Model of hydraulic turbine taking into account the impact of efficiency

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Abstract: Research, optimization and practical implementation of a Small Hydropower Plants as a source of clean electricity are one of the actual tasks in the current energetics, which is virtually impossible to solve without powerful computer support due to the strongly nonlinear nature of such systems. The article presents an overview of the most common simulation model schemes of Small Hydropower Plants, whereas explores the sub-models of its individual subsystems. The hydraulic turbine is considered as a core of the hydropower plant, whose efficiency calculation is analytically demanding and dependent on parameters that are often obtained only by theoretical estimation. In the article a fuzzy system was used to create its model based on measured operating data on the turbine flow rate and the height of its water column without the need to know the turbine parameters. Such a model is applicable in practice for the design of an energy-optimized hydro-turbine control as well as to the effective determination of the deterioration rate. The correctness of the results was verified by simulation measurements in the MATLAB program.

Keywords: HYDRAULIC TURBINE, SMALL HYDROPOWER PLANT, FUZZY SYSTEM, EFFICIENCY

1. Introduction

Hydroelectric power generation complies with ecological criteria aimed at environment protection. There is a lot of renewable energy accumulated in the surrounding water resources (lakes, rivers, artificial water reservoirs, etc.). This hydropower can first be converted into rotational mechanical power using hydraulic turbines, then, using synchronous and asynchronous power generators, it can be converted into electrical energy, and finally this whole energy conversion process can be controlled taking into account the requirements of the electrical energy end user.

In the energy conversion process, an essential role is played by the efficiency of the hydraulic turbine, both in terms of the overall efficiency of the conversion, and in terms of wear and possible maintenance or replacement.

2. Hydraulic Turbine Modeling

The model of a hydraulic turbine is a relatively complex non-linear dynamic system, some of the parameters of which are difficult to compute. However, correct and accurate modelling of the turbine is crucial in terms of accurate diagnosis and specification of the response of the entire hydropower plant to external influences, as well as in terms of correct tuning of its control.

Several approaches to hydraulic turbine modelling have been listed in the literature, differing in the manner they take into account the efficiency of the turbine or its impact on the final mechanical output of the turbine \( P_m \) [1-2].

2.1. Model Taking into Account Turbine Efficiency Through Turbine Flow Value \( Q_{al} \) at Zero Load

The development of a hydraulic turbine model is based mainly on theory and on the mathematical description given in expert publications [3] and [4]. The resulting block diagram of the hydraulic turbine structure is shown in Fig.1.

![Fig. 1 Hydraulic turbine model](image)

The penstock has been modelled on the assumption that the fluid is incompressible and that the penstock is sufficiently rigid, of \( L \) length and \( A \) diameter. The level loss due to the influence of the penstock has been disregarded here. We therefore start from the equation:

\[
\frac{d\bar{q}}{dt} = \left(\bar{h}_0 - \bar{h} - \bar{h}_1\right)g \frac{A}{L}
\]

where:

- \( \bar{q} \): standardized turbine flow
- \( A \): penstock cross-section area
- \( L \): penstock length
- \( g \): acceleration due to gravity
- \( \bar{h}_0 \): static height of level
- \( \bar{h} \): level at point of entry into turbine
- \( \bar{h}_1 \): level loss due to friction in the penstock

The above equation can be rewritten into the standardised form:

\[
\frac{dq}{dt} = \frac{(1-h-h_1)}{T_w} q_{base}
\]

where parameter \( h \) denotes the level in the turbine area and parameter \( h_1 \) accounts for the level losses that are being disregarded.

The equation also includes a previously unused time constant \( T_w \), referred to in the literature as the water time constant or the water start time. Its value can be obtained using the relationship (according to 1)

\[
T_w = \left(\frac{L}{A}\right) q_{base}^{-\beta}
\]

where parameter \( \beta \) is the turbine flow at maximum opening of the inlet valve \((G=1)\) and \( q_{base} \) is the value of the total available static height of level.

By introducing values \( h_{base} \) and \( q_{base} \) we are able to express, by means of the turbine characteristic, the value of water flow at \( G=1 \) as

\[
q = G\sqrt{h}, \quad \text{and thus} \quad q_{base} = 1\sqrt{h_{base}}
\]

The turbine flow \( q \) is therefore the function of the position of servo valve \( G \) (inlet valve position) and the level \( h \).

The mechanical output of the turbine would in an ideal case be characterised as turbine flow multiplied by the level.

