1. Introduction

The sensitivity analysis is currently receiving considerable interest in the area of the performance evaluation of different stochastic models. The sensitivity analysis consists of investigating which individual input parameter drives most of the uncertainty on the model output. In this regard, we estimate the Sobol’s indices for global sensitivity analysis of stationary distribution in the GI/M/1/N queueing models. Specifically, when we estimate the Sobol’s indices, we consider the more influential parameters are uncertain. In this case, we estimate the expectation and the variance of the stationary distribution under the uncertainty.

Recently there has been a rapid increase in the literature on queueing systems with negative arrivals. Queues with negative arrivals, called G-queues, were first introduced by Gelenbe [5]. When a negative customer arrives, it immediately removes an ordinary (positive) customer if present. Negative arrivals have been interpreted as inhibiter and synchronization signals in neutral and high speed communication network. For example, we can use negative arrivals to describe the signals, which are caused by the client, cancel some proceeding. There is a lot of research on queueing system with negative arrivals. But most of these contributions considered continuous-time queueing model: Boucherie and Boxma [6], Jain and Sigman [8], Bayer and Boxma [2], Harrison and Pitel [9] all of them investigated the same M/G/1 model but with the different killing strategies for negative customers; Harrison, Patel and Pitel [10] considered the M/M/1 G-queues with breakdowns and repair; Yang [11] considered GI/M/1 model by using embedded Markov chain method.

The remainder of this paper is organized as follows. In Section 2, we introduce the necessary notations: the sensitivity analysis and uncertainty analysis. In Section 3, we outline description of the model and we finish by the numerical framework to illustrate the applicability of this analysis. Concluding remarks are provided in Section 4.

2. Sensitivity Analysis

Mathematical models always approximate the real phenomena. The uncertainty of their input parameters described the incapacity to envisage precisely her issues, from which the uncertainty also of the output parameters.

Thus, the precision of the output parameters will depend on the quality of the available information. These uncertainties often correspond to the errors made by measuring instruments, manufacturing processes or limited data.
customer. A sequence of random variables \( L_k: k = 1, 2, \ldots, N \) constitutes a Markov chain. Its transition probabilities matrix is given by:

\[
P = \\
\begin{pmatrix}
  b_0 & a_0 & 0 & 0 & 0 & \ldots & 0 \\
  b_1 & a_0 & a_0 & 0 & 0 & \ldots & 0 \\
  b_2 & a_2 & a_1 & a_0 & 0 & \ldots & 0 \\
  b_3 & a_3 & a_2 & a_1 & a_0 & \ldots & 0 \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots \\
  b_{N-1} & a_{N-1} & a_{N-2} & a_{N-3} & a_{N-4} & \ldots & a_0 \\
  b_{N-1} & a_{N-1} & a_{N-2} & a_{N-3} & a_{N-4} & \ldots & a_0
\end{pmatrix}_{(N-1) \times (N-1)}
\]

\[
a_j = \int_0^{+\infty} e^{\lambda t} t^{j-1} \mu \lambda^j \int_0^{\infty} \frac{e^{-t\lambda} t \mu}{\mu^j} d\lambda
\]

Where

\[
b_j = 1 - \sum_{i=0}^{j} a_i.
\]

4. Numerical Results

Consider this application

\[
\pi_l: \mathbb{R}^n \rightarrow \mathbb{R},
\]

\[
\theta \rightarrow \pi_l(\theta),
\]

where \( \pi_l, l = 0, 1, \ldots, N \) is the stationary distribution of a such model, and \( \theta = (\theta_1, \theta_2, \ldots, \theta_m) \) is a vector of all parameters of the model. The first order Sobol’s indices are compute by this formula

\[
S_i = \frac{V(E[\pi_l]/\theta_l)}{V(\pi_l)}, \quad i = 1, \ldots, m; \quad l = 0, 1, \ldots, N.
\]

The M/M/1/4

Consider the M/M/1/4 system with negative arrivals, where the inter-arrivals times and the service times are exponentials.

Let the input vector parameters

\[
\theta = (\theta_1, \theta_2, \theta_3, \theta_4) = (\lambda_1, \lambda_2, \mu, \zeta).
\]

The H^2/M/1/4

Consider the H^2/M/1/4 system with negative arrivals, where the inter-arrivals times are hyper-exponentials and the service times are exponentials.

Let the input vector parameters

\[
\theta = (\theta_1, \theta_2, \theta_3) = (\lambda, \mu, \zeta).
\]
According to this figure, we note that the values of the highest indices those, which correspond to the parameters $\lambda_1$, $\mu$, $\zeta$, and $\xi$, therefore these two last, are more influents on the stationary distribution, and as the parameter $\lambda_1$ is less influent, so it is considered deterministic (constant).

According to the analysis carried previously, we obtained $\mu$ and $\zeta$ like the most influential parameters for each component $\pi_l$, $l = 0, 1, \ldots, 4$ of the stationary distribution $\pi (\mu, \zeta)$, and the parameter $\lambda$ is considered like a deterministic parameter.

To simulate the expectation and the variance of the stationary distribution, we present a new formula the two parameters $\mu$ and $\zeta$:

$$\mu = \bar{\mu} + \sigma_\mu \epsilon_\mu,$$

$$\zeta = \bar{\zeta} + \sigma_\zeta \epsilon_\zeta,$$

According to the analysis carried in this table, we note that a disturbance of 10% of each input parameter involve a maximum variance of $4.63 \times 10^{-04}$, which proves the robustness of the M/M/1/4 model with negative arrivals, compared to the uncertainty inflicted in the influential parameters. In other words, a small disturbance on the input parameters generates a small disturbance in the output parameter.

5. References


