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Matlab as a teaching, learning and motivating tool for engineering mathematics

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Abstract: The present paper describes the fact that revised and pedagogically justifiable implementation of CAS systems (e.g. Matlab system) is a great contribution to teaching higher mathematics. It brings practical applications to mathematics teaching and thus enhances motivation and students' interest in the discussed topic. The Computer Algebra System, popularly known as CAS, is a modern computing technology that provides great learning and teaching opportunities to both students as well as instructors. MATLAB is one of the most popular CAS programs that is widely used in mathematics classrooms especially at the university level. As more and more universities are adopting its use in education, it is necessary to study and analyze how the software could be used in teaching and learning of mathematics. It has been shown to be a technology that could be used for developing mathematical thinking, concepts and skills, and a source of motivating students for learning mathematics. It is also a very useful tool in demonstrating connections in various underlying mathematical concepts in the teaching and learning processes.

Keywords: LINEAR DIFFERENTIAL EQUATION, PROGRAM MATLAB, VISUALIZATION AND MOTIVATION OF THE MATHEMATICAL TERMS

1. Introduction

In this paper we show the examples of Matlab application to the solution of linear n -th order constant coefficient differential equations. The assigned differential equation is solved with the help of simple commands of the system Matlab. The examples can serve as a simple manual for students – only the basic knowledge of Matlab and the knowledge of mathematics on the level of the first form of bachelor study is sufficient.

During seminars we teach our students how to understand differential equations and be able to solve them. From our experience we know that if the students are shown a concrete application of differential equations in the field, they study their interest in the defined problem is higher. However, these application examples are sometimes computationally demanding and hence time consuming. But if these examples are solved with the help of the system Matlab they can be used in the teaching process to enhance the motivation of students.

We do not want to replace a traditional mathematical teaching by using a Matlab system. However, we would like to motivate students to be able to solve also more complicated exercises from practice. Matlab system is a tool not only for elimination of routine and mechanical calculations but predominantly for enhancement of students' motivation during the process of this thematic unit learning.

2. Computer algebra systems (CAS) and their place in teaching mathematics

Computer algebra systems (including Matlab) are strong computational and visual tools with wide practical application. Using of CAS systems in mathematics teaching at the university has both supporters and opponents.

Bernhard Kutzler from ACDCA (Austrian Centre for Didactics of Computer Algebra) speaks about it in a very pertinent way: "There exist thousands of ways – good and bad – how to use CAS in teaching. Inappropriate approaches usually come from technical enthusiasts – teachers who use CAS just because they exist. But CAS should never manage mathematics teaching – mathematics teaching (didactic intention) should manage their application" [4].

Helmut Heugl, the director of an Austrian project Derive and TI-89/92 adds: "If CAS application is not pedagogically justified then it is pedagogically justifiable not to use it" [5].

Systems CAS were created to make routine, long-winded and complicated calculations instead of people, to make their work easier and provide desired results. But their usage only for this purpose is from a didactic viewpoint (when we emphasize the

development of students' mathematical education) insufficient and inappropriate.

We cannot be satisfied with the fact that our students are able to obtain the result by application of suitable mathematical software. Our ambition is to lead students to mathematical terms understanding and subsequent application of the received knowledge in practice.

Therefore, we consider using CAS with comprehension to be the most appropriate one. In the first phase of the educational process we want our students to manage the discussed topic in a theoretical way and to learn the principle of the solution of specific exercises. This will help them to obtain a critical detached view necessary for the correct choice of a solution method and interpretation and evaluation of the obtained results. Only theoretically educated students are offered to use mathematical software suitable for the solution of more complicated application exercises.

We do not want to replace a traditional mathematical teaching by using a Matlab system. However, we would like to motivate students to be able to solve also more complicated exercises from practice.

Matlab possesses a powerful library of functions serving for the solution of ordinary differential equations. What is more, Matlab has also extensive graphical possibilities which can be used in a teaching process. Ordinary differential equations, which often occur in engineering practice, are linear n th order constant coefficient differential equations.

3. Basic terms

Differential equations have a remarkable ability to predict the world around us. They are used in a wide variety of disciplines, from biology, economics, physics, chemistry and engineering. They can describe exponential growth and decay, the population growth of species or the change in investment return over time. A differential equation is one which is written in the form $dy/dx = \dots$. Some of these can be solved (to get $y = \dots$) simply by integrating, others require much more complex mathematics.

Linear n th order constant coefficient differential equations have the form

$$y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \dots + a_{n-1} y' + a_n y = f(x) \quad (1)$$

where $a_1, a_2, \dots, a_n$ are real numbers, $f(x)$ is a continuous function on the interval $\langle a, b \rangle$ and y is an unknown function. This equation can be put in the form

$$L(y) = f(x) \quad (2)$$

where $L(y) = y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \dots + a_{n-1} y' + a_n y$.

Theorem. Let $Y(x)$ is a solution of a linear differential equation with the right side $L(y) = f(x)$. Then each solution of this linear differential equation is in the form

$$y = z + Y \quad (3)$$

where $z = c_1 y_1 + c_2 y_2 + \dots + c_n y_n$ is a general solution of the corresponding linear differential equation without a right side

$L(y) = 0$ and $c_1, c_2, \dots, c_n$ are arbitrary real numbers.

Generally, the solution Y can be found by the method of variation of constants.

Theorem. If $y_1, y_2, \dots, y_n$ is a fundamental system of solutions of the differential equation $L(y) = 0$ than the solutions of a differential equation with the right side $L(y) = f(x)$ is the form

$$Y = \sum_{i=1}^n y_i(x) \int \frac{W_i(x)}{W(x)} dx,$$

where $W(x)$ is the Wronsky determinant and $W_i(x)$ is a determinant which results from Wronsky determinant by replacing i -th column by the column of elements $0, 0, \dots, f(x)$.

4. Examples of solved exercises

With the help of Matlab we can solve a differential equation symbolically by using the command `dsolve`. Its application is presented in the following exercises:

Exercise 1. Let's find a general solution of the differential equation

$$y'' - 7y' + 10y = -(6x + 7) \cdot e^{2x} \quad (1)$$

Let's find a solution of this differential equation that satisfies initial conditions

$$y(0) = 0, \quad y'(0) = 1. \quad (2)$$

Solution. We solve a linear second-order constant coefficient and special right side differential equation. For its solution we use a command `dsolve` where we put the differential equation in apostrophes. A letter `D` indicates a derivative of the function. Generally notation `Dny` means n -th derivative of the function $y = y(x)$ of the variable x .

So `Dy` means the first derivative of the function and `D2y` means the second derivative of the function $y = y(x)$ of the variable x .

```
>>y=dsolve('D2y-7*Dy+10*y=-(6*x+7)*exp(2*x)','x')
```

```
y =
exp(2*x)*C2+exp(5*x)*C1+x*(3+x)*exp(2*x)
```

The second way how to set a differential equation is its defining with the variable called `dr`. This way is useful if we solve more differential equations.

```
>> dr='D2y-7*Dy+10*y=-(6*x+7)*exp(2*x)';
```

```
>> y=dsolve(dr,'x')
```

```
y =
```

```
exp(2*x)*C2+exp(5*x)*C1+x*(3+x)*exp(2*x)
```

```
>>
```

Thus a general solution of the differential equation (1) is

$$y = x(x + 3)e^{2x} + c_1 e^{5x} + c_2 e^{2x}$$

where are c_1, c_2 arbitrary real numbers.

Let's find the solution of the differential equation (1) which satisfies initial conditions (2). The differential equation (1) is defined by the variable called `dr`, initial conditions (2) will be

defined by the variable `zp`. We also use a command `pretty` with the help of which a symbolic expression of a solution in the form similar to a mathematical notation is noted.

```
>> dr='D2y-7*Dy+10*y=-(6*x+7)*exp(2*x)';
```

```
>> zp='y(0)=0,Dy(0)=1';
```

```
>> y=dsolve(dr,zp,'x')
```

```
y =
```

```
2/3*exp(2*x)-2/3*exp(5*x)+x*(3+x)*exp(2*x)
```

```
>> pretty(simple(y))
```

```
2/3 exp(2 x) - 2/3 exp(5 x) + x (3 + x) exp(2
x)
```

```
>>
```

Thus, the solution of the differential equation (1) satisfying initial conditions (2) is

$$y = x(x + 3)e^{2x} - \frac{2}{3}e^{5x} + \frac{2}{3}e^{2x}.$$

With the help of the following commands we draw a graph of the solution on the interval $(0,1)$.

```
>> x=linspace(0,1,20);
```

```
>> z=eval(vectorize(y));
```

```
>> plot(x,z)
```

```
>>
```

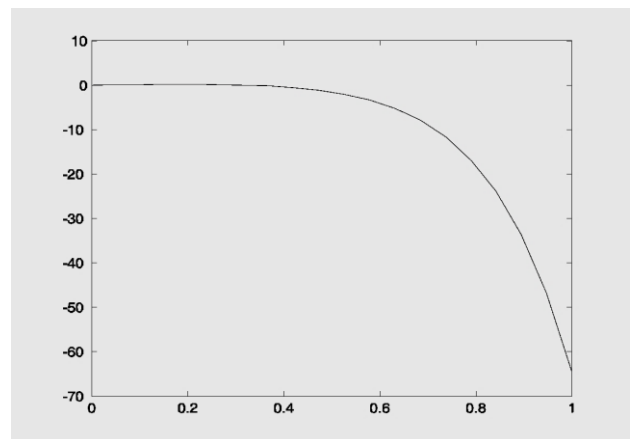


Fig. 1 Graph of the solution of (1) on the interval $(0,1)$.

Exercise 2. Let's find a general solution of the differential equation

$$y'' + y = \frac{1}{\sin^3 x} \quad (3)$$

Solution. We solve a linear second-order constant coefficient differential equation. we will use a newer method of calculation with the command `diff(y,x,n)` for the n -th derivative of the function $y = y(x)$ of the variable x .

```
>> syms y(x)
```

```
>> eqn = diff(y,x,2) == -y+1/(sin(x)^3);
```

```
>> de2c = dsolve(eqn)
```

```
de2c = cos(x)*cotg(x)+c1*cos(x)-
(cotg(x)^2*sin(x))/2-c2*sin(x)
```

```
>> pretty(de2c)
```

Thus, the general solution of the differential equation (3) is

$$y = c_1 \cos x - c_2 \sin x - \frac{\cot^2 x \cdot \sin x}{2} + \cos x \cdot \cot x$$

where are c_1, c_2 the arbitrary real numbers.

Exercise 3. For an electric circuit in Figure 2 it is true that

$$RCL \frac{d^2 i_2}{dt^2} + L \frac{di_2}{dt} + Ri_2 = E(t) \quad (4)$$

Let's find a general solution of this differential equation if $L = 1 \text{ mH}$, $R = 10 \Omega$, $C = 1 \mu\text{F}$ and $E(t) = 2 \sin(100\pi t)$.

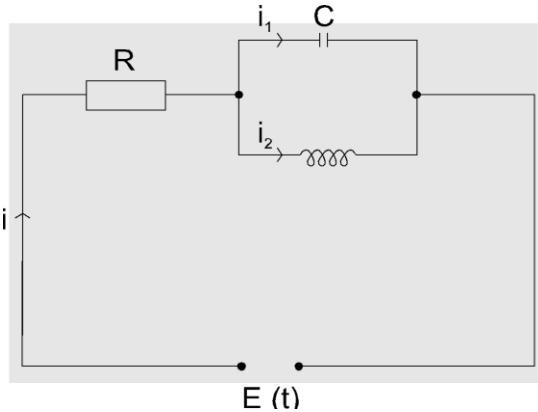


Fig. 2 Graph of an electric circuit.

Solution. We solve a second-order linear constant coefficient differential equation. From the example it is obvious that solving this type of differential equation is time-consuming. Let's define a differential equation for a variable called dr .

```
>> dr='10*D2i2+Di2+10*i2=2*sin(100*pi*t)';
>> i2=dsolve(dr, 't')
i2 =
exp(-1/20*t)*sin(1/20*399^(1/2)*t)*C2
+exp(1/20*t)*cos(1/20*399^(1/2)*t)*C1
+(sin(100*pi*t)-10000*pi^2*sin(100*pi*t)
-10*cos(100*pi*t)*pi)/(5-
99500*pi^2+500000000*pi^4)
>> pretty(i2)
```

Thus a general solution of the differential equation (4) is

$$i_2 = \frac{(1 - 10000\pi^2) \sin(100\pi t) - 10\pi \cos(100\pi t)}{5 - 99500\pi^2 + 500000000\pi^4} + c_1 e^{-\frac{1}{20}t} \cos \frac{\sqrt{399}}{20} t + c_2 e^{-\frac{1}{20}t} \sin \frac{\sqrt{399}}{20} t,$$

where are c_1, c_2 the arbitrary real numbers.