2.2 Hydraulic Turbine Model Taking into Account Penstock Losses

In real operation, a turbine does not operate at 100% efficiency and hence the requirement to consider this fact in the process of hydraulic turbine modelling. There are several ways of
incorporating the impact of turbine efficiency into a simulation model [5].

One of the simplest ways is to use the so-called zero-load turbine flow. This flow, referred to as $q_{nl}$, is a constant that characterizes the value of the turbine's constant power losses, and in the formula this constant is subtracted from the actual turbine flow. It is then this final flow value that is multiplied by the level $h$ as in

$$ P_m = A_t h(q - q_{nl}) - DG\Delta\omega $$  \hspace{1cm} (5)

Another factor present during the operation of the hydro power plant is the turbine damping effect, which is a function of the differences in speeds, servo position and water supply valve position. The $\Delta\omega$ variable represents the change in mechanical shaft speed, $D$ is the penstock diameter (also as the turbine damping coefficient), and $G$ is the servo position as a percentage.

The $A_t$ parameter is considered to be the hydro turbine amplification. Its quantification differs in various scientific publications, e.g. according to [3] it can be calculated as follows:

$$ A_t = \frac{\text{Turbine Output [MW]}}{\left(\text{Generator Output [MVA]} \cdot h_r(q_r - q_{nl})\right)} $$ \hspace{1cm} (6)

where the quantities with the subscript $r$ can be considered as rated. This means that $q_r$ is the turbine flow at rated load, and $h_r$ is the level required for achieving the mentioned rated flow of $q_r$. These parameters are usually part of the technical documentation of the particular systems in a hydro power plant. The model for computing hydraulic turbine losses is illustrated in Fig. 2.

![Fig. 2 Model for computing hydraulic turbine losses](image)

### 2.3. Model Taking into Account Turbine Efficiency by Means of Its Efficiency Computation

This option of developing a simulation structure of a hydraulic turbine is based on the basic relationship describing the computation of its output mechanical power $P_m$

$$ P_m = \eta pqgh $$ \hspace{1cm} (7)

where $\eta$ represents the hydraulic turbine efficiency value; $\rho$ is the water density (value of 1000 kg/m$^3$ is used mostly); $q$ is the actual turbine flow; $g$ is acceleration due to gravity (9.81 m/s$^2$); and $h$ is the actual level of the hydro power plant. Standardised quantities were applied in the model. For the expression of the efficiency of the hydraulic turbine in this case we applied the computation quoted in 5. The efficiency computation presents a separate simulation block as it defines efficiency $\eta$ as a function of $\lambda$ and turbine flow $q$, i.e. $\eta=f(\lambda,q)$.

#### Turbine efficiency computation $\eta_\lambda(q)$

As already mentioned, there are several ways of expressing hydraulic turbine efficiency. In this case we present the computation quoted in [5], which requires knowledge of turbine parameters such as the turbine blade radius - $R_{\text{blades}}$ [m] and the size of the area swept by the blades - $A_{\text{blades}}$ [m$^2$]. Other data required for the computation includes the actual values of turbine flow $q$ and turbine mechanical angular velocity $\omega_{m}$

$$ \eta_\lambda(\lambda, q) = \left[\frac{1}{2}\left(\frac{90}{A_t} + q + 0.78 \exp\left(-\frac{50}{A_t}\right)\right)\right] (3.33q) $$ \hspace{1cm} (8)

where $\lambda_i$ is

$$ \lambda_i = \left[\frac{1}{\left(\lambda + 0.089\right)} - 0.0035\right]^{-1} $$ \hspace{1cm} (9)

and $\lambda$ is defined as

$$ \lambda = \frac{R_{\text{blades}} A_{\text{blades}} \omega_{m}}{q} $$ \hspace{1cm} (10)

In this case the input into the computation of hydraulic turbine efficiency would be the actual turbine flow $q$ and also the mechanical angular velocity of the turbine $\omega_{m}$. The mathematical description is represented in the block diagram in Fig. 3.

![Fig. 3 Hydraulic turbine efficiency computation model](image)

The block scheme of hydraulic turbine overall model taking into account losses and taking into account efficiency is shown in Fig. 4.

