RESEARCH OF MATHEMATICAL MODELS OF LITHIUM-ION STORAGES

ИССЛЕДОВАНИЕ МАТЕМАТИЧЕСКИХ МОДЕЛЕЙ ЛИТИЙ-ИОННЫХ НАКОПИТЕЛЕЙ

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Abstract: *In modern industry self-powered devices are an important component. For such devices, the most important component is the energy storage device used, most often based on lithium-ion technology. The article presents an equivalent circuits of lithium-ion batteries and a mathematical description of charge-discharge processes. Investigated in Matlab / SimPowerSystems built-in library component of lithium-ion battery. Mathematical models of equivalent circuits of different types of lithium-ionic batteries have been analyzed.*
***KEYWORDS**: CHARGE-DISCHARGE CHARACTERISTICS, SIMULATION, LITHIUM-ION TECHNOLOGY, ENERGY STORAGE.*

1. Introduction

Lithium-ion batteries are one of the most popular energy sources for a wide range of autonomous devices from mobile phones to electric vehicles [1]. At the same time mathematical models of lithium-ion batteries are one of the key issues in the modeling of autonomous devices, as they determine the capacity and time of the batteries, voltage stability during discharge, as well as the battery charge rate [2].

Nowadays, electric vehicles are attracting a lot of attention from researchers because of their properties, such as reducing fuel consumption and greenhouse gas emissions [3, 4].

As an energy storage, the battery is one of the basic elements on which the development of electric vehicles depends. The lithium-ion battery is known for its advantages such as high energy density, high charge and discharge speed, safety, etc. [5].

Until recently, a major drawback of electric vehicles was the high cost of lithium-ion batteries. However, there is a tendency to reduce the cost of lithium-ion batteries (Fig. 1). So, by the year 2030, the cost of 1 kW·h of a lithium-ion drive will cost $ 62 [6].

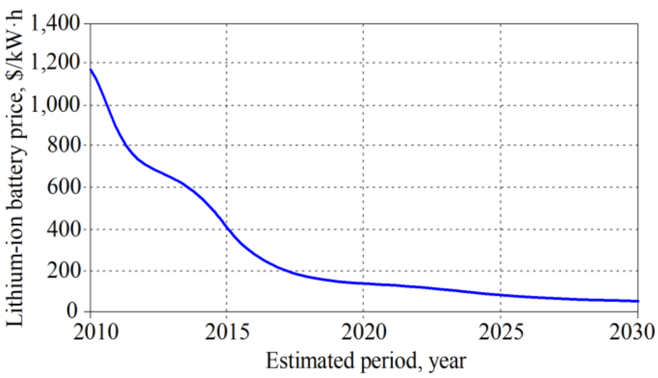


***Fig. 1.** The existing trend and forecast for the decrease in the cost of lithium-ion batteries*

Due to the increasing research in simulation processes in lithium-ion batteries of electric vehicles, it is important to ensure a high accuracy of charge and discharge modeling [7, 8].

The purpose of this work is to analyze to validity of mathematical models of lithium-ion batteries on the example of the battery type NCR-18650b, namely, with the characteristics stated in the documentation for the battery.

2. Volt-ampere characteristics of lithium-ion batteries in the charging process

Fast charging depends on the transfer of energy to the battery at very high power levels. It is not only the chemical composition of the battery that determines the power level at which the cell can take charge, but also the method used to charge the battery [9].

The most popular battery charging procedure is the CC–CV (Constant Current – Constant Voltage) [10] (Fig. 2).

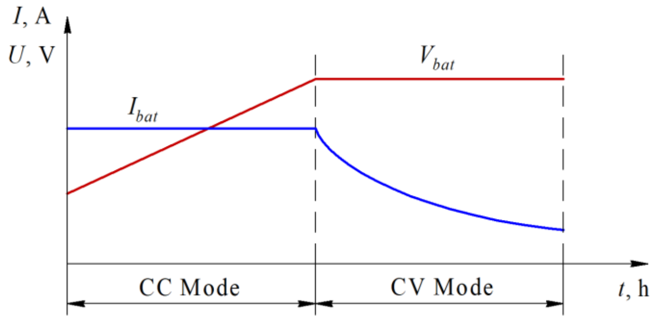


***Fig. 2.** The charging characteristics of the battery*

The main idea behind the CC–CV method is that the battery charges a constant maximum current, usually determined by the element manufacturer, to some cut-off voltage and then charges at that voltage until current consumption decreases to about 0.1C or less, providing full charge [11, 12].