Finally, we present the example from real life for better motivation of our students.

Exercise 4. Time of a death. At midnight, a body with a temperature of 35.6°C was discovered in the hotel room. At 1:00, its temperature dropped to 34.8°C . Determine the time of death if the room temperature has been set to 22°C by the air conditioner and the living person's temperature is 36.8°C .

Solution. Let's choose the time $t = 0$ the time of finding the body. The time of death is an unknown quantity. We compile a differential equation

$$y' = k \cdot (22 - y) \quad (5)$$

Let's find a general solution of this differential equation with the initial condition

$$y(0) = 35.6.$$

We use the condition to calculate the constant k

$$y(1) = 34.8.$$

The Matlab program will give us these results

```
>> syms k y(t)
>> eqn=diff(y,t)==k*(22-y);
>> cond=y(0)==35,6;
>> cond = y(0) == 35
>> ode=dsolve(eqn,cond)
ode =
13*exp(-k*t) + 22
>> pretty(ode)
13 exp(-k t) + 22
```

Thus a general solution of the differential equation (5) is

$$y = 22 + 13e^{-kt}.$$

For constant k we receive $k = 0.0618$ and the form of the function that expresses the dependence of the temperature y of the found body on the time t is

$$y = 22 + 13e^{-0.0618t}.$$

We calculate the time of death after implantation $y = 36.8^\circ\text{C}$.

From the equation

$$36.8 = 22 + 13e^{-0.0618t}$$

we have $t = -1.26 \text{ hod}$. We estimate the time of death to be approximately 1 hour and 16 minutes.

5. Conclusion

MATLAB symbolic math solves engineering problems that can be represented as equations or sets of equations, including differential and integral equations. It is one of the most versatile tools in MATLAB. It can be used to replace many specialized MATLAB Toolboxes with a single Toolbox applicable across multiple engineering disciplines. The advantage is that skill developed in using a single MATLAB Toolbox can be applied across multiple engineering disciplines. The disadvantage of using the MATLAB Symbolic Math Toolbox stems from the requirement for a deeper and more fundamental understanding of the engineering principles underlying the problem solution than is required for using other MATLAB toolboxes.

In summary, the MATLAB Symbolic Math Toolbox creates a successful learning environment for assimilating the underlying thought processes supporting engineering design and analysis.

In several exercises of this paper the possibility of Matlab implementation in the linear differential equations teaching was presented. Matlab system is a tool not only for elimination of routine and mechanical calculations but predominantly for enhancement of students' motivation during the process of this thematic unit learning.

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Comparative analysis of pseudorandom sequences using modified Barker codes and M – sequences of the same length

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Abstract: The paper discusses the correlation properties of pseudo-random sequences (PRS) used to form noise-like signals in high-speed data transmission systems. The most frequently used pseudo-random sequences are considered: modified Barker codes, M-sequences having the same length. In the Matlab medium a comparative analysis of the correlation properties of the PRS is done. It is shown, that modified Barker codes have unsatisfactory correlation properties. It is proved, that the use of M - sequences allows to obtain signals with the required correlation properties for communication systems, including systems with code division of channels. The directions of further research are determined.

KEYWORDS: NOISE-LIKE SIGNAL, PSEUDO-RANDOM SEQUENCES, MODIFIED BARKER CODES, M-SEQUENCES, AUTOCORRELATION FUNCTION, CROSS-CORRELATION FUNCTION

1. Introduction

Currently, in radio systems (RS) is increasingly used a broadband (BB) signals or as they also called noise-like (NL) signals, which are based on pseudo-random sequences (PRS) [1-5]. Depending on the size of the alphabet and the method of construction, binary (size of the alphabet $p = 2$) and multi-value (size of the alphabet $p > 2$) PRS are distinguished.

Communication systems with NL signals are widely used for three reasons:

Firstly, the broadband signals generated by different PRSs can have the same center frequency, i.e. be transmitted in the same band.

The second reason why the use of NL signals is very profitable is its high resistance to the effects of both broadband and narrowband interference, which is very important in conditions of a tense electromagnetic environment in modern communication systems.

The third reason is the high energy secrecy of systems with NL signals and, as a result, the high confidentiality of the transmitted data. The essence of the above is that a broadband signal is not only difficult to decode - it is difficult to simply detect, i.e. identify the fact of the subscriber station.

In RS channels, as a rule, either a signal detection task or a signal discrimination task can be solved, which can be considered as a special case of the detection problem.

To solve these problems, the correlation technique should be applied in an optimal way. The correlation technique is the more effective, the more complex is the useful signal [7, 9].

The best signals are those in which the ratio of the main peak of the autocorrelation function (ACF) to the lateral is the largest.

However, most of the PRS, which found practical application in broadband systems, is not free from a number of disadvantages.

These and other factors prompt not only to search for ways to optimize ensembles of sequences with acceptable correlation properties, but also to explore new classes of SRP with the required correlation properties.

2. Preconditions and means for resolving the problem

2.1. Theoretical part

It is known that the rate of information transmission over the communication channel with additive Gaussian white noise is determined by the Shannon-Hartley theorem:

$$C = W_s \log_2 \left(1 + \frac{P_s}{P_n} \right), \quad (1.1)$$

where W_s is the bandwidth of the communication channel (Hz),

P_s is the average signal power (W),

P_n is the average noise power (interference) (W).

From "Eq. (1)" it follows that the same bandwidth C (bit / s) of the communication channel can be provided either by using a wide bandwidth W_s (Hz) with a small signal-to-noise ratio P_s/P_n , or - a narrow band W_s with higher signal-to-

noise ratio P_s/P_n . Consequently, between the bandwidth of the communication channel W_s and the signal-to-interference ratio P_s/P_n interchange is possible.

An important parameter of a system using noise-like signals is processing gain. The processing gain (PG) shows the degree of improvement of the signal-to-noise ratio when converting the noise-like signal received by the receiver into the desired information signal. According to the classical definition, PG is equal to:

$$PG = 10 \log (C_{ch}/C_{inf}), \quad (1.2)$$

where C_{ch} is the repetition rate of the pseudo-random sequence chips, chip/s.,

C_{inf} - information transfer rate, bits/s.

By this definition, a system that has a data transfer rate of 1 Mbps. and a chip repetition rate of 13 Mchip/s. (this means that each bit of information is encoded by a 13-bit Barker pseudo-random code sequence), will have a PG of 11.14 dB.

This result means that the performance of the information transmission system will be maintained with the same BER if the useful signal at the input decreases by 11.14 dB. If you increase the length of the code sequence to 64 chips per bit, then at the same information transfer rate of 1 Mbps, the processing gain will be $10 \lg (64) = 18.0$ dB.

In general, the following requirements are imposed on pseudo-random sequences (PSP) used to expand the spectrum of signals [1, 3, 4]:

- a large volume of the ensemble of sequences formed using a single algorithm;
- "good" auto- and cross-correlation properties of the sequences included in the ensemble;
- balance of structure;
- the maximum period for a given length of the shift register that forms the sequence;
- unpredictability of the sequence structure over its undistorted segment of limited length.

The autocorrelation function (ACF) of discrete signals is calculated by the formula:

$$R_u(n) = \sum_{j=-\infty}^{\infty} u_j u_{j-n}, \quad (1.3)$$

where n is an integer, positive, negative or zero.

The study of ACF plays an important role in the selection of code sequences from the point of view of the least probability of establishing false synchronization.

The cross-correlation function (CCF), on the other hand, is of great importance for systems with code division of subscribers, such as CDMA. The cross-correlation function between two discrete signals is calculated using the same "Eq. (1.3)" formula

$$R_{uv}(n) = \sum_{j=-\infty}^{\infty} u_j v_{j-n}, \quad (1.4)$$

Correlation properties of code sequences used in BB systems depend on the type of code sequence, its length L , the frequency of

its characters and its character structure [3,9 -13].

Let us perform a comparative analysis of the correlation characteristics of the SRP, which are used to obtain noise-like signals. The characteristics of the PSP are the functions of autocorrelation (ACF) and cross-correlation (CCF), which are divided into periodic and aperiodic. ACF and CCF are calculated by counting the difference in the number of matching and non-matching bits of a PRS when one of them shifts. Periodic ACF (PACF) and CCF (PCCF) are calculated during the cyclic shift of the PRS, and aperiodic ACF (AACF) and CCF (ACCF) are calculated during the usual shift of the PRS (parts of the PRS of various lengths are compared - from maximum to minimum).

We study PRSs that have a length $L \approx 63$.

Barker codes. Barker signals (codes) can be attributed to discrete signals with the best ACF structure. The Barker signal code sequence consists of N characters ± 1 and is characterized by a normalized ACF of the form:

$$R_u(n) = \begin{cases} 1, & \text{для } n = 0, \\ 0, & \text{для } n = 2l + 1, \\ \pm 1/N, & \text{для } n = 2l, \end{cases}$$

(1.5)

where $l = 0, 1, \dots (N-1)/2$.

The sign in the last line depends on the value of N .

These signals have a unique property: regardless of the number of positions N in the code combination, the ACF values calculated by formula (1.3) do not exceed unity for all $n \neq 0$. At the same time, the energy of these signals, i.e. the value of $R_u(0)$ is numerically equal to N . Only seven Barker signals are known, the most complex of them consists of 13 characters and has a ratio of the height of the main ACF peak to the side peak equal to 13. This property allows reliable detection of such a signal at signal-to-noise ratios $P_s/P_n < 1$.

In particular, these codes are used in systems with the spread spectrum of the IEEE 802.11 standard.

However, in data transmission channels in which significant interference is present, even Barker signals do not provide the required reliability of their detection.

In [8, 11], to increase the signal-to-noise ratio and increase the probability of correct detection, it is proposed to use modified Barker signals — Barker – Volynskaya signals.

The method of obtaining such signals is based on the combination of Barker signals. The Barker sequence is taken as the “maternal” sequence, and then each symbol of the “maternal” sequence is replaced by a direct or inverse Barker “daughter” sequence, depending on whether zero or one in the “maternal” sequence.

In [8], it is stated that out of all possible pairwise combinations of “mother” and “daughter” sequences of Barker codes with the highest ratio of the central peak of ACF to the side lobes, only 10 sequences satisfy: 3×4.1 ; 3×3 ; 3×7 ; 3×11 ; 7×3 ; 7×7 ; 7×11 ; 11×3 ; 11×7 ; 11×11 . However, combinations of pairwise combinations of Barker codes 13×3 ; 3×13 ; 5×13 ; 13×5 were not considered.

This paper investigates pseudo-random sequences that have a length of approximately 63. PRS, which has a length $L \approx 63$, can be obtained using the proposed, modified Barker codes. They can have the following structures:

$$(5 \times 13), (13 \times 5), ((5 \times 3) \times 4), ((3 \times 5) \times 4), ((2 \times 11) \times 3), ((11 \times 2) \times 3).$$

The last four combinations are a combination of the “maternal” sequence in the form of Barker – Volynskaya codes, in which each character of this sequence is replaced by a direct or inverse “daughter” sequence of the Barker code. And this, in turn, depends on what value a symbol (zero or one) takes in the “maternal” sequence of the Barker-Volynskaya code. For example, for a 3×4.1 sequence, the “maternal” sequence is - 1 1 0, and the “daughter” sequence is - 1 0 1 1, then the new sequence has the form:

$$\begin{array}{ccc} \overbrace{1011} & \overbrace{1011} & \overbrace{0100} \\ \text{«1»} & \text{«1»} & \text{«0»} \end{array}$$

M - sequences. It should be noted the following - Barker codes are mainly used for high-speed BB systems designed to transmit information, but not for code separation of subscribers. Barker code sequences longer than 13 characters are unknown. Modified Barker codes allow increasing the length of the PRS, but at the expense of some deterioration in the correlation properties. Therefore, to obtain greater noise immunity, as well as for code division of channels, sequences of greater length are used, a significant part of which form M-sequences.

The sequences of maximum length or M - sequences are the sequences formed by shift registers with linear feedback and having a period $L = 2^n - 1$, where n is the length of the register. The most important feature of M - sequences is that their periodic autocorrelation function is optimal in the class of possible autocorrelation functions of binary sequences of length $L = 2^n - 1$.

