Study the work of specialized car service as queue theory

Assist. Prof. Grozev D. PhD. 1, Assist. Prof. Milchev M. PhD. 2 M.Sc. Georgiev I. PhD. 2
Faculty of Transport –University of Ruse “Angel Kanchev”, Bulgaria 1 dgrozev@uni-ruse.bg
Faculty of Natural sciences and education –University of Ruse “Angel Kanchev”, Bulgaria 2

Abstract: Study the refusal of requests received in a specialized automotive service workshop in the city of Rousse was investigated. The present work analyses the average monthly requests from customers of the workshop and the number of the actual repairs is determined. The work in the service have been seen as a mass service system with a non-stationary mid-month incoming stream with queries. Under non-stationary conditions, the basic values of the system parameters were calculated and Mat Lab application was created. The proposed approach can serve as a methodology for analysing and optimizing the activity of other universal automotive service.

KEYWORDS: QUEUE THEORY, REFUSALS OF REQUESTS, MODELING, SPECIALIZED AUTOMOTIVE SERVICE, CORRECTION, OPERATING MODE, ORGANIZATION OF WORK

1. Introduction

Various studies have shown the importance of tires for driving a vehicle safely and economically [3]. This is especially important in bad weather conditions combined with wet and slippery roads [5]. In the Republic of Bulgaria, such conditions occur mainly during the autumn-winter period of the year. To ensure road safety, the Bulgarian Road Traffic Act (Article 139, Paragraph 1, Item 4) states that during the period (November 15-March 1) motor vehicles, trailers and semi-trailers must be fitted with tires designed for winter conditions complying with the requirements of paragraph 4, or tires with a tread depth of at least 4 mm. [12]. From what has been said so far, there is no clear definition in the Bulgarian legislation of what kind of tires should be used during the winter months of the year. The decision on what kind of tires the vehicles will be equipped with is left to the fleet owners and managers. It is the responsibility of the same persons to decide when to change the tires. Therefore, in the tire repair shops can be expected a peak in the performed activities around the date of entry into force of Art. 139, para. 1, item 4 of the Road Traffic Act.

This report examines the specialized service of ARGO Ltd. in city of Rousse. The service was opened in 1997 as additional part of the company’s business [13]. In the surveyed service, for a period of one year, more than 2000 cars enter the service and repair of the wheels, and more than 1100 are returned (about 52%) due to their inability to be serviced. The high percentage of returned cars raises the question of whether it is possible to reduce returned vehicles. Exploring this possibility is implemented as the work of the service is viewed as a system of mass service. The main activities carried out in this specialized service are: fixing flat tires, balancing of automobile wheels, replacement of worn tires with new or appropriate for the season.

2. Exposition

In order to describe the operation mode of the car service, considered as a mass service, it is necessary to know the characteristics of the incoming flow of cars considered as a stochastic process, the service intensity, the maximum length of the tail and the number of service units stochastic process, the service intensity, the maximum length of the tail and the number of service units.

For the inbound flow of freight we can make the following assumptions:

- ordinary flow - The probability of two or more cars occurring for an elementary time interval is infinitely small compared to the probability of occurrence of only one car. The normality feature means that the cars come as single, not in group of two, three and so on at the same time. [6,9,14,15];

- flow without consequences - the number of cars arriving in the system for time interval Δt does not depend on how many vehicles have already arrived. ie. does not depend on the history of the studied phenomenon (the flow without action afterwards (Poisson flow).

- stationarity/non-stationarity of the flow [8,11,14] - for sufficiently long periods of time - 1 month, 6 months, 1 year, etc. it is possible to assume the steady-state of the incoming stream, that is to say, with certain conventions, the probability of occurrence of a certain number of cars in a given, sufficiently long interval depends only on the length of that interval. Generally, in arbitrary periods, the λ stream is non-stationary λ = λ (t). This non-stationarity is clearly distinguishable over a period of one business year (about 300 working days).

Research service has three working places with six workers who work normal working hours.

An investigation of the flow of cars for the period from October 2018 to September 2019 (Table 1, Figure 1) was made. For the period under review, 2105 vehicles have passed. There are clearly two major peaks, in orders, in the months of October-November and April-May.