The discharge characteristics of the NCR-18650b battery according to the technical documentation are shown in Fig. 3.

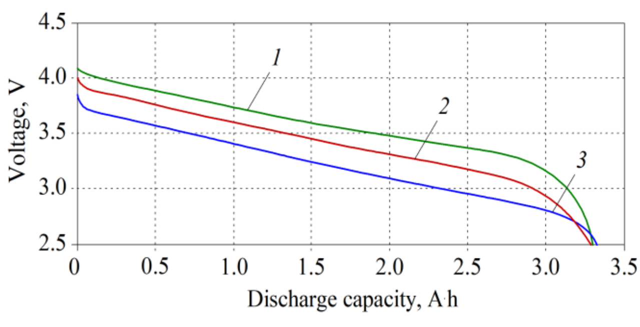


***Fig. 3.** The discharge characteristics of the NCR-18650b battery at load current: 1 – 0.65 A; 2 – 3.2 A; 3 – 6.5 A*

3. Equivalent circuits and mathematical models of lithium-ion storages

There are various mathematical models and equivalent circuits describing the processes in lithium-ion batteries, such as an active-resistive battery model, a dynamic resistive-capacitor model, the first and second order Thevenin model and others [13, 14]. These models give different accuracy in the description of charge-discharge characteristics.

A model of a lithium-ion battery that takes into account the active resistance of the battery (active-resistive model of the battery) is shown in Fig. 4.

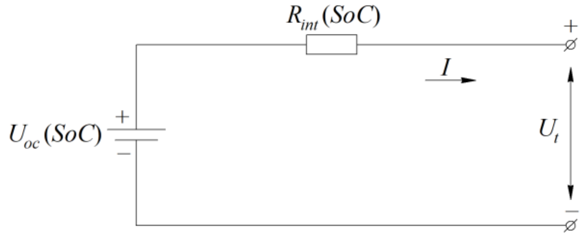


Fig. 4. *Active-resistive battery model taking into account power loss*

There are various options for implementing the model, in which the value of the internal resistance of the battery is either constant R_{int} = const, or depends on the percentage of battery charge $R_{int} = f(SoC)$, where SoC is the state of charge, the percentage of battery charge. Similarly, in various models, the internal EMF of the battery can be constant U_{oc} = const, or it can depend on the percentage of battery charge $U_{oc} = f(SoC)$.

In the case where the U_{oc} voltage and resistance depend on the percentage of charge, the output voltage at the battery terminals are expressed as [15]:

$$\begin{cases} U_t = U_{oc}(SoC) - R_{int}(SoC) \cdot I; \\ U_{oc}(SoC) = U_0 - k \cdot SoC; \\ R_{int}(SoC) = R_{int} - k_r \cdot SoC, \end{cases} \tag{1}$$

where I is the battery current; U_0 is the open circuit voltage when the battery is fully charged; k, k_r are the empirically derived coefficients.

Of the drawbacks, the model does not reduce throughput when the load increases, so it is not suitable for dynamic systems or transition states [16].

Based on the dynamic characteristics and operating principles of the battery, an equivalent circuit model was developed using resistors, capacitors as voltage sources (resistive-capacitor model) (Fig. 5).

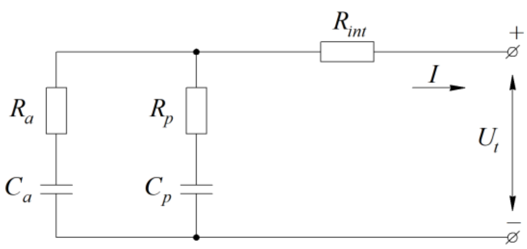


Fig. 5. *Resistive-capacitor battery model*

This model includes the capacitor C_a, which represents the accumulated capacitance, the series resistance R_a, which represents the polarization effect, the capacitor C_p, and the current-dependent resistance R_p, which simulates the effects of polarization and power dissipation on the internal resistance R_{int}. The C_p value is very small, while the C_a value usually takes on very large values. The SoC value is represented by the change in voltage across the capacitors C_a and C_p.

