Methods of solving non-standard inequalities

Методы решения нестандартных неравенств

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Abstract:*This article discusses the problems of teaching non-standard tasks at school. Currently, one of the main tasks of secondary schools is to identify creative thinking abilities of students and solve problems of their development. The report was identified as the main tool for developing students ' mental and thinking abilities. Problem solving is the most productive form of learning mathematics, and this process should be a necessary component of all extracurricular activities conducted in mathematics. Problems at the mathematical Olympiad are non-standard problems. Olympic tasks will usually help you learn a lot, find independent non-standard methods of solving problems. The high school curriculum provides for the use of the definition of inequality, proving using known inequalities, using the relationship between the arithmetic mean and geometric mean of positive numbers, and proving them using methods other than reverse-trip methods.*

KEYWORDS: *NON- STANDARD TASK, OLYMPIC TASK, HIGHLY PRODUCTIVE FORM.*

1. Introduction

Currently, one of the main tasks of secondary schools is to identify creative thinking abilities of students and solve problems of their development.

Non-standard tasks develop students ' ability to think, leads to thinking and increases motivation, interest in accounting.Non-standard tasks do not allow students to solve problems correctly. Therefore, non-standard tasks are the main means of developing students ' mathematical thinking.

The degree of mathematical development of students is obvious in their ability to solve problems. Report the education of each student mentally steady fixed assets. The solution of unusual, interesting problems occupies an important place in the mathematical creativity of children. First of all, we must remember that to learn to solve problems – to find its solution. D. poya wrote the book " mathematical knowledge "(M. 1976.13):» If you want to learn to swim, dive into the water without secrets, and find its solution to learn how to solve problems " [1].

Solving any difficult test required a lot of work, great strength and perseverance from the student. These qualities are enhanced when the child wakes up with a desire to take into account. Since interesting tasks set your mental energy in motion, it becomes easier to solve them. This is why the teacher should choose tasks that will be interesting and independently solved by students.

To develop the mathematical abilities of students and foster interest in mathematics, it is advisable to use humorous problems and mathematical puzzles. The student's confidence that they can output a report is one of the important factors for success. If the calculation is too difficult, the student loses goals, reduces the effectiveness of thinking, and undermines further learning. The teacher strives for the solution of his students, strive for its solution, believe in himself-these are the necessary prerequisites for success. In the process of solving each problem, you should distinguish four stages: 1) understand the conditions of the task; 2) make a plan; 3) implement the plan; 4) "Open your eyes back", i.e., study the solution found.

Creative understanding of the learned material of the student and the development of new ways of activity is due to the presence of the following three components of thinking: 1) a high level of formation of simple mental operations, such as analysis and synthesis, comparison, analogy, classification; 2) a high level of thinking activity, expressed in the presentation of multi-faceted solutions and unexpected ideas; 3) a high level of cohesion and purposefulness, expressed in ways of independent thinking.

2. Preconditions and means for resolving the problem

The formation of these qualities of thinking contributes to the development of the creative personality of the student, overcoming difficulties in the development of educational material. The essence of this is that the student uses theoretically based methods of education and activity, or independently finds new approaches to solving the problem.The task of the teacher is to form these components of thinking. And the key to it is the release of a creative report. Solving creative tasks of students is carried out through their knowledge, skills and abilities. In addition, the lesson plays a role of motivation in maintaining a high active mental activity, the desire of the student to their work. Therefore, we can say interesting tasks (tasks, tasks, puzzles, logical tasks) as a means of developing the mind, adapting the student to creative activity. They have the opportunity to improve creative activity, mentally train, rationalize as an auxiliary, additional way.

Although such materials are diverse, they have common properties.

1) the method of solving interesting problems is unknown. The achievement of their solution occurs as a "Brownian movement of thought", i.e. by the method of awakening, errors. Petropavlovsk hosted the regional competition " state language-my language!", dedicated to the 70th anniversary of Victory in the great Patriotic war.;

2) interesting tasks are the basis of the student's interest and activity in the subject.;

3) Interesting tasks are made up on the basis of knowledge of the laws of thinking.

