Optimization of pneumatic vane motor based on mathematical modeling and computer simulation

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Abstract: Pneumatic vane motors are important actuators in industry, their optimization is difficult without the best mathematic model and computer simulation. In this article we present a mathematical model for pneumatic vane type motors, construction of a simulation program associated with constructive engine parameters, experimental confirmation, and pneumatic motor performance optimization based on this model. The article begins with the description of the working principle of the vane type pneumatic motors, the geometric parameters that characterize it. The set of mathematical model equations consists in: the equations of geometry, the equation that describe the rotating moment, and equations expressing the mass flow into the motor. For the construction of the simulation program we have used the programming language of the G type LabView. Matching the results obtained from the simulation with the measured ones experimentally indicate that the built mathematical model is accurate and can be used to optimize pneumatic motors.

KEYWORDS: PNEUMATICS, PNEUMATIC VANE MOTOR, AUTOMATIZATION, FLUID-POWER, LABVIEW, COMPUTER SIMULATION.

1. Introduction

The principle of the vane motor is that a rotor with a number of vanes is enclosed in a rotor cylinder. The motor is supplied with compressed air through one connection and air escapes from the other connection.

To give reliable starting, the air pressures press the vanes against the rotor cylinder. The air pressure always bears at right angles against a surface. This means that the torque of the motor is a result of the vane surfaces and the air pressure.

To study the performance of vane motor is difficult without the best mathematic model and computer simulation. In this article we present a mathematical model for pneumatic vane type motors and the way to optimize them.

2. Preconditions

To build a mathematic model, assuming a polytrophic process and an ideal gas.

2.1 Mathematic Model

To construct the mathematical model of the pneumatic vane motor we will refer to the set of equations consisting of: equations describing geometry, equations expressing the rotational momentum of the rotor, and equations describing the mass flow of fluid passing through the motor.

Equations describing geometry

\[ \gamma = \frac{2\pi}{z} \]  

(2.1.1)

Where \( z \) is the number of vanes which is usually between 3 and 8.

The eccentricity \( \varepsilon \) is the difference between the inner radius of the cylinder and the radius of the rotor:

\[ \varepsilon \varepsilon = R_S - R_R \]  

(2.1.2)

the rotor angle \( \beta \) is a function of the stator angle \( \alpha \).

\[ \beta = \alpha - \arcsin \left( \frac{\varepsilon R_S \sin(\alpha)}{R_S} \right) \]  

(2.1.3)

Geometric volume of chambers between vanes from point d to point c. See fig.2.1.2 given by:

\[ V_{\alpha} = \frac{L}{2} \left( R_S^2 \cdot \beta - R_R^2 \cdot \alpha - \varepsilon \cdot R_S \cdot \sin(\beta) \right) \]  

(2.1.1)

Geometric volume \( V_w \) of work chamber, volume between two vanes is given by:

\[ V_w = \begin{cases} V_{\alpha} - V_{\alpha-i\gamma} & \text{for} \quad 0 < \alpha \leq \gamma \\ V_{2\pi} - V_{\alpha+i\gamma} & \text{for} \quad i\gamma < \alpha \leq 2\pi \end{cases} \]  

(2.1.2)

The filling volume \( V_{\text{fill}} \) and the expanded volume \( V_{\text{exp}} \) if the geometric data is available, see Fig. 2.1.1. The ratio between the expanded and filling volume determines to what extent the internal energy of the air is used and is called the expansion ratio \( \varepsilon \).

\[ \varepsilon = \frac{V_{\text{exp}}}{V_{\text{fill}}} \]  

(2.1.3)

If a high expansion ratio is used, the engine power increases, but it may be that the air temperature in the supply equipment drops so much that it results in freezing of the water in the air by blocking the engine.

The nominal displacement volume is given by:

\[ V_{\text{disp,nom}} = V_{\text{fill}} \cdot z \]  

(2.1.4)

Ideal engine torque

To construct the mathematical model of the rotational moment, we studied the thermodynamic processes. They include the equations for calculating the work, whose derivatives with
respect to time will give us the model of engine power and torque.

Figure 2.1.1. shows the volume of one compartment as a function of the rotation angle. Fig. 2.1.4. the corresponding PV diagram (assuming clockwise rotation of the rotor). State a is the beginning of the rotation when the vane opens the compartment and air fills the dead volume. In state b the compartment is completely filled with air of pressure p1. While the rotor turns to c, the volume of the compartment increases. This process is assumed to be polytropic and the air pressure can be calculated by:

\[ p_{1-e} = p_1 \left( \frac{V_{exp}}{V_{fill}} \right)^n = p_1 e^n \]  

(2.1.5)

Where \( n \) polytropic index, \( 1 \leq n \leq \kappa \), \( \kappa = 1.4 \) for air.