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>October 2018</td>
<td>352</td>
</tr>
<tr>
<td>November 2018</td>
<td>361</td>
</tr>
<tr>
<td>December 2019</td>
<td>120</td>
</tr>
<tr>
<td>January 2019</td>
<td>86</td>
</tr>
<tr>
<td>February 2019</td>
<td>72</td>
</tr>
<tr>
<td>March 2019</td>
<td>61</td>
</tr>
<tr>
<td>April 2019</td>
<td>348</td>
</tr>
<tr>
<td>May 2019</td>
<td>353</td>
</tr>
<tr>
<td>June 2019</td>
<td>133</td>
</tr>
<tr>
<td>July 2019</td>
<td>87</td>
</tr>
<tr>
<td>August 2019</td>
<td>69</td>
</tr>
<tr>
<td>September 2019</td>
<td>63</td>
</tr>
</tbody>
</table>

Fig. 1 Number of cars in service
a car in the busiest months is about 40 minutes and in the nonbusiest months it is about 15–20 minutes. This is explained by the fact that with fewer clients, some of the installers who are not busy servicing the channel (the channel is free) help the busy channels.

**Tab. 2 Average time to service one car**

<table>
<thead>
<tr>
<th>Month</th>
<th>Average time to service one car, [min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>October 2018</td>
<td>40</td>
</tr>
<tr>
<td>November 2018</td>
<td>40</td>
</tr>
<tr>
<td>December 2019</td>
<td>30</td>
</tr>
<tr>
<td>January 2019</td>
<td>25</td>
</tr>
<tr>
<td>February 2019</td>
<td>20</td>
</tr>
<tr>
<td>March 2019</td>
<td>15</td>
</tr>
<tr>
<td>April 2019</td>
<td>40</td>
</tr>
<tr>
<td>May 2019</td>
<td>40</td>
</tr>
<tr>
<td>June 2019</td>
<td>30</td>
</tr>
<tr>
<td>July 2019</td>
<td>25</td>
</tr>
<tr>
<td>August 2019</td>
<td>20</td>
</tr>
<tr>
<td>September 2019</td>
<td>15</td>
</tr>
</tbody>
</table>

**Fig. 2 Average time to service one car**

The average service intensity \( \mu \) per month is calculated as the number of vehicles that can be serviced by all channels for a given month. Regarding the number of service channels (servers) in the service, it is 3. Two workers are required to ensure continuous operation for 8 hours per channel. Again, according to the manager, given the nature of the service, with more than \( m = 1 \) waiting clients in the queue, in most cases the customer refuses to wait. To test the performance of the system, it is necessary to find the probability in the system that has the \( k \) number of vehicles at the moment \( t \) with \( n \) servers running, ie. [2,7].

\[
P_k(t) =? , k = 0, n + m, t \in [0, T],
\]

where for one period \( T \) is taken one full working month. The beginning of the first working month (initial state) \( t = 0 \) coincides with the astronomical beginning of the year, the end of the last working month is \( t = 11 \), which is the end of the astronomical year.

For a model such as queue model system (QMS), the following can be summarized as follows: a queuing flow of density \( \lambda(t) \) enters a queuing system with \( n \) serving channels. The query service time is a random variable with an exponential distribution and a parameter \( \mu \) - in part constant. A car arriving at the moment when all channels are occupied, enters the waiting line and “patiently” waits to be serviced, unless there is more than \( m = 1 \) car in the service queue, ie. the system is with limited number of cars in the queue. The main indicator will be the intensity of returned requests, as well as during which time of the year these peaks are highest, as well as the average number of unprocessed requests in these peaks.

From what has been said up to now it can be said that the system is of type (M / M / s) in non-stationary mode. The following system of Kolmogorov (Erlang-Kolmogorov) differential equations is valid for describing a system of this type. [1,7,15]:

\[
dP_m(t) = -\lambda P_m(t) + \mu P_m(t)
\]

\[
dP_k(t) = \lambda P_{k-1}(t) - (\lambda + k\mu)P_k(t) + \mu(k + 1)P_{k+1}(t)
\]

In the general case, at large maximum queue length when all servers are busy. In some cases (with endless waiting) the system of differential equations is open and for the numerical solution it is necessary to take the additional algebraic condition \( \sum_{i=0}^{\infty} P_i(t) = 1 \) for normality.

The input stream \( \lambda(t) \), in general, is different every day, and some seasonality is highlighted. The following table 1 provides statistics on requests received and service intensity by month for the period from 10.2018 to 09.2019. To model \( \lambda(t) \), it is appropriate to choose a relatively elementary function which has periodicity. This is appropriate given the seasonal variations. The least squares method (LSM) was used to approximate the averages values for the period 10.2018 -09.2019. The model should be as elementary as possible but reflect the most characteristic behavior of the real flow. The following trigonometric row, nonlinear to the coefficients sought, is selected as the model:

\[
\lambda(t) = a_0 + a_1 \cos(\omega t) + b_1 \sin(\omega t) + a_2 \cos(2\omega t) + b_2 \sin(2\omega t)
\]

(3)

Coefficients \( a_0, a_1, b_1, a_2, b_2, \omega \) are calculated according to the LSM:

\[
a_0 = 175.7846, a_1 = -122.5684, b_1 = -104.5706, a_2 = 39.6417, b_2 = 70.3940, \omega = 1.0508.
\]

(4)

The coefficient of determination is \( R^2 = 0.9966 \) (statistically significant). The coefficients \( a_0, a_1, b_1, a_2, \omega \), are also statistically significant.