So the systematic use of these types of tasks contributes to the development of these mental operations, the formation of mathematical representations in children. Solving interesting problems often occurs during a trial search. Skillful prediction of the exhibits ingenuity and resourcefulness in children. Resourcefulness is a special type of creativity that includes analysis, comparison, generalization, identification, understanding, and the ability to determine relationships, on the basis of which the counter comes to a single conclusion and groups the game. It is an indicator that they are able to utilize their knowledge. Resourcefulness and inflexibility, which can foresee the solution of interesting problems, is nothing else. The success of such mental activity can and should be developed in the learning process.

In any case, a thorough analysis is made to predict the solution of the problem: to distinguish the main properties of the problem, the spatial location and grouping of figures, their features, and similar features. However, the method of baiting and errors for solving interesting problems is not decimal reliable and versatile. The most effective method is to equip children with such important methods as analysis and synthesis, comparison, identification, and qualification of mental activity.

Among the problems that have a cognitive significance and play a leading role in the system of mathematical education are inequalities.

Special attention is not paid to the irrepressibility of textbooks for secondary schools: students can not form skills to prove mathematical failures. In order to draw attention to these shortcomings and methodological difficulties, the article is devoted to school teachers, students of the physics and mathematics faculty of pedagogical universities.

3. Methods for solving real inequality

The high school program uses evidence on the use of the definition of inequality, proving using known inequalities, using the relationship between the arithmetic environment and the geometric environment of positive numbers, and the method of reverse Hiking. Let's look at other ways.

3.1 Let's focus on the inequalities proved by the discriminant of a square triangle.

Many inequalities are proved by the symbol of a quadratic three-dimensional discriminant. The given quadratic triple must be in order to accept a positive value. $a>0, d<0$ .We need to prove the inequality below.

$a \succ 0, b \succ 0, c \succ 0$ by

$$a^2b^2+c^2b^2+a^2c^2 \geq abc(a+b+c)$$

$$a^2b^2+c^2b^2+a^2c^2-a^2bc-b^2ac-c^2ab=$$

$$=(b^2-bc+c^2)a^2-bc(b+c)a+c^2b^2 \geq 0$$

This inequality can be considered as a square three dimensional one:

$$D=b^2c^2(b+c)^2-4b^2c^2(b^2-bc+c^2)$$

$$=b^2c^2(b^2+2bc+c^2)-4b^4c^2+4b^3c^3-4b^2c^4$$

$$=b^4c^2+2b^3c^3+b^2c^4-4b^4c^2+$$

$$+4b^3c^3-4b^2c^4=6b^3c^3-3b^2c^4-3b^4c^2$$

$$=-3b^2c^2(b^2+c^2+2bc)=-3b^2c^2(b-c)\leq 0$$

a in any value of the number the specified inequality will be correct. we arrive at this conclusion by getting a square three-dimensional by b and c. x - in any value $kx^2-(5k+1)x+3(2k+1)$ does a quadratic triangle have a positive value?

$$k>0, D=(5k+1)^2-12k(2k+1)=$$

$$=25k^2+10k+1-24k^2-12k=$$

$$=k^2-2k+1=(k-1)^2 \geq 0$$

Therefore, in any value of x, the square cannot take a positive value of three squares.

Consider the inequalities that are proved using the "amplification" method» :

if $A>B, B\geq C$, then $A>C$ the properties of inequalities can be proved by relying on many things. $\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+...+\frac{1}{n^2} \prec 1 (n \in N, n>1)$ prove the inequality.

$$\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+...+\frac{1}{n^2} \prec \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+$$

$$+\frac{1}{3\cdot 4}+...+\frac{1}{(n-1)^n}=\frac{2-1}{1\cdot 2}+\frac{3-2}{2\cdot 3}+$$

$$+\frac{4-3}{3\cdot 4}+..+\frac{n-(n-1)}{(n-1)^n}=(1-\frac{1}{2})+$$

$$+(\frac{1}{2}-\frac{1}{3})+(\frac{1}{3}+\frac{1}{4})+...+(\frac{1}{n-1}-\frac{1}{n})$$

$$=1-\frac{1}{n} \prec 1$$

$$A=\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+...+\frac{1}{2n-1}+\frac{1}{2n} \prec \frac{3}{4}$$

prove the inequality.