Typically a value of \( n = 1.3 \) is used (Daser 1969; Sbahi 1992). When the vane opens, the air discharges and the pressure falls to the surrounding pressure \( p_e \). This is the pressure at state d after the vane has closed the compartment and a reduction of the compartment volume begins. The air is compressed until at e the vane opens and releases most of the air to the second working port. Some air remains, state f. The compression from d to e is often neglected.

The work from the whole process is the sum of the expansion work, the displacement work and the compression work. Assuming a polytropic process and an ideal gas, the work is given by:

\[ W = W_{exp} + W_{disp} + W_{compre} \]  

(2.1.6)

The power \( P \) of the motor can be calculated by differentiating the work \( W \) with respect to time. For the simulation model the torque \( T \) at the motor shaft is needed which is given by:

\[ T = \frac{p}{\omega} = \frac{1}{\omega} \frac{dW}{dt} \]  

(2.1.10)

The working radius of the shovel referred to in figure 2.1.5 can be calculated from the relation:

\[ X_\alpha = ex \cdot \cos \phi + \sqrt{B^2 - ex^2 \cdot \sin^2 \phi} \]  

(2.1.11)

The area \( A_v \) between the two shovels shown in light purple in Figure 2.1.6. is calculated by the following equation:

\[ A_v = \frac{1}{2} \int_{\phi_1}^{\phi_2} X_\alpha (\phi) d\phi \]  

(2.1.12)
Fig. 2.1.7. Vane surface where rotational force is created

Referring to figure 2.1.7, the rotational moment will be described by the relation:

\[ M = (p_a - p_b) \cdot (\frac{X_a^2}{2} - r^2) \frac{L}{2} \]  

(2.1.13)

Mass flow

Before the air can enter a compartment, it has to flow through long and narrow ducts whose resistance cannot easily be calculated analytically. There is also considerable leakage between the ports and through the bearing at the front side of the rotor.

Fig. 2.1.8. Mass flow in the ideal motor

Referring to Figure 2.1.8. we will express the mass flow through the equation:

\[ \dot{m}_1 = -\dot{m}_{from\_1} + \dot{m}_{to\_vol\_1} \]  

(2.1.14)

And through the equation of the ideal gas state:

\[ p_1 = \frac{m_1 \cdot R \cdot T_0}{V_1} \]  

(2.1.15)

The mass flow rate \( \dot{m}_{to\_vol\_1} \) depends on the pressure at port 1. The mass flow through the motor can be calculated from the compartment volume and the pressure \( p_1 \). Assuming clockwise rotation, \( \omega > 0 \), the mass flow rate from inlet 1 of the stator to inlet of \( \dot{m}_{from\_1} \) is given by:

\[ \dot{m}_{from\_1} = \frac{p_1 \cdot \omega \cdot V_{disp\_nom}}{2\pi R \cdot T_0} \]  

(2.1.16)

For the mass flow rate \( \dot{m}_{to\_2} \) it follows accordingly:

\[ \dot{m}_{to\_2} = \frac{p_2 \cdot \omega \cdot V_{disp\_nom}}{2\pi R \cdot T_0} \]  

(2.1.17)

The mass flow \( \dot{m}_{to\_e} \) can be calculated from the equation:

\[ \dot{m}_{to\_2} + \dot{m}_{to\_e} = \dot{m}_{from\_1} \]  

(2.1.18)

The mathematical model of the ideal engine does not take into account some important phenomena, such as the leakage flows between vanes and stator, system inertia and mechanical friction. Therefore, additional flow paths with nozzles, inertia and bearing friction have to be added to this ideal motor, see Fig. 2.1.9 for nozzles. To find parameter values for the nozzles and the friction model, a numerical estimation scheme can be used (Beater 2004).