**Tab. 3 Intensity of service in the surveyed service by months**

<table>
<thead>
<tr>
<th>Months of the year</th>
<th>Service intensity for month ( \mu ) (25 working days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.2018</td>
<td>300</td>
</tr>
<tr>
<td>11.2018</td>
<td>300</td>
</tr>
<tr>
<td>12.2018</td>
<td>400</td>
</tr>
<tr>
<td>01.2019</td>
<td>480</td>
</tr>
<tr>
<td>02.2019</td>
<td>600</td>
</tr>
<tr>
<td>03.2019</td>
<td>800</td>
</tr>
<tr>
<td>04.2019</td>
<td>300</td>
</tr>
<tr>
<td>05.2019</td>
<td>300</td>
</tr>
<tr>
<td>06.2019</td>
<td>400</td>
</tr>
<tr>
<td>07.2019</td>
<td>576</td>
</tr>
<tr>
<td>08.2019</td>
<td>132</td>
</tr>
<tr>
<td>09.2019</td>
<td>120</td>
</tr>
</tbody>
</table>

In the general case, at large maximum queue length, the system has a large dimension. The computational features related to system (2) after the eventual introduction of an algebraic equation are as follows:

- large dimension system (generally);
The system is of the "stiff system" type (rigid system of equations). This can be summarized as: a system of rigid, differential-algebraic equations and in some cases of large dimension system. Special numerical methods have been developed to overcome these difficulties. A Matlab program for solving system (2) was made using the built-in "ode15s" solver implementing Gere's method. When entering λ(t), μ, n, m, the application returns a numerical solution to \( P_k(t) \). The accuracy of the default for the solver is \( 10^{-6} \) up to \( 10^{-8} \) for an absolute mistake and by \( 10^{-3} \) up to \( 10^{-5} \) for relative error [4,10].

For the service intensity \( μ(t) \), it is also appropriate to select a periodic function. Considering the data in Table 1, a more complex trigonometric row is selected for the model relative to (3):

\[
\mu(t) = a_0 + a_1 \cos(\omega t) + b_1 \sin(\omega t) + a_2 \cos(2\omega t) + b_2 \sin(2\omega t) + a_3 \cos(3\omega t) + b_3 \sin(3\omega t) + \cdots
\]  

(5)

The reason for choosing model (5) is in the better approximation. The coefficients \( a_0, a_1, b_1, a_2, b_2, w \) are calculated by LMS in equation (5) are:

\[
a_0 = 390.6947, a_1 = 71.0263, b_1 = 159.3549, a_2 = 86.5555, b_2 = 100.3834, a_3 = -146.9048, b_3 = -16.4436, w = 0.5746
\]  

(6)

Figure 3 shows the graph of \( λ(t) \) with coefficients calculated by the LMS.

The coefficient of determination is \( R^2 = 0.8963 \) (statistically significant).

Figure 4 shows the graph of \( μ(t) \) with coefficients calculated by the LMS.

The initial state of the system \( P_k(t_0) \) is unknown. It is known that these types of processes are stable and after a long time enter into regular mode of operation. Therefore, an arbitrary initial state can be taken. The integration of the system needs to be done not for a single period of time, but for a sufficiently large number of periods. Thus the probability functions \( P_k(t) \) hey start to tend to their regular values. After several integrations with different end times, it is found that only after 5-6 periods the functions \( P_k(t) \) enter regular mode (for two adjacent periods, they remain the same). The accuracy is also increased here as the integration is done for 20 periods, with the difference between all \( P_k(t) \) in the last and the penultimate period being less than \( 10^{-8} \) for each t.

The following graphs show the results of the solution of system (2) at the following values:

- \( λ(t) \) calculated respectively by the coefficients of (4) and \( μ(t) \) by coefficients of (6),
- \( n = 3 \) (3 running servers),
- \( m = 1 \) (up to 1 place in the queue).

Figure 5 shows the probabilities of having exactly \( k \in [0; 4] \) vehicles in the system. It is noteworthy that the most likely values for small or zero vehicles in workshop are around the end of February, the beginning of March, and also in September.

\[
\text{Fig. 5 Graph of the probability of having exactly } k \text{ vehicles in the system}
\]

It is also essential to know the density of rejected requests in Fig. 6. They are given with:

\[
P_f(t) = λ(t)P_{n+m}(t)
\]

(7)

\[
\text{Fig. 6 Graph of density rejected requests}
\]

It is noteworthy that there are two large peaks of rejected requests density - one is the end of the aryl month, the beginning of May, the other is shortly after the beginning of October and November.

