences or activities that are observed; and on their basis reflections are provided, and additionally, concepts and principles suitable for use in different situations are formed (Fig.1).

According to this model, we didn't start by teaching definitions, propositions or formulas. We designed particular activities (which included use of mathematical software), where the students manipulate, observe and then analyze and generalize mathematical concepts. We used information technologies, in the first place the PC software GeoGebra. We created multiple simulations and animations of mathematical concepts. Students could literally touch them, experiment with them, getting to know them and test their properties.

Fig. 1 *Kolb's Experiential Learning Cycle*

3. Solution of the examined problem

Experimental teaching was executed with chosen set of first year students at FBI Žilinskej Univerzity v Žiline. Every week during the winter semester we carried out two innovative exercises according to created scenarios that used ICT. After this we checked the impact of this new teaching method on attitude towards math of our students. Now we present selected example activities implemented during the course of this experiment.

Problem 1 The security case

A metal workshop has received an order for a security case. This case will have a square bottom and is required to have a volume of exactly 800 liters = 0.8 m3. The material for the sides and the top cost 25 € per square meter and the material for the bottom, which needs to be thicker, is 50 € per square meter. What dimensions should the box have in order for the material costs to be as low as possible?

When you have a first solution to this problem, you should attempt to improve on your model, using Figure 2 as inspiration.

Fig. 2 *Type of security locker case*

Proposed solution 1

A routine algebraic solution to this typical optimization problem could look something like this:

Assume the sides of the square bottom to be *a* meters and the height to be *v* meters.

Then $a^2.v = 0{,}8$ (cubic meters), leading to $v = \dfrac{0{,}8}{a^2}$. The cost for the material can then be divided into the following parts:

Top: $25.a^2$,

Sides: $4..a.v.25 = 4.a.\dfrac{0{,}8}{a^2}.25 = \dfrac{80}{a}$.

Bottom: $50.a^2$.

The total cost sums to: $C(a) = 75a^2 + \dfrac{80}{a}$.

It is obvious that there exists a minimum because the function is increasing both as it approaches and for large values of *a*. The minimum is found by setting $C'(a) = 0$, we leading to the equation

(1) $150a - \dfrac{80}{a^2} = 0$,

which can be reduced to

(2) $\dfrac{150a^3 - 80}{a^2} = 0 \Leftrightarrow 150a^3 - 80 = 0 \Leftrightarrow 15a^3 = 80.$

So $a^3 = \dfrac{8}{15} \Rightarrow a \approx 0{,}811$ meter, giving a height of 1.217 meter and a total cost of 148 €.

Proposed solution 2

GeoGebra is an excellent tool for modeling situations like these. We can then solve the problem without the use of derivatives. Begin by noting that $v = \dfrac{0{,}8}{a^2}$, and let $x = a$ and $y = v$. We might want to draw a graph, but we should instead make a geometric model of the locker case. Because the case has a square bottom, we can make a reasonable model using two dimensions just by constructing a scale figure of one side.

Start by creating a point on the *x*- axis. We can do this using the point tool. Just click on the and then on the *x*- axis. Next press **Escape** to avoid producing more points by mistake. We could rename the point by right- clicking on it or by simply typing "P" **Immediately** after creating it, before pressing **Escape**.

Alternatively we could insert slider *a* using the button and create the point P by typing P = [*a*, 0] in the **Input Bar**. The P will

locate itself at the origin, but it can be dragged anywhere on the *x*- axis. Our work will be easier if we create *a* and *v*, so type in

$$a = x(P) \text{ and } v = \dfrac{0{,}8}{a^2}$$

where the function $x(P)$ gives the *x*- coordinate of the point P. Then create another three points by typing

$Q = [0,0]$ and $R = [0,v]$ and $O = [a,v]$, finally Polygon [O, P, R, Q].

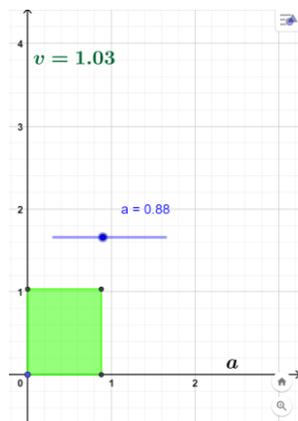We can now drag P and see how the case changes its appearance, as indicated in Figure 3.