Optimality here is understood in the sense of the minimum of the maximum value of the lateral outliers of the autocorrelation function. It is the good autocorrelation properties of M-sequences and the simplicity of their formation that led to their wide application in communication systems [1,3-5].

The law of the formation of linear PRS is determined by the linear recurrence relation:

$$a_j = \sum_{i=1}^n C_i a_{j-i},$$

(1.6)

where the addition is made by mod 2.

The coefficients C_i take the values 0 or 1 and are determined by the characteristic polynomial:

$$f(x) = x^n \oplus C_{n-1}x^{n-1} \oplus \dots \oplus C_1x \oplus 1. \quad (1.7)$$

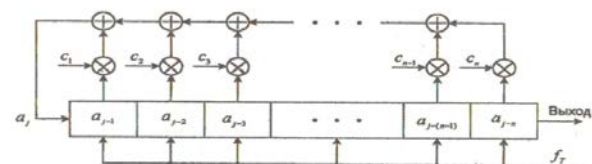


Fig. 1. 1. Scheme of PRS generator with linear feedback (LFSR)

A necessary condition for obtaining an M-sequence using the characteristic polynomial (1.7) is its irreducibility. A polynomial $f(x)$ of degree “ n ” is called irreducible if it cannot be decomposed into cofactor polynomials of lesser degree. For example, the polynomial $f(x) = x^5 + x + 1$ is reducible, since $x^5 + x + 1 = (x^3 + x^2 + 1) \times (x^2 + x + 1)$. If $2^n - 1$ is a prime number, then an irreducible polynomial generates an M-sequence.

An irreducible polynomial $f(x)$ of degree “ n ” is called primitive if the period of coefficients $1/f(x)$ is $2^n - 1$. The primitiveness of the polynomial $f(x)$ is a necessary and sufficient condition for obtaining an M - sequence. Primitive

polynomials exist for all $n > 1$. the number of such polynomials is defined by the following expression:

$$N_p(n) = F_p(L)/n = (1/n) \prod_{i=1}^k (p_i - 1) \cdot p_i^{n_i-1}, \quad (1.8)$$

where $F_p(L)$ is the Euler function that determines the number of integers that are coprime and do not exceed L ; p_i -multipliers of numbers 2^n-1 .

If L can be represented as a product of non-multiple multipliers, i.e. $n_i = 1$, then expression (1.8) takes the form:

$$N_p(n) = F_p(L)/n = (1/n) \prod_{i=1}^k (p_i - 1). \quad (1.9)$$

For example, for $n = 8$, we get $L = 255 = 3 \times 5 \times 17$ and the number of irreducible polynomials is $N_p(n) = (1/8) (3-1) (5-1) (17-1) = 16$.

Table 1.1 shows some data regarding the number and numbers of taps of generators of M-sequences for a different number of bits of the shift register.

Table 1.1. Numbers of taps and the number of M - sequences

Number of digits, n	M – sequence period	Quantity M-sequences	Tap numbers register for the chain feedback
2	3	1	[2,1]
3	7	2	[3,2]; [3,1]
4	15	2	[4,3], [4,1]
5	31	6	[5,3], [5,2]
6	63	6	[6,5], [6,1]

As follows from the data given in table 1.1, the number of M-sequences increases with increasing "n".

Table 1.2 shows some irreducible polynomials and their binary equivalents.

Table 1.2 Irreducible polynomials and their equivalents

Degree	Irreducible polynomial	Binary sequence
4	$x^4 + x + 1$	10011
	$x^4 + x^3 + 1$	11001
5	$x^5 + x^2 + 1$	100101
	$x^5 + x^3 + 1$	101001
	$x^5 + x^3 + x^2 + x + 1$	101111
	$x^5 + x^4 + x^2 + x + 1$	110111
	$x^5 + x^4 + x^3 + x + 1$	111011
	$x^5 + x^4 + x^3 + x^2 + 1$	111101
6	$x^6 + x + 1$	1000011
	$x^6 + x^4 + x^2 + x + 1$	1010111
	$x^6 + x^4 + x^3 + x + 1$	1011011
	$x^6 + x^5 + 1$	1100001
	$x^6 + x^5 + x^2 + x + 1$	1100111
	$x^6 + x^5 + x^3 + x^2 + 1$	1101101
	$x^6 + x^5 + x^4 + x + 1$	1110011

The autocorrelation function of an M-sequence can be defined as:

$$R_u(n) = (k - l)/L = (L - 2d)/L, \quad (1.10)$$

where k is the number of matches;

l – number of mismatches;

L is the total number of characters;

d is the Hamming distance.

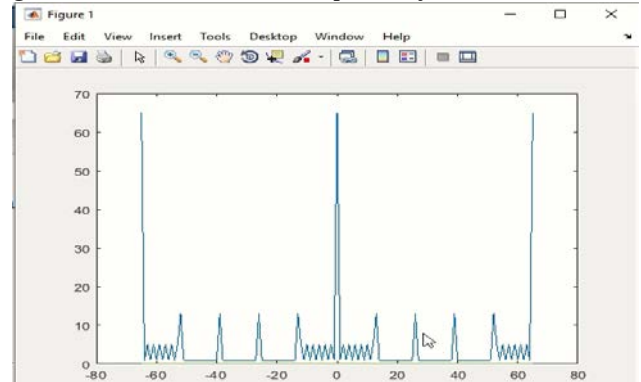
Therefore, the ACF can only take two values:

$$R_u(n) = \begin{cases} 1, & n = 0 \bmod L, \\ -1/L, & n \neq 0 \bmod L. \end{cases} \quad (1.11)$$

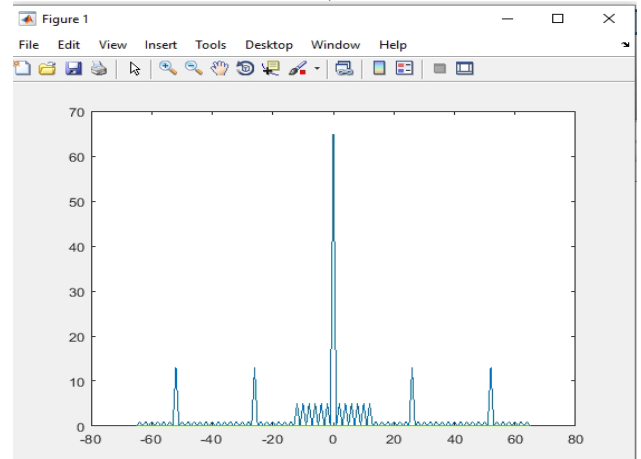
2.2. Experimental data

The correlation characteristics of PRSs are better studied in the Matlab - Simulink environment, especially if the PRSs have a long length. This can be done in two ways: using blocks from the Simulink library to build a generator for the corresponding pseudo-random sequence, or programmatically.

The correlation properties of broadband signals of modified Barker codes, which are presented above, were studied in the Matlab environment [6], the results of which are presented in figures 1.2, 1.3, 1.4 and 1.5, respectively.

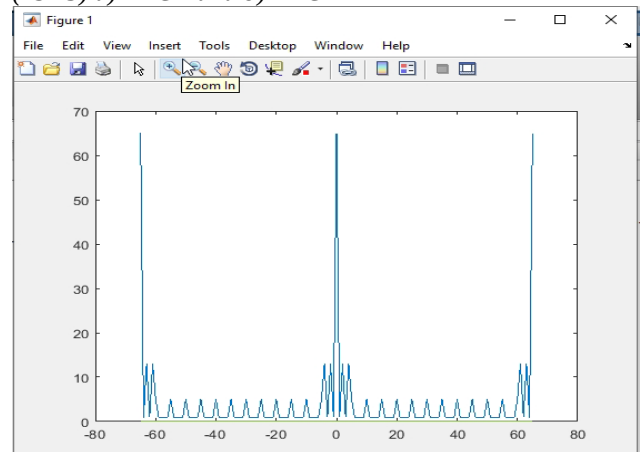


a)

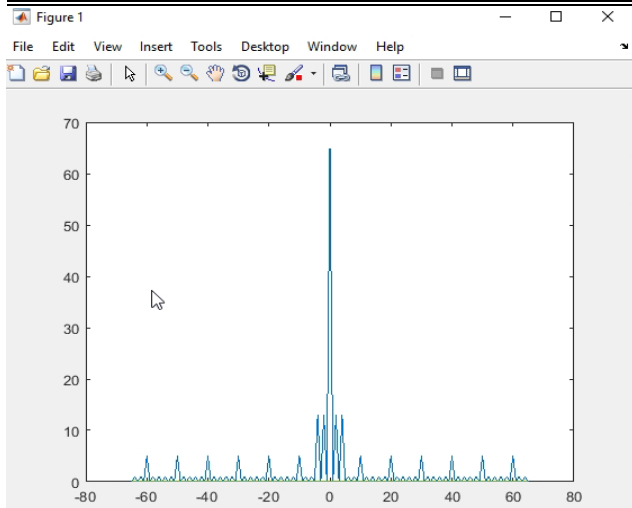


b)

Fig. 1.2. Correlation functions of modified Barker codes (13×5) a) PACF and b) AACF



a)



b)

Fig. 1.3. Correlation functions of modified Barker codes (5×13) a) PACF and b) AACF

Fig. 1.4 shows the cross- correlation function of these two signals.

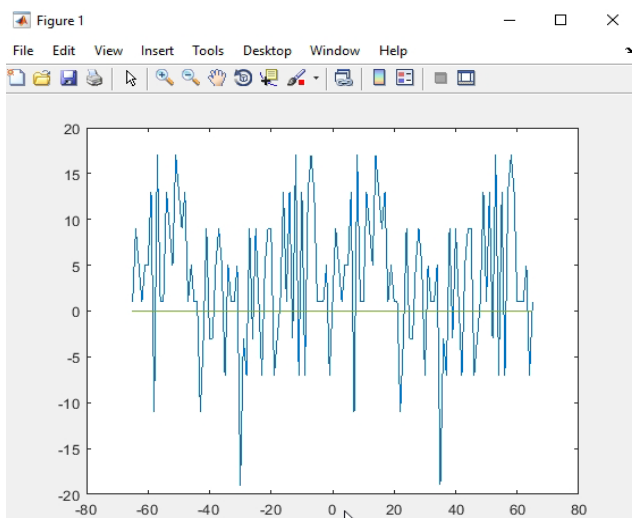
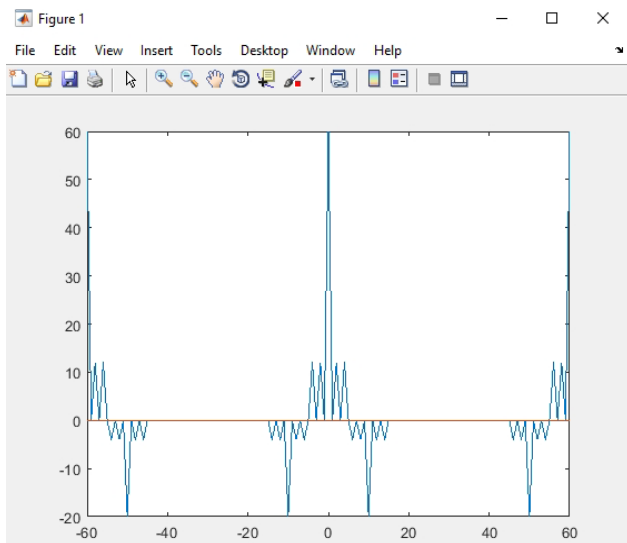
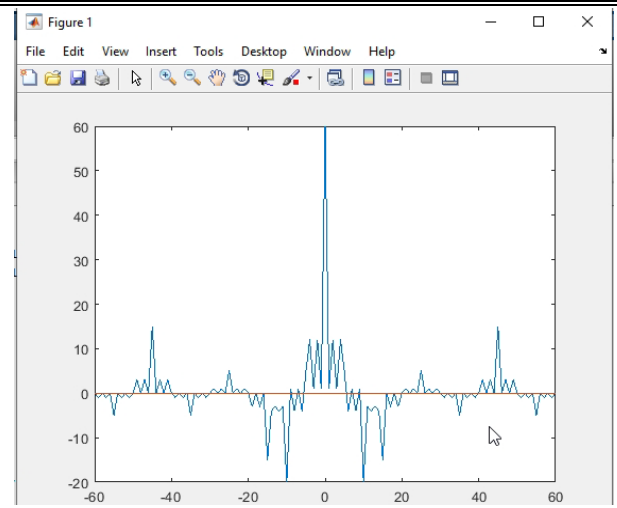


Fig. 1.4. CCF modified Barker codes (13×5) and (5×13)



a)



b)