$$A=\frac{1}{2n}\left[(\frac{1}{n}+\frac{1}{2n})+(\frac{1}{n+1}+\frac{1}{2n-1})+...+(\frac{1}{2n}+\frac{1}{n})\right]$$

$$=\frac{1}{2}(\frac{3n}{2n^2}+\frac{3n}{2n^2}+\frac{3n}{2n^2}+...+\frac{3n}{2n^2})$$

$$\prec \frac{1}{2}(\frac{3n}{2n^2}+\frac{3n}{2n^2}+\frac{3n}{2n^2}+...+\frac{3n}{2n^2})$$

$$=\frac{1}{2}(n+1)\frac{3}{2n} \prec \frac{3}{4}+\frac{3}{4n} \prec \frac{3}{4}$$

3.2 Method for reducing the number 1 in two parts of inequalities.

This method is used if there is more than 1 correct fraction on both sides of the sea. To do this, you need to convert two parts of the inequality yourself, compress, and then subtract two parts of them by 1. This way, the result can be compared.[2]

$\frac{2n-1}{2n} \prec \frac{2n}{2n+1}$ prove the inequality.

$$1-\frac{2n-1}{2n}=\frac{1}{2n}$$

$;1-\frac{2n}{2n+1}=\frac{1}{2n+1}$ яғни $\frac{1}{2n} \succ \frac{1}{2n+1}$

if $1-A \succ 1-B$, then $B \succ A$

Therefore $\frac{2n-1}{2n} \prec \frac{2n}{2n+1}$.

$\frac{33331}{33334} \succ \frac{44441}{44445}$ it is necessary to prove the inequality.

$$A=\frac{33334-3}{33334}=1-\frac{3}{33334}\bullet\frac{4}{4};$$

$$B=\frac{44445-4}{44445}=1-\frac{4}{44445}\bullet\frac{3}{3}$$

$1-A=\frac{12}{133336}; 1-B=\frac{12}{133335}$, $A>B$.

Give large numbers from any zero. The relations between these numbers are as follows: $a_1, a_2, ...a_n$.

$\frac{a_1+a_2+...+a_n}{n}=A_n$ - the arithmetic mean,

$\sqrt[n]{a_1 \bullet a_2 \bullet ... \bullet a_n}=\sigma_n$ - the geometric mean,

$\frac{\sqrt{a_1^2+a_2^2+...+a_n^2}}{\sqrt{n}}=Q_n$ - square mean,

$\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}}=H_n$ - it's called the harmonic mean.

This is between the average $H_n \leq \sigma_n \leq A_n \leq Q_n$ the ratio is performed. In most cases, when inequalities are proved, these relations are applied.

$a_1, a_2, ...a_n$ the arithmetic mean of positive numbers will be greater than or equal to their geometric center:

$$\frac{a_1+a_2+...+a_n}{n} \geq \sqrt[n]{a_1 \bullet a_2 \bullet ... \bullet a_n}$$

This inequality implies Cauchy's inequality. Proof of this inequality.

$\sqrt[n]{a_1 \bullet a_2 \bullet ... \bullet a_n} = \sigma_n$ numbers in a geometric tool $a_1 \geq a_2 \geq a_3 \geq ... \geq a_n$ to satisfy the inequality. $a_1 a_2 = (\sqrt{a_1 a_2})^2 = \sigma^2, a_1 a_2 a_3 = (\sqrt[3]{a_1 a_2 a_3})^3 =$ $= \sigma^3, ..., a_1 a_2 ... a_n = (\sqrt[n]{a_1 a_2 ... a_n})^n = \sigma^n$

Because $\frac{a_1}{\sigma} = 1, \frac{a_1 a_2}{\sigma^2} = 1, ..., \frac{a_1 a_2 ... a_n}{\sigma^n} = 1$

$\frac{\sigma}{a_1} = 1, \frac{\sigma^2}{a_1 a_2} = 1, ..., \frac{\sigma^n}{a_1 a_2 ... a_n} = 1$ will be. This is

$$\frac{a_1}{\sigma} \bullet \frac{\sigma}{a_1} + \frac{a_1 a_2}{\sigma^2} \bullet \frac{\sigma^2}{a_1 a_2} + ... + \frac{a_1 a_2 ... a_n}{\sigma^n} \bullet \frac{\sigma^n}{a_1 a_2 ... a_n} =$$

$$= n \leq \frac{a_1}{\sigma} \bullet 1 + \frac{a_1 a_2}{\sigma^2} \bullet \frac{\sigma}{a_1} + ...$$

$$+ \frac{a_1 a_2 ... a_n}{\sigma^n} \bullet \frac{\sigma^{n-1}}{a_1 a_2 a_3 ... a_{n-1}} =$$

$$\frac{a_1}{\sigma} + \frac{a_2}{\sigma} + \frac{a_3}{\sigma} + ... + \frac{a_n}{\sigma} = \frac{a_1 + a_2 + ... + a_n}{\sigma}$$

That is $\frac{a_1 + a_2 + ... + a_n}{n} \geq \sigma = \sqrt[n]{a_1 a_2 ... a_n}$ this was supposed to prove it.