![Fig. 4 Overall model of hydraulic turbine a) taking into account losses b) taking into account efficiency](image)
3. Hydraulic turbine efficiency fuzzy model

As it follows from the previous sections, an exact determination of the efficiency of a particular turbine is rather complicated. For example, according to equation (10), this efficiency depends on the parameters of the blades of a particular turbine, which can change significantly throughout operation. Furthermore, this dependence is strongly non-linear (see equations (8-10) and is dependent on several only approximately determined parameters. For example, the efficiencies at various turbine flows and mechanical angular velocities for the blade radius \( R_{\text{blades}} = 1.9 \text{ m} \) and the area swept by the blades \( A_{\text{blades}} = 9 \text{ m}^2 \), are illustrated in Fig. 5.

![Fig. 5 Turbine efficiency for concrete blade conditions \( R_{\text{blades}} =1.9 \text{m} \) and \( A_{\text{blades}}=9\text{m}^2 \)](image)

When the circumference area of the blades changes (e.g. as a result of wear) to \( A_{\text{blades}} = 8 \text{ m}^2 \), this turbine "efficiency image" will change, as shown in Fig. 6. The maximum efficiency of the turbine in this case is approx. 77%, while in the previous case it was approx. 88%.

![Fig. 6 Turbine efficiency for concrete blade conditions \( R_{\text{blades}} =1.9 \text{m} \) and \( A_{\text{blades}}=8\text{m}^2 \)](image)

An image of turbine efficiency can also be generated by measuring flow \( q \), speed \( \omega_m \), and turbine efficiency in steady state operation, e.g. according to Tab. 1.

<table>
<thead>
<tr>
<th>Measured turbine efficiency ( h )</th>
<th>Flow ( q ) ( 0.2 )</th>
<th>( 0.4 )</th>
<th>( 0.6 )</th>
<th>( 0.8 )</th>
<th>( 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.4 )</td>
<td>12.77%</td>
<td>30.36%</td>
<td>36.23%</td>
<td>37.83%</td>
<td>37.97%</td>
</tr>
<tr>
<td>( 0.4 )</td>
<td>2.74%</td>
<td>26.40%</td>
<td>49.60%</td>
<td>64.41%</td>
<td>72.24%</td>
</tr>
<tr>
<td>( 0.6 )</td>
<td>0.35%</td>
<td>13.64%</td>
<td>40.90%</td>
<td>66.92%</td>
<td>87.22%</td>
</tr>
<tr>
<td>( 0.8 )</td>
<td>0.64%</td>
<td>5.86%</td>
<td>27.20%</td>
<td>56.26%</td>
<td>84.31%</td>
</tr>
<tr>
<td>( 1 )</td>
<td>0.00%</td>
<td>2.11%</td>
<td>16.07%</td>
<td>42.03%</td>
<td>72.49%</td>
</tr>
</tbody>
</table>

The data from Table 1 can be used in searching for the fuzzy inference structure (FIS) of the modelled efficiency describing the measured relations between \( [w_m, q] \rightarrow h \).

The fuzzy model can be designed based on this database using standard cluster analysis tools with adaptive approaches resulting in improved quality of modelling and reduced development time. In terms of the adaptive approach, a hybrid arrangement based on a fuzzy inference tool in connection with a neural network was used [6]. From the large number of methods for adaptive fuzzy system development [7] we chose the adaptive neuro-fuzzy inference system (ANFIS). The basic characteristics of cluster analysis are the reduction of the fuzzy rules number and good initial rule parameters setting. We used subtractive clustering [8] as a fast and robust data analysis method, with the following parameters: Range of influence = 0.5, Squash factor = 1.1, Accept ratio = 0.45, Reject ratio = 0.005.

As a result, we obtained a static Sugeno type fuzzy system with twelve rules, as shown in Fig. 7.

![Fig. 7 Efficiency fuzzy model – Sugeno type with 12 rules](image)

From the data in Table 1, using the Anfisedit tool of MATLAB, we compiled a static fuzzy system which approximates turbine efficiency, without the need to know its specific parameters, as illustrated in Figure 8.

![Fig. 8 Fuzzy image of turbine efficiency based on measured data](image)

4. Discussion and conclusion

Computer models of power systems make it possible to significantly simplify and speed up their design, as well as save costs in their implementation. The computation of hydro turbine efficiency is analytically demanding and dependent on parameters which are often obtained only by theoretical estimation. Knowledge of the "efficiency image" of a particular hydro turbine is essential for its optimal power control.

The paper describes a fuzzy model of a hydraulic turbine that takes into account its efficiency. The model can be obtained from measured data on turbine flow and head. It is shown that the model obtained in this way practically equals the analytical models obtained by analytical methods. The model is applicable in practice in the design of optimal hydro turbine power control as well as in the effective determination of its wear rate.
Acknowledgement
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5. References