The disadvantage of this model is that the description of the discharge process has sufficiently large errors in the full discharge zone. The SoC zone of 20…80 % is described quite accurately by this model.

Equations that describe battery operation:

$$\begin{cases} U_t = U_{Cp} - I_{Cp} \cdot R_p - I \cdot R_{int}; \\ U_t = U_{Ca} - I_{Ca} \cdot R_a - I \cdot R_{int}; \\ I = I_{Cp} + I_{Ca}; \\ E_C = C \cdot U^2 / 2; \\ I_C = C \cdot dU / dt. \end{cases} \tag{2}$$

The first-order Thevenin model describes charge-discharge characteristics as an active-resistive model with an additional RC circuit, which are connected in parallel. RC parameters depend on SoC, current and temperature. The first-order Thevenin model is shown in Fig. 6.

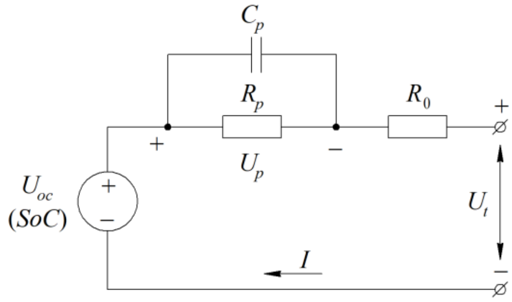


Fig. 6. *Equivalent scheme of the first-order Tevenin model*

The equations that describe the processes of charge-discharge of the battery are expressed as:

$$\begin{cases} U_t = U_{oc} - U_p - I \cdot R_0; \\ U_p = \dfrac{I}{C_P} - \dfrac{U_p}{C_p \cdot R_p}; \\ U_{oc} = K_0 + K_1 \cdot \ln SoC + K_2 \cdot \ln(1 - SoC), \end{cases} \tag{3}$$

where R_0 is the ohmic resistance; R_p is the polarization resistance; C_p is the polarization capacity, which is used to describe the transient during the charge-discharge of the battery.

Resistor R_0 provides the internal resistance of an element affected by SoC, temperature, and aging.

For a more accurate description of charge-discharge processes, the second-order Tevenin model is used (Fig. 7).

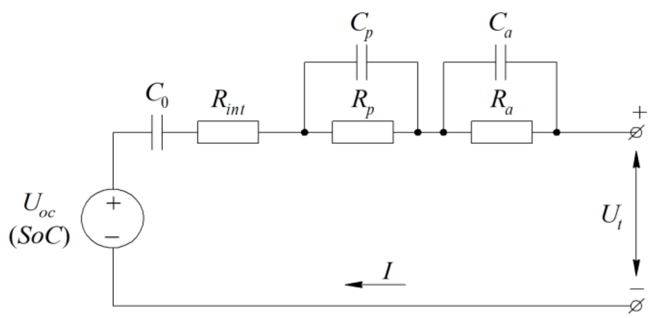


Fig. 7. *Equivalent scheme of the second-order Tevenin model*

The second-order Thevenin model has one additional RC component compared to the first-order model. With the optional RC component, a second-order model can achieve greater accuracy in terms of describing the transient behavior of the cell, but at the same time, the processing power increases.

The first RC circuit has a low time constant for describing short-term transient effects. These transient effects are associated with electrochemical and concentration polarization effects, including charge transfer, diffusion, and other factors.