The volumes of rejected requests \( V_f \) calculated between arbitrary times \( t_1 \) and \( t_2 \) are given by:

\[
V_f = \int_{t_1}^{t_2} λ(t)P_{n+m}(t) \, dt = \int_{t_1}^{t_2} P_f(t) \, dt
\]

(8)

The average volume of rejected requests for the whole period is about 234,2671. The months of April, May, October and November are interesting, as the highest failure rates occur in them. The year is divided into three periods. The first period includes the months of April and May, the second - the months of October and November, and the third group includes all other months. After numerically solving the integral (8) for each group of months, the results are reported in Table 5.
3. Conclusion

The mode of operation of a specialized car service described is described as a queuing system. The characteristics of the incoming flow of vehicles, considered as stochastic process, the intensity of service, the maximum allowable length of the queue, and the number of service units are determined. There is a two peak load of the service load in the incoming car flow with requests, which are in the months of October-November (713) and April-May (701). During these same months there is an increase in the service time of each request (about 40 min.). The model of the system describing the work in the specialized service is also defined, namely the system of Kolmogorov differential equations (Erlang-Kolmogorov).

After solving the system, the differential equations (2) are calculated the probabilities of the system to be in its many states. On the basis of these probabilities, key characteristics are calculated. One of them is the density of failures.

Table 5 shows that almost 87% of the returned requests are for a period of 4 months - from April, May, October and November.

### Table 5 Results of the calculations performed

<table>
<thead>
<tr>
<th>Months</th>
<th>Average number of rejects</th>
<th>% of all rejects</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,5</td>
<td>102.3368</td>
<td>43.6838</td>
</tr>
<tr>
<td>10,11</td>
<td>98.9257</td>
<td>42.2277</td>
</tr>
<tr>
<td>1,2,3,6,7,8,9,12</td>
<td>33.0046</td>
<td>14.0884</td>
</tr>
</tbody>
</table>

After solving the system, the differential equations (2) are calculated the probabilities of the system to be in its many states. On the basis of these probabilities, key characteristics are calculated. One of them is the density of failures.

The flow of returned (unaccounted for) cars was established for the study period, which amounts to 234 units. The discrepancy between the theoretical average number and the actual number of failures for the months of April, May, October and November diverges due to the accumulated errors in the approximation of the incoming flow λ and the speed of service μ, as well as factors not accounted for in the model used.

Table 5 shows that almost 87% of the returned requests are for a period of 4 months - from April, May, October and November.

3. Conclusion

The mode of operation of a specialized car service described is described as a queuing system. The characteristics of the incoming flow of vehicles, considered as stochastic process, the intensity of service, the maximum allowable length of the queue, and the number of service units are determined. There is a two peak load of the service load in the incoming car flow with requests, which are in the months of October-November (713) and April-May (701). During these same months there is an increase in the service time of each request (about 40 min.). The model of the system describing the work in the specialized service is also defined, namely the system of Kolmogorov differential equations (Erlang-Kolmogorov).

After solving the system, the differential equations (2) are calculated the probabilities of the system to be in its many states. On the basis of these probabilities, key characteristics are calculated. One of them is the density of failures.

The flow of returned (unaccounted for) cars was established for the study period, which amounts to 234 units. The discrepancy between the theoretical average number and the actual number of failures for the months of April, May, October and November diverges due to the accumulated errors in the approximation of the incoming flow λ and the speed of service μ, as well as factors not accounted for in the model used.

Different performance indicators can be calculated based on the model used to describe the work of the specialized service. Further exploration of a service center with that model can also provide a better option to handle the large number of returned (rejected) requests.

The study was supported by a contract of University of Ruse “Angel Kanchev”, № BG05M2OP001-2.009-0011-C01, “Support for the development of human resources for research and innovation at the University of Ruse “Angel Kanchev””. The project is funded with support from the Operational Program “Science and Education for Smart Growth 2014-2020” financed by the European Social Fund of the European Union.

### References

6. M. Zukerman, Introduction to queuing theory and stochastic teletraffic models, EE Department, City University of Hong Kong (2016)
10. V. Pavlov, I Georgiev, Optimization methods with MATLAB. (Avangard print, 2016)
14. Представяне на фирма Арго ООД в бизнес каталоз (аудио и визуална информация), (последно посетен 14.11.2019г.)
15. Т. Савън. „Елементи теории массового обслуживания и ее приложения“ (Книжный дом "Либроком", Москва, 2010)