Fig. 3 *Dynamic model of the locker case*

Calculate the total material cost by typing

$$C(a) = 75a^2 + \dfrac{80}{a}$$

then drag the result from the **Algebra window** to the graphics view. We can now simply drag P to find the minimum cost. By selecting P, we can control it using the arrow keys. The step size is set to 1 for points by default, but this can be changed in the properties of the point. On the **Algebra** tab we set the **Increment** to be 0.001.

We have now built a dynamic simulation of the locker case. In Figure 4, **Graphics view 2** is open where a point (*a, c*), having its **Trace On**, has been created.

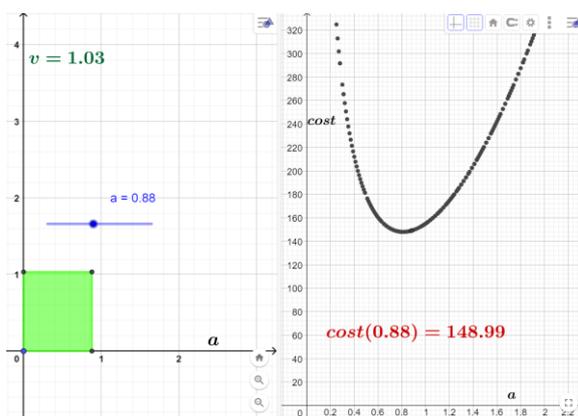Moving P will now result in a trace, clearly showing the minimum.

Fig. 4 *Plot of cost c against side a*

As the example in this paper show, the ease and versatility of GeoGebra make it an ideal tool for creating dynamic worksheets for both classroom demonstration and student exploration. GeoGebra's close connection between the algebraic and geometric representations of functions helps students visualize concepts from calculus from multiple perspectives.

Moreover, since it is extremely simple to make dynamic Java applets available on the web with GeoGebra, you can easily make it

possible for your students to continue interacting with mathematical concepts beyond the classroom.

After using the new teaching method with the alternative forms of teaching in the learning process of the subject Mathematics 1 we carried out a qualitative pedagogical experiment. The aim of this experiment was to find out whether this implementation would have a positive influence on the student's attitudes to mathematics. The students expressed their attitude to a new education method in the submitted questionnaire. We got a lot of interesting material which was subjected to the qualitative statistical analysis. In this article we present only a few partial results.

We used χ^2 – test for contingency table $k \times m$ to verify dependence of each pair of the qualitative characters A and B. The character A was acquiring k categories and the character B was acquiring m categories.

We tested the null hypothesis:
H_0: the characters A and B are independent, versus
H_1: the characters A and B are dependent.

The test statistics is:

(1) $\chi^2 = \sum_{i=1}^{5} \sum_{j=1}^{5} \frac{\left(f_{ij} - o_{ij}\right)^2}{o_{ij}}$, where f_{ij} are observed frequencies,

o_{ij} are expected frequencies;

(2) $o_{ij} = \dfrac{f_i^A \cdot f_j^B}{n}, n = 1,2,...,k; \ j = 1,2,...,m.$

The rejection region is

(3) $\chi^2 \geq \chi_\alpha^2\left((k-1)(m-1)\right)$, where $\chi_\alpha^2\left((k-1)(m-1)\right)$ is the critical value of χ^2 distribution with $((k-1)(m-1))$ degree of freedom. The degree of statistical dependence between the observed qualitative characters A and B is assessed using the contingency coefficient C and Cramer coefficient V which are defined as

(4) $C = \sqrt{\dfrac{\chi^2}{n + \chi^2}}$, $V = \sqrt{\dfrac{\chi^2}{n(h-1)}}$, where n is a sample size and

$h = min\{k, m\}.$

Table 1: *The contingency table of the observed frequencies.*

$A \setminus B$	B_1	B_2	...	B_m	f_i^A
A_1	$f_{11}(o_{11})$	$f_{12}(o_{12})$	...	$f_{1m}(o_{1m})$	f_1^A
A_2	$f_{21}(o_{21})$	$f_{22}(o_{22})$	...	$f_{2m}(o_{2m})$	f_2^A
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_k	$f_{k1}(o_{k1})$	$f_{k2}(o_{k2})$	...	$f_{km}(o_{km})$	f_k^A
f_j^B	f_1^B	f_2^B	...	f_m^B	n

4. Verification of formulated hypothesis

We formulated hypothesis: ***The style of teaching mathematics determines the level of its understanding.***

To determine where or not the style of teaching mathematics and the level its understanding by students are independent we used the χ^2 – test of independence. We use $\alpha = 0,05$, $n = 200$. We observed the following characters: the character A – the style of teaching mathematics and character B – the level of understanding mathematics by students.