Fig. 1.5. Correlation functions of modified Barker codes (5×13) $\times 4$ a) PACF and b) AACF

Fig. 1.6 shows the CCF of modified Barker codes ($(3 \times 5 \times 4)$ and $((5 \times 3 \times 4)$

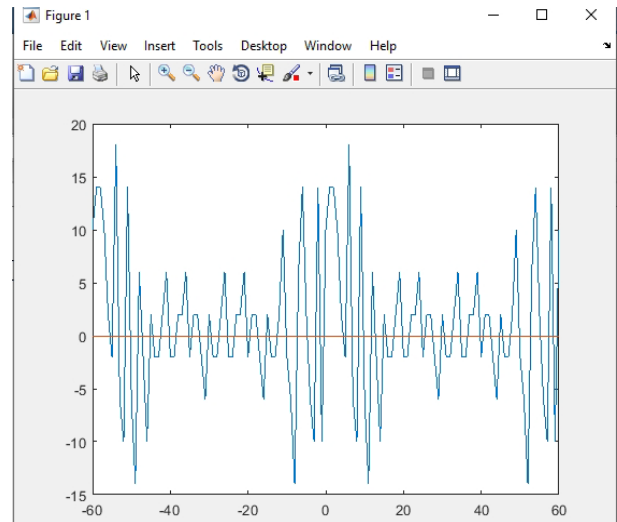
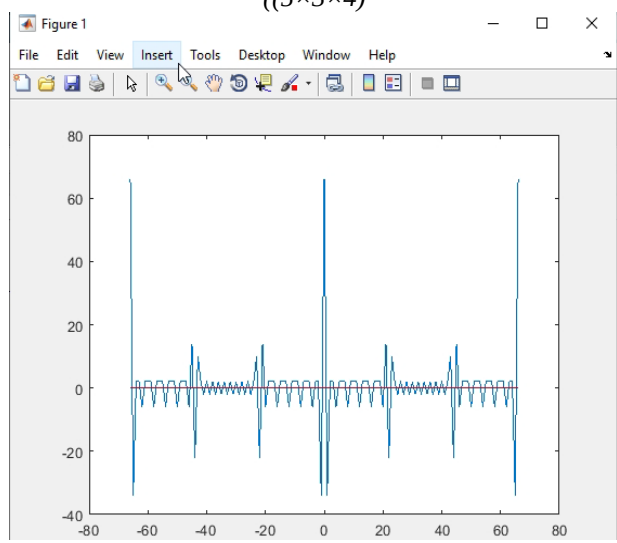


Fig. 1.6. CCF modified Barker codes ($(3 \times 5 \times 4)$ and $((5 \times 3 \times 4)$



a)

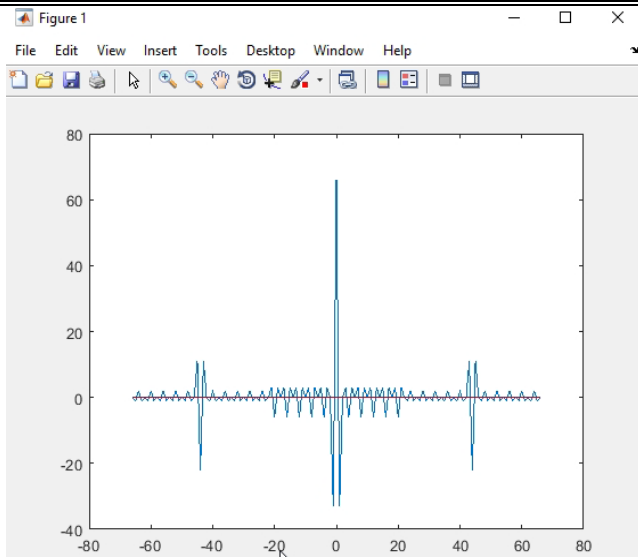


Fig. 1.7. Correlation functions of modified Barker codes $(2 \times 11) \times 3$ a) PACF and b) AACF

Fig.1.8 shows the CCF of the modified Barker codes $((2 \times 11) \times 3)$ and $((11 \times 2) \times 3)$

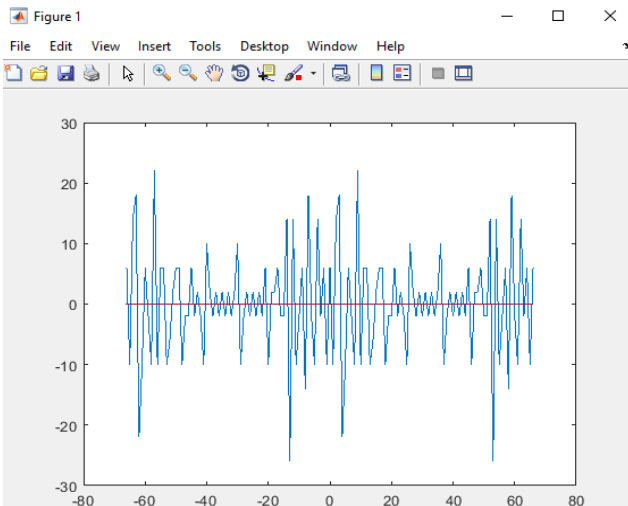


Fig. 1.8. CCF modified Barker codes $((2 \times 11) \times 3)$ and $((11 \times 2) \times 3)$

From Fig. 1.2, ..., Fig.1.8 it can be seen that the use of modified Barker signals leads to an increase in the amplitude of the central peak of the ACF, but there are side outliers that can lead to errors in the processing of input signals. The cross-correlation functions of the considered signals have a large unevenness, which can lead to an increase in the level of interference of multiple access.

The Matlab environment also investigated the correlation properties of M - sequences with polynomials $f_1(x) = x^6 + x + 1$, $f_2(x) = x^6 + x^4 + x^3 + x + 1$, $f_3(x) = x^6 + x^5 + 1$ and $f_4(x) = x^6 + x^5 + x^4 + x + 1$, which have a length equal to $2^6 - 1 = 63$.

The graphs of the aperiodic autocorrelation functions listed above for M-sequences are shown in Fig.1.9, ..., Fig.1.12, respectively.

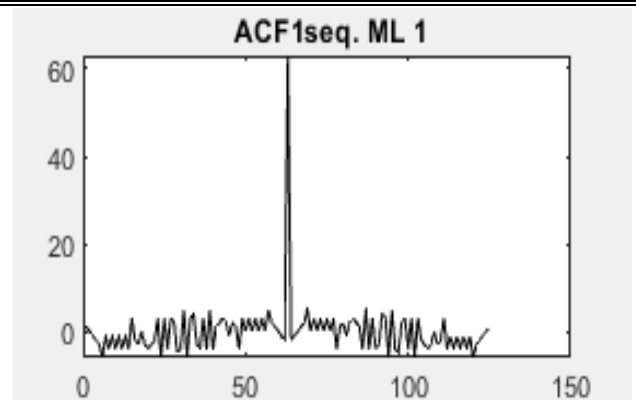


Fig. 1. 9. Aperiodic ACF M-sequences with a verification polynomial $f_1(x) = x^6 + x + 1$

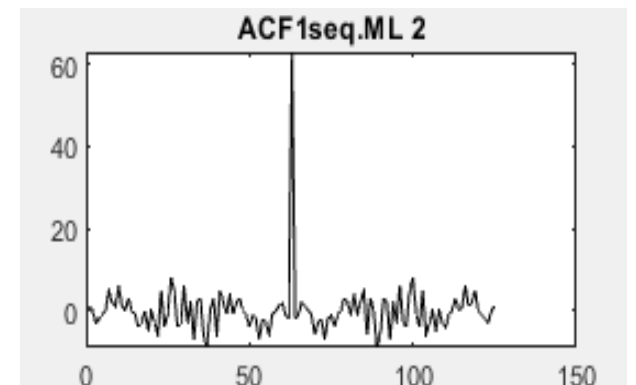


Fig. 1. 10. Aperiodic ACF M-sequences with a verification polynomial $f_2(x) = x^6 + x^4 + x^3 + x + 1$

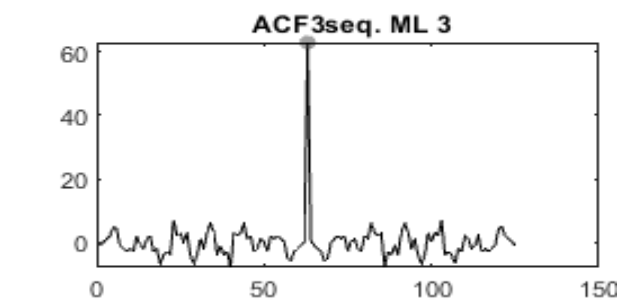


Fig. 1. 11. Aperiodic ACF M-sequences with a verification polynomial $f_3(x) = x^6 + x^5 + 1$

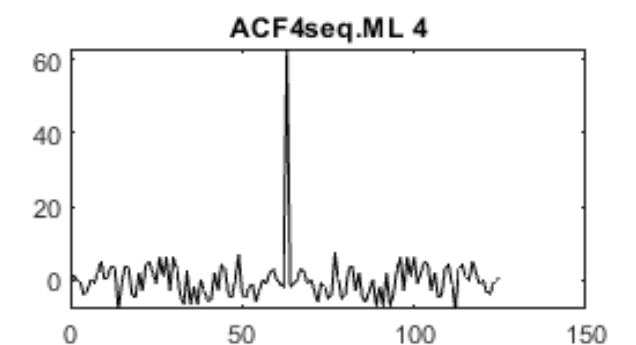


Fig. 1. 12. Aperiodic ACF M-sequences with a verification polynomial $f_4(x) = x^6 + x^5 + x^4 + x + 1$

Fig. 1.13 and Fig.1.14 show the cross-correlation functions of M-sequences $f_1(x) = x^6 + x + 1$, $f_2(x) = x^6 + x^4 + x^3 + x + 1$ and $f_3(x) = x^6 + x^5 + 1$, $f_4(x) = x^6 + x^5 + x^4 + x + 1$, respectively.

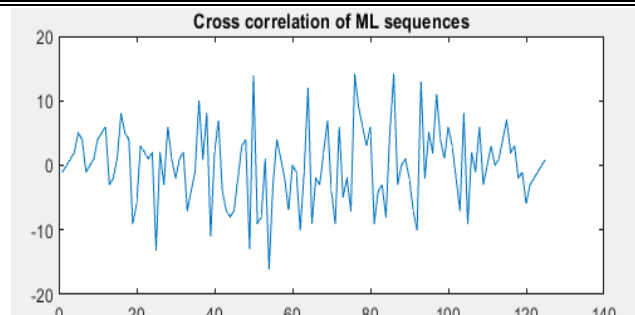


Fig. 1.13. CCF M - sequences with verification polynomials $f_1(x) = x^6 + x + 1$ and $f_2(x) = x^6 + x^4 + x^3 + x + 1$

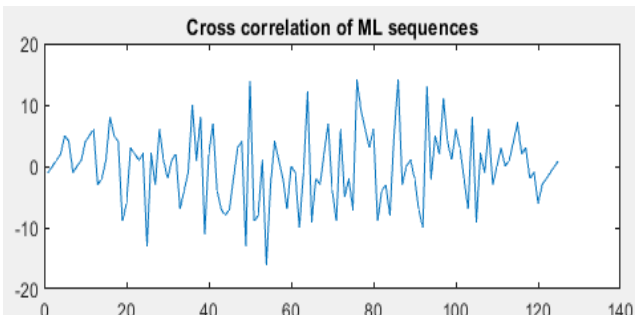


Fig. 1.14. CCF M - sequences with verification polynomials $f_3(x) = x^6 + x^5 + 1$ and $f_4(x) = x^6 + x^5 + x^4 + x + 1$

3. Conclusion

Analysis of the correlation properties of modified Barker codes allows us to draw the following conclusions:

- Increasing the length of the modified Barker sequence increases the amplitude of the central peak of the ACF, but also increases the amplitudes of the side outliers of the ACF. This can lead to a failure of synchronization, errors in the processing of input data.
- Modification of the Barker sequence leads to complication of circuit solutions in the construction of pseudo-random sequence generators based on such Barker codes.
- Modified Barker codes can be used in high-speed data transmission systems, taking into account the influence of the side outliers of the ACF.

The investigated M sequences, which have the same length as the modified Barker codes, have better autocorrelation and cross-correlation properties. M-sequences have a better balance of zeros and ones than modified Barker codes, and also have the least equivalent linear complexity. Therefore, M-sequences are preferable for high-speed data transmission systems.