Example. $n! \leq \left(\frac{n+1}{2}\right)^n, (n \geq 2)$ to prove the inequality.

$$\sqrt[n]{n!} = \sqrt[n]{1 \bullet 2 \bullet 3 ... n} \leq \frac{1+2+3+...+n}{n} = \frac{(n+1) \bullet n}{2n} = \frac{n+1}{2}$$

This is an inequality on both sides **n**-as for the degree $n! \leq \left(\frac{n+1}{2}\right)^n$ that was what had to be proved.

3.3 Chebyshev inequality.

a) If $a_1 \geq a_2 \geq a_3 \geq ... \geq a_n \geq 0$ and $b_1 \geq b_2 \geq b_3 \geq ... \geq b_n \geq 0$,

there $(a_1 b_1 + a_2 b_2 + ... + a_n b_n \geq (a_1 + a_2 + ... + a_n) \bullet (b_1 + b_2 + ... + b_n)$

Add the separation of the **n** inequality

$$(a_1 b_1 + a_2 b_2 + ... + a_n b_n \geq (a_1 b_1 + a_2 b_2 + ... + a_n b_n)$$

$$(a_1 b_1 + a_2 b_2 + ... + a_n b_n \geq (a_1 b_2 + a_2 b_3 + ... + a_n b_{n-1})$$

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$$(a_1 b_1 + a_2 b_2 + ... + a_n b_n \geq (a_1 b_n + a_2 b_1 + ... + a_n b_{n-1})$$

We know a idea Chebyshev's inequality are true.

б) If $a_1 \geq a_2 \geq a_3 \geq ... \geq a_n \geq 0$ and $b_1 \leq b_2 \leq b_3 \leq ... \leq b_n \leq 0$, there

$$n(a_1 b_1 + a_2 b_2 + ... + a_n b_n) \leq (a_1 + a_2 + ... + a_n) \bullet (b_1 + b_2 + ... + b_n)$$

the following inequality is satisfied.[3]

Example. If $a_1 \geq a_2 \geq a_3 \geq a_4 \geq 0$ and $b_1 \geq b_2 \geq b_3 \geq b_4 \geq 0$, there

$$4(a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4) \geq (a_1 + a_2 + a_3 + a_4) \bullet (b_1 + b_2 + b_3 + b_4)$$

to prove the inequality.

When the report conditions are met, there

$$a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 \geq a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$$

$$a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 \geq a_1 b_2 + a_2 b_3 + a_3 b_4 + a_4 b_1$$

$$a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 \geq a_1 b_3 + a_2 b_4 + a_3 b_1 + a_4 b_2$$

$$a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 \geq a_1 b_4 + a_2 b_1 + a_3 b_2 + a_4 b_3$$

If you perform a separate join of expressions to the left and right of these inequalities

$$4(a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4) \geq a_1 \bullet (b_1 + b_2 + b_3 + b_4) + a_2 (b_1 + b_2 + b_3 + b_4) + a_3 (b_1 + b_2 + b_3 + b_4) + a_4 (b_1 + b_2 + b_3 + b_4)$$

this was supposed to prove it.

4. Conclusions

In elective classes in mathematics, it is necessary to instill in the formation of students ' interest, to improve their work skills, to develop the knowledge obtained in their school curriculum, to determine their life needs.

The meaning of mathematics in a school course is its versatility, that the main objects of yagni are based on real life. Therefore, solving non-standard tasks outside the program develops the system of knowledge and thinking of students.

Problem solving is the most productive form of learning mathematics, and this process should be a necessary component of all extracurricular activities conducted in mathematics. Problems at the mathematical Olympiad are non-standard problems. Olympic tasks, as a rule, will help you learn a lot, find independent non-standard methods of solving problems.

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