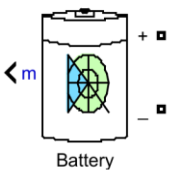
The equations that describe the operation of the second-order Tevenin model are expressed as:

$$\begin{cases} U_t = U_{oc} - I \cdot R_{int} - U_{C_1} - U_{C_2}; \\ \dot{U}_{C_1} = -\frac{1}{C_1 \cdot R_1} \cdot U_{C_1} + \frac{1}{C_1} \cdot I; \\ \dot{U}_{C_2} = -\frac{1}{C_2 \cdot R_2} \cdot U_{C_2} + \frac{1}{C_2} \cdot I; \\ U_{oc} = K_0 + K_1 \cdot \ln SoC + K_2 \cdot \ln(1 - SoC). \end{cases} \quad (4)$$

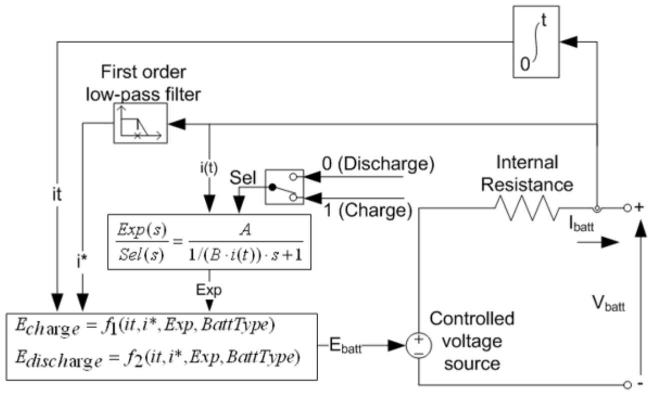
Thus, the second-order Thevenin model is more accurate and at the same time quite simple. In accordance with the requirement for model accuracy, the number of *RC* components added to the model can be increased even to infinity. However, as indicated above, the complexity of the model increases with the number of *RC* components. A model is always selected based on a compromise between accuracy and computational complexity [17].

4. Mathematical model of lithium-ion battery in the Matlab program

In Matlab / Simulink / SimPowerSystems, there is a library component of the lithium-ion battery (Fig. 8, a). The block diagram of the battery block is shown in Fig. 8, b.



a



b

Fig. 8. *Component of the lithium-ion battery:*
a – the appearance; b – the internal structure of the subsystem

The subsystem is the equivalent circuit of a simple linear battery model in which the internal resistance is not variable and does not depend on *SoC*.

In Fig. 8, b the following notation is inserted: E_{batt} is the nonlinear voltage; *Exp* (*s*) is the dynamic exponential zone; *Sel* (*s*) is the battery mode (*Sel* (*s*) = 0 during discharge, *Sel* (*s*) = 1 during charge).

The charge and discharge characteristics of the lithium-ion battery, which is given in Matlab, is described are expressed as:

$$f_1(it, i^*, i) = E_0 - K \cdot Q / (Q - it) \cdot i^* - K \cdot Q / (Q - it) \cdot it + A \cdot \exp(-B \cdot it); \quad (5)$$

$$f_2(it, i^*, i) = E_0 - K \cdot Q / (it + 0{,}1 \cdot Q) \cdot i^* - K \cdot Q / (Q - it) \cdot it + A \cdot \exp(-B \cdot it), \quad (6)$$

where *it* is the amount of charge consumed, A·h; *i** is the low-frequency component of the current, A; *i* is the battery current, A; E_0 is the constant voltage, V; *K* is the polarization constant, V/A·h, it is also the polarization resistance, Ohm; *Q* is the maximum battery charge, A·h; *A* is the exponential voltage, V; *B* is the capacity in the exponential zone, A·h^{-1}.

5. Simulation research of lithium-ion battery in the Matlab program

The main specifications of the battery type NCR-18650b, stated in the datasheet are shown in Table 1 [10].

Table 1

The main specifications of the battery type NCR-18650b

Characteristic		Value
Nominal capacity, A·h		3.2
Nominal voltage, V		3.6
Full charge time, h		4
Weight, g		48.5
Temperature, ℃	charge	0…+45
	discharge	–20…+60
Energy density, W/kg		243

Setting discharge parameters in the battery block in Matlab / Simulink is shown in Fig. 9.