However, for multichannel data transmission systems with code division multiplexing, it is necessary to use a large number of PRSs that differ from each other. Modified Barker codes cannot provide such an amount of PRS. The number of distinguishable M-sequences increases only with increasing bit depth n of the shift register. For example, if $n = 6$ there are only 6 such sequences, and if $n = 10$ there are 60 such sequences.

In addition, to reduce interference from multiple access, these generators of the PRS must have certain properties - the cross-correlation function of the generated pseudo-random sequences should be relatively uniform with minimal levels of side lobes.

However, only a small number of the entire ensemble of M-sequences with a given period has satisfactory correlation properties. An increase in the degree of the polynomial leads

to an increase in the number of "good" sequences, however, the period of the M - sequence is significantly increased. And this, in turn, leads to an increase in chip speed and the expansion of the width of the spectrum of a noise-like signal more than acceptable. In order to compromise between the level of interference of multiple access and the bandwidth of the communication channel in some cases use "truncated" M - sequence. These sequences have slightly worse correlation properties, but allow you to achieve the desired compromise. The scope of M-sequences is huge and diverse. By selecting the appropriate properties of the M - sequence, a satisfactory result can be achieved in most cases of the operation of broadband systems. Therefore, obtaining ensembles of M-sequences of arbitrary length is an actual practical task. Therefore, further careful study of the properties of M-sequences is required to solve the corresponding applied problems.

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Methods of solving non-standard inequalities

Методы решения нестандартных неравенств

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Abstract: This article discusses the problems of teaching non-standard tasks at school. Currently, one of the main tasks of secondary schools is to identify creative thinking abilities of students and solve problems of their development. The report was identified as the main tool for developing students' mental and thinking abilities. Problem solving is the most productive form of learning mathematics, and this process should be a necessary component of all extracurricular activities conducted in mathematics. Problems at the mathematical Olympiad are non-standard problems. Olympic tasks will usually help you learn a lot, find independent non-standard methods of solving problems. The high school curriculum provides for the use of the definition of inequality, proving using known inequalities, using the relationship between the arithmetic mean and geometric mean of positive numbers, and proving them using methods other than reverse-trip methods.

KEYWORDS: NON- STANDARD TASK, OLYMPIC TASK, HIGHLY PRODUCTIVE FORM.

1. Introduction

Currently, one of the main tasks of secondary schools is to identify creative thinking abilities of students and solve problems of their development.

Non-standard tasks develop students' ability to think, leads to thinking and increases motivation, interest in accounting. Non-standard tasks do not allow students to solve problems correctly. Therefore, non-standard tasks are the main means of developing students' mathematical thinking.

The degree of mathematical development of students is obvious in their ability to solve problems. Report the education of each student mentally steady fixed assets. The solution of unusual, interesting problems occupies an important place in the mathematical creativity of children. First of all, we must remember that to learn to solve problems – to find its solution. D. poya wrote the book "mathematical knowledge" (M. 1976.13): » If you want to learn to swim, dive into the water without secrets, and find its solution to learn how to solve problems " [1].

Solving any difficult test required a lot of work, great strength and perseverance from the student. These qualities are enhanced when the child wakes up with a desire to take into account. Since interesting tasks set your mental energy in motion, it becomes easier to solve them. This is why the teacher should choose tasks that will be interesting and independently solved by students.

To develop the mathematical abilities of students and foster interest in mathematics, it is advisable to use humorous problems and mathematical puzzles. The student's confidence that they can output a report is one of the important factors for success. If the calculation is too difficult, the student loses goals, reduces the effectiveness of thinking, and undermines further learning. The teacher strives for the solution of his students, strive for its solution, believe in himself-these are the necessary prerequisites for success. In the process of solving each problem, you should distinguish four stages: 1) understand the conditions of the task; 2) make a plan; 3) implement the plan; 4) "Open your eyes back", i.e., study the solution found.

Creative understanding of the learned material of the student and the development of new ways of activity is due to the presence of the following three components of thinking: 1) a high level of formation of simple mental operations, such as analysis and synthesis, comparison, analogy, classification; 2) a high level of thinking activity, expressed in the presentation of multi-faceted solutions and unexpected ideas; 3) a high level of cohesion and purposefulness, expressed in ways of independent thinking.

2. Preconditions and means for resolving the problem

The formation of these qualities of thinking contributes to the development of the creative personality of the student, overcoming difficulties in the development of educational material. The essence of this is that the student uses theoretically based methods of education and activity, or independently finds new

approaches to solving the problem. The task of the teacher is to form these components of thinking. And the key to it is the release of a creative report. Solving creative tasks of students is carried out through their knowledge, skills and abilities. In addition, the lesson plays a role of motivation in maintaining a high active mental activity, the desire of the student to their work. Therefore, we can say interesting tasks (tasks, tasks, puzzles, logical tasks) as a means of developing the mind, adapting the student to creative activity. They have the opportunity to improve creative activity, mentally train, rationalize as an auxiliary, additional way.

Although such materials are diverse, they have common properties.

1) the method of solving interesting problems is unknown. The achievement of their solution occurs as a "Brownian movement of thought", i.e. by the method of awakening, errors. Petropavlovsk hosted the regional competition "state language-my language!", dedicated to the 70th anniversary of Victory in the great Patriotic war.;

2) interesting tasks are the basis of the student's interest and activity in the subject.;

3) Interesting tasks are made up on the basis of knowledge of the laws of thinking.

So the systematic use of these types of tasks contributes to the development of these mental operations, the formation of mathematical representations in children. Solving interesting problems often occurs during a trial search. Skillful prediction of the exhibits ingenuity and resourcefulness in children. Resourcefulness is a special type of creativity that includes analysis, comparison, generalization, identification, understanding, and the ability to determine relationships, on the basis of which the counter comes to a single conclusion and groups the game. It is an indicator that they are able to utilize their knowledge. Resourcefulness and inflexibility, which can foresee the solution of interesting problems, is nothing else. The success of such mental activity can and should be developed in the learning process.

In any case, a thorough analysis is made to predict the solution of the problem: to distinguish the main properties of the problem, the spatial location and grouping of figures, their features, and similar features. However, the method of baiting and errors for solving interesting problems is not decimal reliable and versatile. The most effective method is to equip children with such important methods as analysis and synthesis, comparison, identification, and qualification of mental activity.

Among the problems that have a cognitive significance and play a leading role in the system of mathematical education are inequalities.

Special attention is not paid to the irrepressibility of textbooks for secondary schools: students can not form skills to prove mathematical failures. In order to draw attention to these shortcomings and methodological difficulties, the article is devoted to school teachers, students of the physics and mathematics faculty of pedagogical universities.

3. Methods for solving real inequality

The high school program uses evidence on the use of the definition of inequality, proving using known inequalities, using the relationship between the arithmetic environment and the geometric environment of positive numbers, and the method of reverse Hiking. Let's look at other ways.

3.1 Let's focus on the inequalities proved by the discriminant of a square triangle.

Many inequalities are proved by the symbol of a quadratic three-dimensional discriminant. The given quadratic triple must be in order to accept a positive value. $a > 0, d < 0$. We need to prove the inequality below.

$$a > 0, b > 0, c > 0 \quad \text{by}$$

$$a^2b^2 + c^2b^2 + a^2c^2 \geq abc(a + b + c)$$

$$\begin{aligned} a^2b^2 + c^2b^2 + a^2c^2 - a^2bc - b^2ac - c^2ab = \\ = (b^2 - bc + c^2)a^2 - bc(b + c)a + c^2b^2 \geq 0 \end{aligned}$$

This inequality can be considered as a square three dimensional one:

$$\begin{aligned} D &= b^2c^2(b + c)^2 - 4b^2c^2(b^2 - bc + c^2) \\ &= b^2c^2(b^2 + 2bc + c^2) - 4b^4c^2 + 4b^3c^3 - 4b^2c^4 \\ &= b^4c^2 + 2b^3c^3 + b^2c^4 - 4b^4c^2 + \\ &+ 4b^3c^3 - 4b^2c^4 = 6b^3c^3 - 3b^2c^4 - 3b^4c^2 \\ &= -3b^2c^2(b^2 + c^2 + 2bc) = -3b^2c^2(b - c) \leq 0 \end{aligned}$$

a in any value of the number the specified inequality will be correct. we arrive at this conclusion by getting a square three-dimensional by b and c . X - in any value $kx^2 - (5k + 1)x + 3(2k + 1)$ does a quadratic triangle have a positive value?

$$\begin{aligned} k > 0, D &= (5k + 1)^2 - 12k(2k + 1) = \\ &= 25k^2 + 10k + 1 - 24k^2 - 12k = \\ &= k^2 - 2k + 1 = (k - 1)^2 \geq 0 \end{aligned}$$

Therefore, in any value of x , the square cannot take a positive value of three squares.

Consider the inequalities that are proved using the "amplification" method»:

if $A > B, B \geq C$, then $A > C$ the properties of inequalities can be proved by relying on many things.
 $\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + \frac{1}{n^2} < 1 (n \in N, n > 1)$ prove the inequality.

$$\begin{aligned} \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + \frac{1}{n^2} &< \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \\ &+ \frac{1}{3 \cdot 4} + \dots + \frac{1}{(n-1)^n} = \frac{2-1}{1 \cdot 2} + \frac{3-2}{2 \cdot 3} + \\ &+ \frac{4-3}{3 \cdot 4} + \dots + \frac{n-(n-1)}{(n-1)^n} = (1 - \frac{1}{2}) + \\ &+ (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + \dots + (\frac{1}{n-1} - \frac{1}{n}) \\ &= 1 - \frac{1}{n} < 1 \end{aligned}$$

$$A = \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n-1} + \frac{1}{2n} < \frac{3}{4}$$

prove the inequality.

$$\begin{aligned} A &= \frac{1}{2n} \left[\left(\frac{1}{n} + \frac{1}{2n} \right) + \left(\frac{1}{n+1} + \frac{1}{2n-1} \right) + \dots + \left(\frac{1}{2n} + \frac{1}{n} \right) \right] \\ &= \frac{1}{2} \left(\frac{3n}{2n^2} + \frac{3n}{2n^2} + \frac{3n}{2n^2} + \dots + \frac{3n}{2n^2} \right) \\ &< \frac{1}{2} \left(\frac{3n}{2n^2} + \frac{3n}{2n^2} + \frac{3n}{2n^2} + \dots + \frac{3n}{2n^2} \right) \\ &= \frac{1}{2} (n+1) \frac{3}{2n} < \frac{3}{4} + \frac{3}{4n} < \frac{3}{4} \end{aligned}$$

3.2 Method for reducing the number 1 in two parts of inequalities.

This method is used if there is more than 1 correct fraction on both sides of the sea. To do this, you need to convert two parts of the inequality yourself, compress, and then subtract two parts of them by 1. This way, the result can be compared.[2]

$$\frac{2n-1}{2n} < \frac{2n}{2n+1} \quad \text{prove the inequality.}$$

$$1 - \frac{2n-1}{2n} = \frac{1}{2n}$$

$$1 - \frac{2n}{2n+1} = \frac{1}{2n+1} \quad \text{яғни} \quad \frac{1}{2n} > \frac{1}{2n+1}$$

if $1 - A > 1 - B$, then $B > A$

$$\text{Therefore } \frac{2n-1}{2n} < \frac{2n}{2n+1}.$$

$$\frac{33331}{33334} > \frac{44441}{44445} \quad \text{it is necessary to prove the inequality.}$$

$$A = \frac{33334-3}{33334} = 1 - \frac{3}{33334} \cdot \frac{4}{4}$$

$$B = \frac{44445-4}{44445} = 1 - \frac{4}{44445} \cdot \frac{3}{3}$$

$$1 - A = \frac{12}{133336}; 1 - B = \frac{12}{133335}, \quad A > B.$$

Give large numbers from any zero. The relations between these numbers are as follows: $a_1, a_2, \dots, a_n$.

$$\frac{a_1 + a_2 + \dots + a_n}{n} = A_n - \text{the arithmetic mean,}$$

$$\sqrt[n]{a_1 \cdot a_2 \cdot \dots \cdot a_n} = \sigma_n - \text{the geometric mean,}$$

$$\frac{\sqrt{a_1^2 + a_2^2 + \dots + a_n^2}}{\sqrt{n}} = Q_n - \text{square mean,}$$

$$\frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}} = H_n - \text{it's called the harmonic mean.}$$

This is between the average $H_n \leq \sigma_n \leq A_n \leq Q_n$ the ratio is performed. In most cases, when inequalities are proved, these relations are applied.