Fig. 2.1.9. Schematic of the main paths for leakage mass flow inside the vane motor

Fig. 2.1.10. Model that takes into account internal fluid leakage

\[ \dot{m}_{leakage} = 0.1 \cdot mm \cdot L \cdot \sqrt{2 \cdot \frac{k}{k-1} \cdot p_1 \cdot \rho_1 \cdot \left( \frac{\bar{E}_1}{\bar{E}_{1_0}} \right)^{\frac{k}{k-1}} \left( \frac{\bar{p}_1}{\bar{p}_{1_0}} \right)^{\frac{k-1}{k}}} \]  

(2.2.6)

2.2. Computer Simulation

Using the mathematical model described above and the LabVIEW G type programming language we have built simulation programs for the pneumatic vane motor.
3. Optimisation

The performance characteristic of the motor is shown in curves as below fig.2, from which torque, power, and air consumption can be read off as a function of speed. Power is zero when the motor is stationary and also when running at free speed (100%) with no load. Maximum power (100%) is normally developed when the motor is driving a load at approximately half the free speed (50%).

Where:
- \( P = \) Power
- \( M = \) Torque
- \( Q = \) Air consumption
- \( N = \) speed

Torque at free speed is zero, but increases as soon as a load is applied, rising linearly until the motor stalls. As the motor can then stop with the vanes in various positions, it is not possible to specify an exact torque. Air consumption is greatest at free speed, and decreases with decreasing speed, as shown in the above diagram.

The performance of an air motor is dependent on the inlet pressure. At a constant inlet pressure, air motors exhibit the characteristic linear output torque / speed relationship. However, by simply regulating the air supply, using the techniques of throttling or pressure regulation, the output of an air motor can easily be modified. The most economical operation of an air motor is reached by running close to nominal speed. By torque of \( M = 0 \), the maximum speed (idle speed) is reached. Shortly before standstill (\( n - 0 \)), the air motor reaches its maximum torque \( (M_{max} = 2 \times M_0) \). At nominal speed (\( n_n \)), for example in the middle of the speed range, air motor reaches its maximum power output \( (P_{max}) \).

**Energy Efficiency**

A pneumatic motor achieves its maximum power when it is operating as close as possible to its rated speed (50% of the rated idle speed). The energy balance is best in this area, because the compressed air is used efficiently.

**Optimization of speed and torque**

The speed and torque can also be regulated by installing a pressure regulator in the inlet pipe. This means that the motor is constantly supplied with air at lower pressure, which means that when the motor is braked, it develops a lower torque on the output shaft.

**Speed regulation, air flow reduction**

Every size reduction or restriction on the air line, whether of the supply hose itself or fittings, before the air motor affects the amount of the supplied air. By throttling you reduce the speed of the motor and simultaneously, the required torque. That means that you reduce the motor performance. The most common way to reduce the speed of a motor is to install a flow control valve in the air outlet, you can set the speed without loss of the torque. When the motor is used in applications where it must reverse and it is necessary to restrict the speed in both directions, flow control valves with by-pass should be used in both directions. If the inlet air is restricted, the air supply is restricted and the free speed of the motor falls, but there is full pressure on the vanes at low speeds. This means that we get full torque from the motor at low speeds despite the low air flow. Since the torque curve becomes "steeper", this also means that we get a lower torque at any given speed than would be developed at full air flow. The benefit of throttling the inlet is that air consumption is...
reduced, whereas throttling exhaust air maintains a slightly higher starting torque.

**Fig.3.3.** Speed regulation, air flow reduction

**Reducing motor leakages and air consumption**

Inside the motor in the starting moment is the air pressure that pushes the vanes to the surface of the stator to create the necessary seal. So in the starting moment the leakages and air consumption are too big versus next moments, because the first moment distance between vanes and stator is too big. If the vanes are adapted with special spring see the figure 3.4 below. The seal is in the necessary level at the starting moment.

**Fig.3.4.** Motor leakages and air consumption reduction way

**Optimization of torque**

Are to way to optimize the torque of the motor, increase the pressure, and increasing the radius of the stator and rotor. Using the program build in LabView we have do the simulation for stator diameter increased 10%, 20% and 30%.

**Fig.3.5.** Three diameters of stator considered in simulation

**Fig.3.6.** The torque of the motor for 3 stator diameters
Blue 10%, red 20%, orange 30% more bigger

**Fig.3.7.** Change of working vanes length in 360° rotation

**Angle**

**4. Conclusion**

Using simulation software for the design and optimization of pneumatic actuators is the best and most economical way. Implementation of simulation programs requires a good and complete mathematical model as well as a programming language that can solve complex hydrostatic and mechanical equations in real time. The mathematical model presented above as well as the built-in simulation software can be used for optimization and precision (PID) command of the pneumatic vane motors.

**5. References**