The character A takes the categories:

A_1 – very good, A_2 – sometimes convenient, sometimes inconvenient, A_3 – mostly inconvenient, A_4 – most not, A_5 – not at all.

The character B takes the categories:

B_1 – the subject matter is understood most of the time, B_2 – the subject matter is understood at 50% of lessons, B_3 – the subject matter is rarely understood, B_4 – little hours, B_5 never.

We have compiled the contingency table of the observed frequencies:

Table 2: *The concrete contingency table of the observed frequencies.*

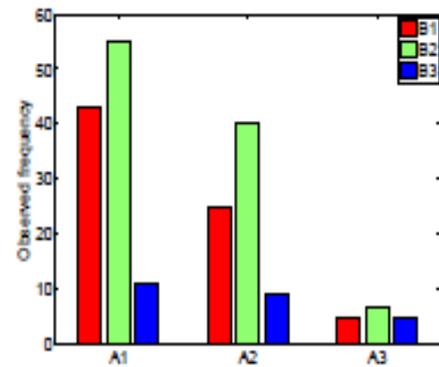| A / B	B_1	B_2	B_3	B_4	B_5	f_i^A
A_1	1 (0,84)	7 (3,54)	3 (6,12)	1 (1,26)	0 (0,24)	12
A_2	8 (6,79)	27 (28,6)	52 (49,5)	9 (10,1)	1 (1,94)	97
A_3	2 (5,18)	23 (21,8)	40 (37,7)	7 (7,77)	2 (1,48)	74
A_4	2 (1,05)	2 (4,4)	7 (7,65)	4 (1,57)	0 (0,3)	15
A_5	1 (0,14)	0 (0,6)	0 (1,1)	0 (0,2)	1 (0,04)	2
f_j^B	14	59	102	21	4	200

Fig. 5 *The bar chart of observed frequencies of characters A and B.*
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5. Results and discussion

We tested the null hypothesis:

H_0: the style of teaching mathematics and the level of its understanding are independent, versus

H_1: the style of teaching mathematics and the level of its understanding are dependent

The test statistics is $\chi^2 = 16,89$.

The critical value with $(k - 1)(m - 1) = 4$ degrees of freedom is $\chi_{0,05}^2(4) = 9,49$. The rejection region is $\chi^2 > 9,49$.

Since $\chi^2 = 16,89 \geq 9,49$, the hypothesis H_0 is *rejected*. It is evident that the style of teaching mathematics and level of its understanding are dependent.

The value of contingency coefficient is **$C = 0,2791$** and the value of Cramer coefficient is **$V = 0,2055$**.

The value of these coefficient indicate that between the analyzed qualitative characters A and B there exists the small degree of connection.

5. Conclusion

The described pedagogical experiment convinced us about the positive influence of implementing the new teaching methods with the technologies in the process of teaching mathematics. For mathematics teachers this means to focus their efforts on creating and applying the chosen modern teaching method in such a way that students can understand the importance of mathematics for everyday practice and they will be more motivated intrinsically.

In this experiment, the GeoGebra software has proven to be an effective tool in enhancing Mathematics teaching and learning.

Students were able to experience a hands- on method of learning which had a positive effect in enabling them to understand the concepts better rather than just being passive learners [2].

The software also gave the teacher and students the opportunity to work through the concepts together through exploration and visualization. This encouraged a more interactive teacher- student interactional environment where everyone worked as a team to guide, help and assist one another to reach the required goals. Overall, GeoGebra is an effective tool in assisting the teacher and students in the mathematics classroom to achieve the principles of constructivist learning. Based on the findings of the our experiment, it is highly recommended that teachers be encouraged to use GeoGebra software in teaching Mathematics.

6. References

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