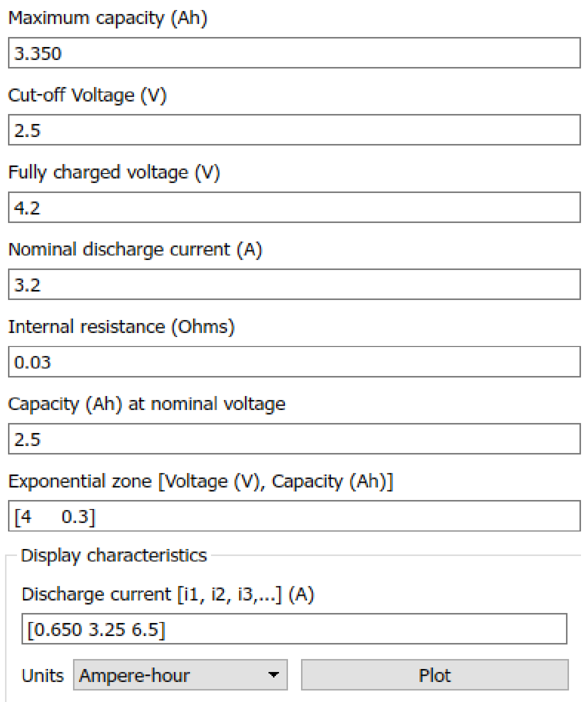


Fig. 9. *Setting discharge parameters in the battery block*

The characteristics of the battery discharge at different load currents obtained in Matlab / Simulink are shown in Fig. 10.

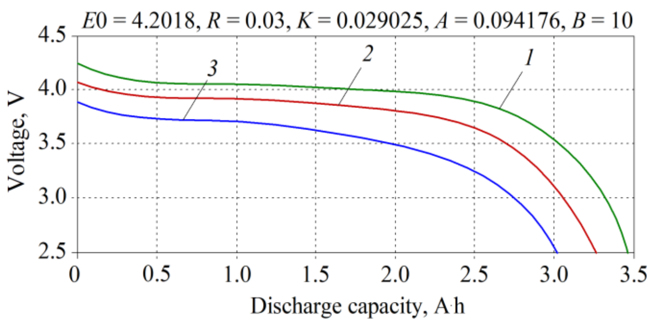


Fig. 10. *Setting discharge parameters in the battery block:*
1 – 0.65 A; 2 – 3.2 A; 3 – 6.5 A

Errors of the discharge characteristics of the lithium-ion battery NCR18650-b of the mathematical models considered and the built-in Matlab model against the data presented in datasheet are shown in Table 2.

Table 2

Errors of discharge characteristics of mathematical models of lithium-ion battery NCR-18650b

Capacity testing	The relative error is given								
	At discharge current 0.65 A			At discharge current 3.2 A			At discharge current 6.5 A		
Depth of discharge range, %	0…5	5…85	85…100	0…5	5…85	85…100	0…5	5…85	85…100
Deviation of battery Matlab-model data against Datasheet	4.5	13.3	11.1	9.4	18.2	5.9	17	23	30
Active-resistive model	2.2	4.3	16.1	3.1	5.8	18.0	4.2	6.4	18.6
Resistive-capacitor model	7.7	1.7	63.6	7.1	2.1	58.1	6.8	3.2	54.2
Model Tevenin	2.1	0.8	21.4	2.0	1.1	20.1	2.0	4.3	19.7

6. Results and discussion

Analysis of the data in Table 2 showed that the most accurate is the Tevenin model in the range of 0…85 %. That is, in the range where the battery is almost completely discharged, the existing mathematical models quite accurately describe the value of the voltage on the battery. In the range of discharge 85…100 % more accurately describes the battery voltage active-resistive model.

The high error of modeling of the discharge characteristics in the built-in Matlab model of the Battery block is caused, first of all, by a non-ideal mathematical model that describes an equivalent circuit of a linear model in which the internal resistance is a constant and the value of the internal EMF depends on the magnitude of the discharge.

Therefore, to improve the accuracy of process modeling in lithium-ion batteries, it is recommended to use other substitution circuits that more accurately describe the actual behavior of the battery, such as the resistive-capacitor model, the Tevenin model, etc.

7. Conclusion

The article presents equivalent circuits of lithium-ion batteries and a mathematical description of charge-discharge processes.

In addition, the built-in library component of the lithium-ion battery was investigated.

An analysis of mathematical models of equivalent circuits of different types of lithium-ion batteries was carried out, which showed that the most accurate are the resistive-capacitor model and the Tevenin model.

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