$a_1, a_2, \dots, a_n$ the arithmetic mean of positive numbers will be greater than or equal to their geometric center:

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 \cdot a_2 \cdot \dots \cdot a_n}$$

This inequality implies Cauchy's inequality. Proof of this inequality.

$\sqrt[n]{a_1 \cdot a_2 \cdot \dots \cdot a_n} = \sigma_n$ numbers in a geometric tool
 $a_1 \geq a_2 \geq a_3 \geq \dots \geq a_n$ to satisfy the inequality.
 $a_1 a_2 = (\sqrt{a_1 a_2})^2 = \sigma^2, a_1 a_2 a_3 = (\sqrt[3]{a_1 a_2 a_3})^3 =$
 $= \sigma^3, \dots, a_1 a_2 \dots a_n = (\sqrt[n]{a_1 a_2 \dots a_n})^n = \sigma^n$

Because $\frac{a_1}{\sigma} = 1, \frac{a_1 a_2}{\sigma^2} = 1, \dots, \frac{a_1 a_2 \dots a_n}{\sigma^n} = 1$

$\frac{\sigma}{a_1} = 1, \frac{\sigma^2}{a_1 a_2} = 1, \dots, \frac{\sigma^n}{a_1 a_2 \dots a_n} = 1$ will be. This is
 $\frac{a_1}{\sigma} \cdot \frac{\sigma}{a_1} + \frac{a_1 a_2}{\sigma^2} \cdot \frac{\sigma^2}{a_1 a_2} + \dots + \frac{a_1 a_2 \dots a_n}{\sigma^n} \cdot \frac{\sigma^n}{a_1 a_2 \dots a_n} =$

$= n \leq \frac{a_1}{\sigma} \cdot 1 + \frac{a_1 a_2}{\sigma^2} \cdot \frac{\sigma}{a_1} + \dots$

$+ \frac{a_1 a_2 \dots a_n}{\sigma^n} \cdot \frac{\sigma^{n-1}}{a_1 a_2 \dots a_{n-1}} =$

$\frac{a_1}{\sigma} + \frac{a_2}{\sigma} + \frac{a_3}{\sigma} + \dots + \frac{a_n}{\sigma} = \frac{a_1 + a_2 + \dots + a_n}{\sigma}$

That is $\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sigma = \sqrt[n]{a_1 a_2 \dots a_n}$ this was
supposed to prove it.

Example. $n! \leq \left(\frac{n+1}{2}\right)^n, (n \geq 2)$ to prove the inequality.

$\sqrt[n]{n!} = \sqrt[n]{1 \cdot 2 \cdot 3 \cdot \dots \cdot n} \leq \frac{1+2+3+\dots+n}{n} = \frac{(n+1) \cdot n}{2n} = \frac{n+1}{2}$

This is an inequality on both sides n -as for the degree

$n! \leq \left(\frac{n+1}{2}\right)^n$ that was what had to be proved.

3.3 Chebyshev inequality.

a) If $a_1 \geq a_2 \geq a_3 \geq \dots \geq a_n \geq 0$ and
 $b_1 \geq b_2 \geq b_3 \geq \dots \geq b_n \geq 0$,

there $(a_1 b_1 + a_2 b_2 + \dots + a_n b_n) \geq (a_1 + a_2 + \dots + a_n) \cdot (b_1 + b_2 + \dots + b_n)$

Add the separation of the n inequality

$(a_1 b_1 + a_2 b_2 + \dots + a_n b_n) \geq (a_1 b_1 + a_2 b_2 + \dots + a_n b_n)$

$(a_1 b_1 + a_2 b_2 + \dots + a_n b_n) \geq (a_1 b_2 + a_2 b_3 + \dots + a_n b_{n-1})$

.....

$(a_1 b_1 + a_2 b_2 + \dots + a_n b_n) \geq (a_1 b_n + a_2 b_1 + \dots + a_n b_{n-1})$

We know a idea Chebyshev's inequality are true.

b) If $a_1 \geq a_2 \geq a_3 \geq \dots \geq a_n \geq 0$ and
 $b_1 \leq b_2 \leq b_3 \leq \dots \leq b_n \leq 0$, there

$n(a_1 b_1 + a_2 b_2 + \dots + a_n b_n) \leq (a_1 + a_2 + \dots + a_n) \cdot$

$\bullet (b_1 + b_2 + \dots + b_n)$

the following inequality is satisfied.[3]

Example. If $a_1 \geq a_2 \geq a_3 \geq a_4 \geq 0$ and

$b_1 \geq b_2 \geq b_3 \geq b_4 \geq 0$, there

$4(a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4) \geq (a_1 + a_2 + a_3 + a_4) \cdot$

$\bullet (b_1 + b_2 + b_3 + b_4)$

to prove the inequality.

When the report conditions are met, there

$a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 \geq a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$

$a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 \geq a_1 b_2 + a_2 b_3 + a_3 b_4 + a_4 b_1$

$a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 \geq a_1 b_3 + a_2 b_4 + a_3 b_1 + a_4 b_2$

$a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 \geq a_1 b_4 + a_2 b_1 + a_3 b_2 + a_4 b_3$

If you perform a separate join of expressions to the left and right of these inequalities

$4(a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4) \geq a_1 \cdot (b_1 + b_2 + b_3 + b_4)$

$+ a_2 (b_1 + b_2 + b_3 + b_4) + a_3 (b_1 + b_2 + b_3 + b_4)$

$+ a_4 (b_1 + b_2 + b_3 + b_4)$

this was supposed to prove it.

4. Conclusions

In elective classes in mathematics, it is necessary to instill in the formation of students' interest, to improve their work skills, to develop the knowledge obtained in their school curriculum, to determine their life needs.

The meaning of mathematics in a school course is its versatility, that the main objects of yagni are based on real life. Therefore, solving non-standard tasks outside the program develops the system of knowledge and thinking of students.

Problem solving is the most productive form of learning mathematics, and this process should be a necessary component of all extracurricular activities conducted in mathematics. Problems at the mathematical Olympiad are non-standard problems. Olympic tasks, as a rule, will help you learn a lot, find independent non-standard methods of solving problems.

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Development of information model of power transmissions in the light of Industry 4.0

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Abstract. The main features of creating an information model (Digital Twin) for a technically complicated item are formulated. These include: a multitude of systemic representations of the item, individualization of its behavior and state under specific operating conditions, fatal uncertainty in the description of component properties, reconstruction of data based on sensors readings, individual lifetime expense in operation. Above features are illustrated by authors' developments in application to power transmissions. The key issues of synthesis, calculation, design and diagnostics of transmissions based on the created complex of lifetime-and-functional models are solved. Transmission is considered as a multicomponent system with a variable structure and states. Creating a single universal model to reproduce basic properties of the technically complicated item is impossible. Principles of constructing models and methods for calculating, monitoring and predicting functional and lifetime properties are described. The basic representations of the transmission (structure, regular mechanical system, kinematic and dynamic diagrams, diagnostic model, schemes of limiting states, lifetime expense model) and the mathematical models serving them are developed. The approaches considered are methodologically typical for complex mechanical and combined objects based on mechanical systems.

1 Introduction

The evolution of concepts, which led to the now accepted term "Digital Twin" (DT) and its modern interpretation, is presented in [1, 2].

Many publications analyze various interpretations of this concept and give their own definitions. In some cases, a statistical analysis of formal features from known publications is used. An example of a such work is [3]. It examines three questions: What are the definitions of DT that have been published in literature so far? What are the main characteristics that should be implemented in a DT? What are the domains in which DT applications have been developed?

As the first result, the following definition is formulated: "DTs can be defined as (physical and/or virtual) machines or computer-based models that are simulating, emulating, mirroring, or "twinning" the life of a physical entity, which may be an object, a process, a human, or a human-related feature. Each DT is linked to its physical twin through a unique key, identifying the physical twin, and therefore allowing to establish a bijective relationship between the DT and its twin". The second result (the main characteristics that DTs are supposed to possess): both the physical and the digital twins must be equipped with networking devices to guarantee a seamless connection and a continuous data exchange either through direct physical communications or through indirect cloud-based connections.

Based on such publications, the following conclusion can be made: The term DT has become quite familiar, and specialists at the subconscious level are usually understood it unambiguously, but when defining this concept, they generate a lot of formulations.

As if confirming this conclusion, work [4] states that "the existing DT implementations are fundamentally different from each other, adjusting to the needs of each use case and created with a wide variety of tools. Especially, there seems to be an unfruitful competition between modeling oriented and information management oriented views of the DT concept". The authors of [4] suggest a feature-based digital twin framework (FDTF) to universally define and structure digital twins. The framework consists of three main principles: 1) the idea that all digital twins consist of a definite set of features, 2) the features can be used to compare digital twin instances to each other, and 3) the features can be combined via a data link feature to construct future digital twins more efficiently. Besides, it was found that the features can be identified in existing digital twin implementations and the feature combinations of the implementations are diverse.

In [5], it was noted that most approaches today lack a comprehensive review to examine DT benefits by considering both engineering product lifecycle management and business innovation as a whole. The work conducts a state-of-the art survey of DT in the frame of innovation direction. As a result, eight future perspectives for DT are identified: modular DT, modeling consistency and

accuracy, incorporation of Big Data analytics in DT models, DT simulation improvements, VR integration into DT, expansion of DT domains, efficient mapping of cyber-physical data and cloud/ edge computing integration. Authors of [5] suppose that this work will become a guide for DT development and application in today's academic and industrial environment.

Moving from the consideration of general issues related to DT to the topic of "gear and transmission", it should be noted the papers, which are directly devoted to DT and related the following questions:

- simulation of the tooth root strength under consideration of material quality, finishing process and size effects [6],
- improved tooth contact analysis by using virtual gear twins [7],
- standardized gear unit model [8],
- Digital Twin of gear measuring center [9],
- combining gear design with manufacturing process [10].

A significant paper for the development of transmissions [8] indicates that nowadays a large number of different CAE tools are available for the design and analysis of a gear unit and its components, each of which has its own strengths. A major milestone for Industry 4.0 is the establishment of industry-wide standards.

FVA e.V. (Research Association for Drive Technology) in close cooperation with industry and research, is developing an industry-wide standard for simple data exchange in transmission development under the name REXS (Reusable Engineering EXchange Standard). The REXS initiative pursues the goal of providing a "digital twin" in transmission development and calculation. REXS defines uniform parametric modeling and nomenclature of gear units and their components across standards and industries, based on detailed terminology from FVA's 25 project committees and 50 years of joint industrial research.

This is meant to reduce the number of interfaces involved in the design process, that covers the following interacting basic procedures (from a REXS point of view): component analysis – first draft design – detailed design-construction (CAD) – FE-analysis – dynamics – customers & external partners – manufacturing simulation.

This is the vision of the REXS initiative: a free, nonproprietary, standardized interface for the exchange of transmission data which reduces the complexity of data exchange in the design process significantly.

REXS has the potential to establish itself on a large scale as a standard model for data exchange in the field of gear unit design and analysis. This would result in a number of advantages for the software manufacturers, for the companies using the tools and for the users: 1) Transmission design and analyses at any level of detail, i.e. from the overall system view via the analysis of individual components to the individual physical phenomena, can always be carried out on the basis of a single data model; 2) A simple data

exchange between classic analytical gear design programs and universal dynamic, FE and CAD systems would be possible. Expenses for additional, specific modeling could be greatly reduced, etc.

Summarizing the above, it can be stated that the process of creating the transmission DT is under development and is aimed at unifying the interface during the most common types of gear calculations. At the same time, a complete list of basic issues is not covered, including analysis and synthesis, special types of kinematic and dynamic calculations, dependability assessment of a transmission as a system, diagnostics, etc.

In this paper, using the example of a transmission, the emphasis is placed on considering the following fundamental points of building an information model (Digital Twin) of a complex object.

- *The transmission is considered as a technically complicated item (TCI).* Such items have a hierarchical structure; their workflows take place under varying conditions, there is an operator or an automatic control system for the item, the item structure can change (gear shifting), the item as a system consists of elements of various types (gears, bearings, shafts, clutches), the lifetimes of these elements is calculated according to different, not related techniques, enshrined in various regulatory documents.

- *Systemic representations of a transmission.* The fundamental issue for building a DT of a complex technical object is the development of a system of its interrelated representations. (As it is noted in [11]: "When solving a number of problems, it turns out to be insufficient to use only one system representation and, therefore, to use only one division of the whole into elements. The task is possible to solve only when using different system representations associated with each other. Moreover, the elements into which the whole is divided are fundamentally different in different systemic representations. The object is as if projected onto several screens. Each screen sets its own division into elements, thereby generating a certain structure of the object. The screens are connected with each other so that the researcher has the opportunity to correlate different pictures, bypassing the object itself. We will call such a "device" synthesizing various system representations a "configurator"). This requires the development and application of various models that relate to different representations of the object, and not one universal model.

- *Individualization.* The main feature of the simplest digital twin of a technical object (after its making and assembly) is modeling the individual behavior and state of the object in specific operating conditions. Moreover, the use of sensors is not always required. For example, it is possible, on the basis of a changing description of the external environment, to reflect a change in the behavior and state of the object accordingly. The use of sensors makes it possible to clarify the condition of the object by load and speed factors, but it does not completely remove the problem of uncertainty in assessing the lifetime, since the load-carrying ability of the elements is of a random nature even with the highest quality of their manufacture. (This is confirmed in some respects in a paper [12] on digital twin approach for damage-tolerant mission planning under uncertainty, and where the proposed methodology includes the quantification of the uncertainty in diagnosis, prognosis, and optimization, considering both aleatory and epistemic uncertainty sources).

- *The problem of obtaining and reconstructing operation data.* It is unrealistic to believe that with the development of the sensor base, all the necessary parameters of the processes occurring with the TCI in operation can be obtained directly from the sensors. Of particular difficulty is the data on force factors in mobile parts. Therefore, the problem of data reconstruction arises: how to obtain the necessary information about the object based on the available sensory data? This problem cannot be solved without the correct calculation models. And the task of rational installation and use of sensors is associated with above problem.

- *Lifetime expense of TCI.* To describe the individual state of life of a complex object that consists of components of various types, with different mileages and ages, the term "lifetime expense" and a new indicator are introduced. This can be seen as a response

to the paper [13], which considers integrated vehicle health on the DT state.

2 Key points of the approach

At the stage of the transmission life cycle, data describing the transmission as a whole are transformed and developed. The transmission is supplemented with new representations in the following way: structure - kinematic diagram - layout - mechanical model - dynamic diagrams for various dynamic calculations - diagnostic models - schemes of limiting states - lifetime expense model.

These representations are accompanied by a set of component models, including a description of their limiting states / failures.

During operation, the level of the external loading of the transmission stabilizes quite quickly. The main variable individual component in operation is the internal dynamics of the transmission. Data on the internal dynamic loading are determined using the diagnostic system. These data are transmitted to lifetime forecasting systems, and also serve as independent indicators of the technical condition of the transmission. At the same time, the most advanced systems provide data for all significant transmission components.

Data obtained promptly from the operation of the object are used to predict the lifetime of individual components and estimate the transmission lifetime expense in general. The techniques used to solve these problems are of universal importance and are applicable to the machine, which includes the transmission.

3 Basic principles of developing an information model

The authors' vision and understanding of the information model of a machine as technically complicated item (TCI) begins with a paper [14], that developed the idea about the need for an information model of the machine [15]. The paper [14] contains the proposition that the creation and use of science-intensive products should be based on new information technology. The technology involves the development of an information model at all stages of the life cycle of the machine.

The information model should be included in the technical documentation accompanying the machine and allow the use of various sources: semantic, structural (logical), parametric (quantitative, mathematical) models; measurement results; expert evaluations; means of simulating the elements and units of the machine (in slow, accelerated and real time scales in relation to the current, retrospective and predicted state).

Thus, the mentioned methodology anticipated the basic principles of Industry 4.0: the model approach and creation of the digital twin.

The newly developed principles based on [16] cover:

- sharing synthesis, analysis and multi-criteria estimates of the transmission;
- interaction of micromechanical models of materials and reliability theory;
- consideration of drives as multi-component systems with variable states and variable power flows depending on the mode of their operation;
- unity of component loading modes (principle of dependent behavior of components in a loaded system);
- localization of loading in damage models of their simplest components;
- construction of diagnostic models using condition assessment by an integral criterion and predictive calculation based on monitoring the accumulation of transmission damage.

The following basic systemic representations are used for transmissions: regular mechanical system, structural diagram, kinematic and dynamic diagram-schemes, normalized dynamic scheme, scheme of limiting states.

The principles of schematization for the transition from a real mechanical object to its symbolic representation using the concept of a regular mechanical system are developed. The universal mapping of multifarious differentials and gear sets along with the typical displaying of other transmission devices (shafts, controls

and a frame) gives the universal representation, formalization and automation of getting equations in the field of transmission kinematics and dynamics. This provides the correct schematization of transmissions of any kind with standard and non-standard components. For the mathematical description of transmissions structures, the developed canonical matrix and structurally-distributive matrix are used.

The presented models and methods are used at all stages of the transmission life cycle to determine loading and qualification of the life state for the limiting components, as well as life expense of the transmission as a whole. Data from the diagnostic system are used to individually synchronize the state of a particular transmission (real object and its digital twin).

4 Typical representations of the power transmission

Considered transmission representations include symbolic mappings and mathematical models serving these mappings. A set of such representations can be considered as typical in a methodical sense for mechanical objects and combined objects, the basis of which are mechanical systems.

4.1 Transmission structure

The initial representation of the transmission is its structure. The stage of structure synthesis is the first in the transmission life cycle. This stage is completed by the development of several promising variants of the transmission (for further developmental work), presented in the form of kinematic diagrams and their parameters (gear ratios and so on).

When describing possible variants of structures, the problem of isomorphism takes place. For unambiguous construction of structures with given parameters (the number of mechanisms and the number of part/links), the canonical matrix method is used.

Fig. 1 shows the canonical matrix and incorrect canonical matrix (selected cells) that correspond to the structure and gear set shown in Fig. 2.

Gear trains	Parts (links)					
	V1	V2	V3	V4	V5	V6
U1	1	1	1			
U2	1	1		1		
U3			1		1	1

Gear trains	Parts (links)					
	V1	V2	V3	V4	V5	V6
U1	1	1		1		
U2	1	1	1			
U3				1	1	1

Fig. 1. The canonical (top) and incorrect (bottom) canonical matrices (own work)

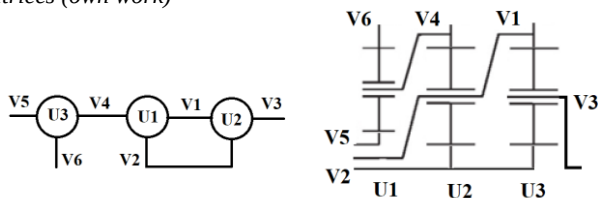


Fig. 2. An example of the structure (left) and the gear set of World Transmission (right, [17]) corresponding to Fig. 1 (own work).

The basic rule that ensures the construction of original canonical matrices is as follows: it is necessary to use parts (links) with the lowest numbers for each mechanism.

4.2 Kinematic diagram

The kinematic diagram is used as the basis for designing the layout, calculating speeds and torques, and also for determining the levels of internal loads when predicting the transmission life. Approaches

to the construction of kinematic diagrams using software packages are presented in Fig. 3 [18].

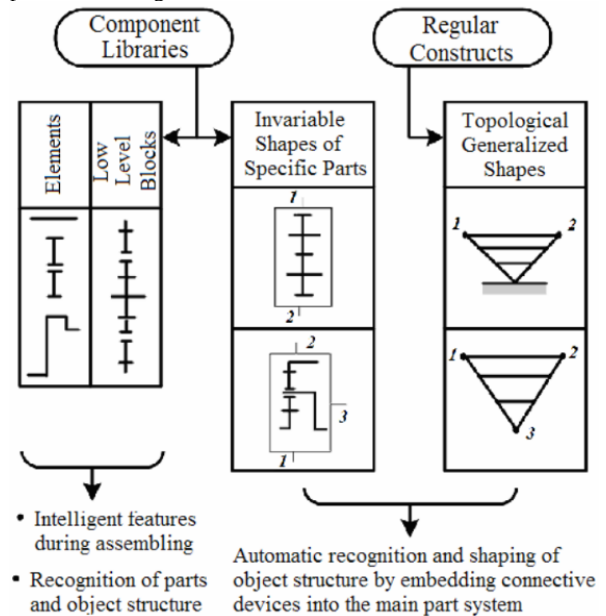


Fig. 3. Approaches to schematization (own work).

Most well-known software packages are universal and contain special modules/component libraries for modeling transmission devices (Class "Component Libraries"). These are: ADAMS / Machinery [19]; Simscape Driveline [20]; MapleSim Driveline [21]; Modelica / Power trains and planetary gearboxes // Gears [22]; KissSoft / KISSsys // GRK [23]).

Transmissions are widely used in mechanical engineering; it is advisable to develop specialized software packages for them. As an example, the World Transmission kinematic diagram made in the specialized package "Kinematics" is shown in Fig. 4. The following IDs are used: I = basic parts (links), F = controls (clutches and brakes), z = numbers of gear teeth. This package allows to build kinematic diagrams with planetary and simple gears, controls (clutches and brakes), several embedded shafts, recognize structural relationships, gear ratios of gear mechanisms, perform calculations and optimize transmission parameters when presenting it with the kinematic diagram.

4.2 Regular constructs for forming kinematic and dynamic diagrams-schemes

The complex and manifold systems need two-levelled schematization and representation. The first specific level is based on the commonly used elements for the family of objects under considerations and on their conventional representation (Class "Component Libraries"). The necessity of the second level stems from the fact that any element base of specific elements has limitation. Hence the second level is more abstract and should use the special symbolic representations to multi-purpose description of structural links in concrete kind of objects. Therefore, for the design of the transmission, it was proposed to create a set of mechanical components that are characteristic for the schematization of mechanical objects (class "Regular Constructs"). The level of abstraction increases with a variety of transmission units. In this case, intuitive symbolic icons for transmission parts/devices are offered. Mathematical models are formally generated from the formed design models for the further kinematic and dynamic calculations [18].

Transmission devices, which have the same mathematical structure and differ only in parameters, can be represented in a generalized form by symbolic regular constructs (Fig. 5).

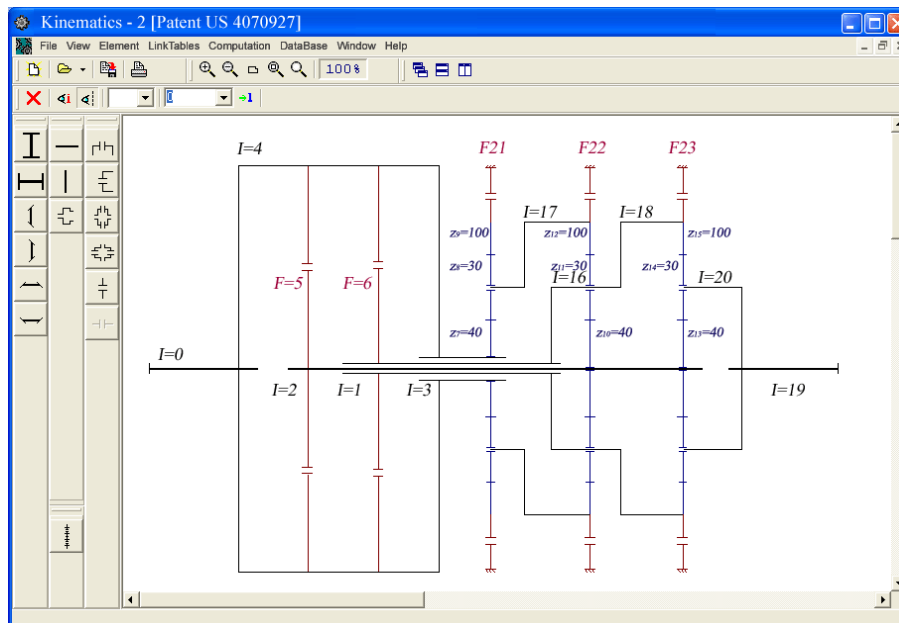


Fig. 4. Software “Kinematics” by an example of World Transmission representation (own work).

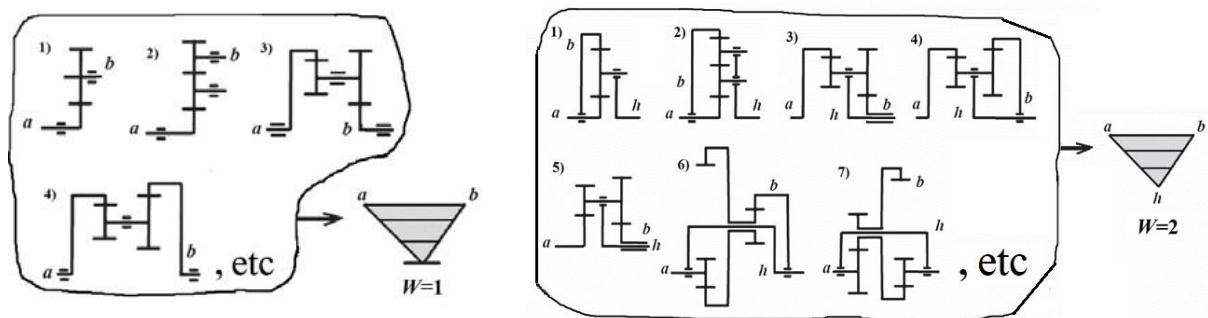


Fig. 5. Symbolic representation of different types of gears (number of degrees of freedom $W=1$, left) and differentials ($W=2$, right) by regular constructs (own work).

Index for device part	Differential D	Train P	Shaft S	Frame R	Clutch F	Brake T
1 (i)	1	1	1	1	1	1
2 (j)	$-u$	$-u$	-1 ($u=1$)	0 ($u=0$)	-1	0
3 (k)	$-(1-u)$	—	—	—	—	—

Fig. 6. Distribution of internal torques among parts (links) of the typical transmission devices (own work).

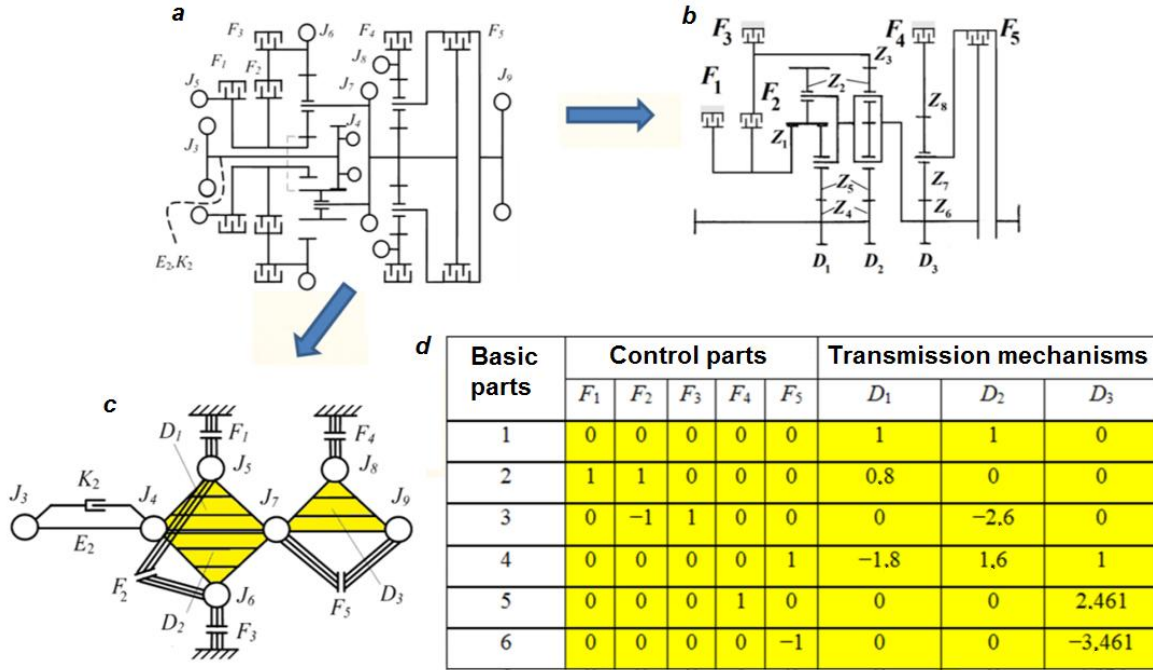


Fig. 7. Schematization of the MZKT-7922 transmission: *a* = mechanical model; *b* = equivalent kinematic diagram; *c* = dynamic scheme; *d* = structural distribution matrix (own work).

To describe the structure and distribution of the internal torques in devices, a structurally-distributive matrix (SDM) is introduced. Each mentioned device is presented at such a matrix in the form of a column, as it is shown in Fig. 6.

The meanings of non-zero elements of k th column ($A_n = A_{jk}$) describe the distribution of torques in the kinematic unit ($A_n = M_n/M_1$).

After describing the structure of the transmission using SDM, the equations for its kinematic and quasi-static calculations are automatically formed. As a result, the speeds of the links, the torques in the devices, and efficiency of the transmission in each of gears are determined [18]. In the latter case, the kinematic gear ratio u is replaced by the power gear ratio $\hat{u} = u\eta^x$ for each device, where η = efficiency; x is equal +1 or -1, depending on direction of power in the device.

An example of forming SDM for the transmission with non-standard planetary gear trains is depicted in Fig. 7. **Note.** Here and further: J = moment of inertia, E = elasticity, k (K) = damping coefficient.

4.3 Regular mechanical systems and dynamic computing

A mechanical system with elementary mechanical components (the concentrated masses and the massless joining links) is essential idealization which imposes certain restrictions on possible combinations of joints (connections) for mentioned items.

The concept of a regular mechanical system (RMS) considers that mechanical system consists of the concentrated masses (inertial components) and massless (non-inertial) devices-connectors: shafts, clutches, brakes, gears, motionless links, and other devices imposing kinematic connections for masses (Fig. 8).

Masses can be in contact interaction. For connecting device, the direct contact (not through inertial component) is prohibited. This is a **principle of regularity**, which is used for the representation of a real object. Its violation leads to wrong schematizations and errors in calculations or impossibility of mathematical model realization by the computer.

Clutches/brakes are the typical devices having variables states (Fig. 9). **Note.** Here and further: M = external torque or internal in the friction device.

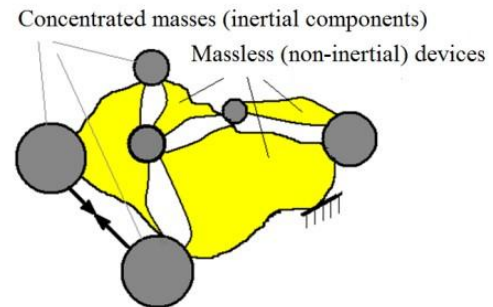


Fig. 8. General representation of RMS (own work).

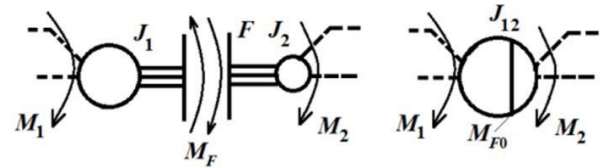


Fig. 9. States of a clutch: slipping (left) and locking (right) (own work).

For getting all-purpose mathematical model for their dynamics, a method of internal torques is used. This one is based on logical variables λ_k , named indicators states, which describe states of the clutches/brakes.

The equations corresponding to the clutches are as follows

$$\dot{\omega}_1 = [M_1 - (1 - \lambda_m)M_F - \lambda_m M_m] / J_1 \quad (1)$$

$$\dot{\omega}_2 = [(1 - \lambda_F)M_F + \lambda_m M_m - M_2] / J_2 \quad (2)$$

where $\lambda_F=0$ under locking, and $\lambda_F=1$ under slippage (unlocking) of the clutch; M_F is the known function describing a friction torque during slipping friction clutch parts. The equations are the same for all states of the clutch.

In order to use **all-purpose equations** a special procedure should be developed for finding the internal torques (like M_{12}) which acting in rigid devices. In a case presented in Fig. 9, the internal torque of the friction clutch can be calculated by formula

$$M_{12} = (J_2 M_1 + J_1 M_2) / (J_1 + J_2) \quad (3)$$

In addition, the conditions for changing the states of friction clutches should be described when solving differential equations. Generally, it is necessary to consider spasmodic change of a

friction torque at transition from slippage to friction clutch locking and inversely.

Generalization of these situations for computing leads to the formulation of **following principle**: if a new condition of contacting inertial parts (for example, slippage or locking of friction clutch/brake) becomes possible then this condition should be necessarily presented at next step of computing process.

In general case, the RMS can be stiff-elastic object that contains rigid and elastic devices. As typical example, RMS with rigid and elastic devices and the corresponding dynamic scheme are shown in Fig. 11.

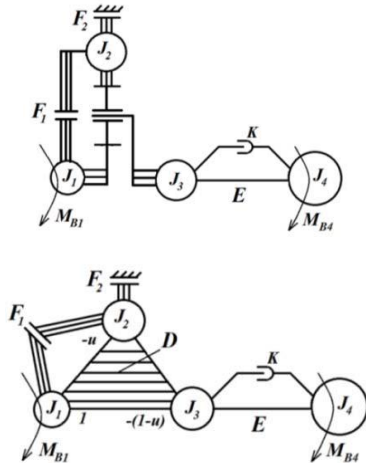


Fig. 10. Stiff-elastic RMS (top) and its dynamic scheme (bottom) with symbolical representation of the differential D (own work).

During computing stiff-elastic object, it is necessary to solve a system of differential equations, and this is accompanied by solution of a system of algebraic equations to determine the internal torques in rigid devices.

To avoid forming and solving algebraic equations, the dynamical system can be **normalized** by replacing rigid devices and links with elasticities E_i (Fig. 11). For brakes external links like E_{22} may be rigid. In the normalized system, masses and elastic links alternate. To determine the internal torques of closed clutches/brakes, a simple formula (3) is used instead of solving the system of algebraic equations.

This approach increases the number of differential equations, but simplifies the formation of mathematical models in general, which is very important.

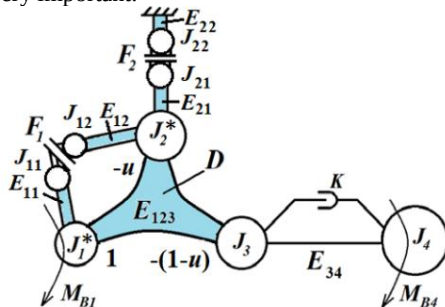
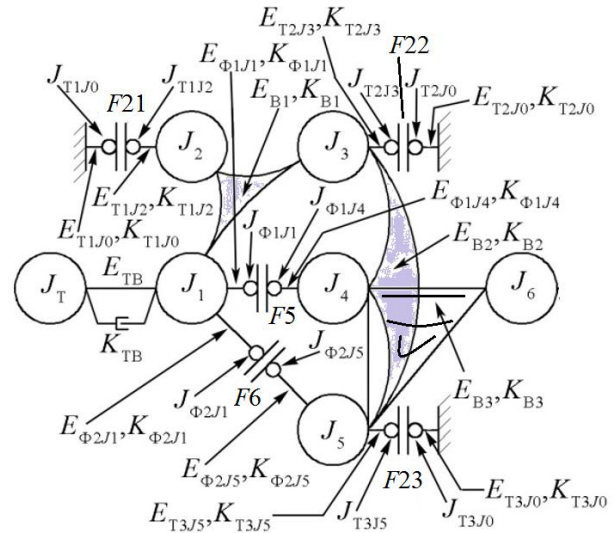


Fig. 11. Normalized dynamic scheme (own work).

Fig. 12 shows the normalized dynamic scheme for modeling the dynamic processes of planetary transmission, the kinematic diagram of which is depicted in Fig. 4. Three differentials are shown by curved triangular figures with elasticities E_{B1} , E_{B2} , E_{B3} , which correspond to the planetary gear sets U1, U2, U3 in Fig. 2 of the same transmission.



The first provision means that loads acting on various components of the assembly (gears, bearings, etc.) must be coordinated. This should be taken into account when calculating damage measures for various components using load data using load data from real operation sources or operational models.

The second provision assumes that the limiting state of a complex item is usually determined by a complicated way, based on combination of limiting states of its components.

To describe and calculate such states, the scheme of limiting states (SLS) is introduced. This is one more systemic representation for the transmission and any TCI. It describes the limiting states of a complex item much easier than the known tools (Failure tree and Reliability block diagram). In addition, SLS is convenient for use in statistical modelling procedures, where it is presented in the form of simple records.

The SLS consists of a hierarchical structural scheme and records describing limiting states for objects except for ones that don't have component parts (descendants). All objects other than the highest-level object are assigned a specific type: the first type, the second type, and so on. Objects whose limiting states have the same significance for a higher-level object are classed as the same type. The object type corresponds to its position (first, second, etc.) in the schematic record, which describes the criterion of the limiting state.

The schematic record (X1, X2, etc.) means that the limiting state of the object occurs if the limiting states are reached with its X1 parts of the first type (here X1 is the number standing in the first position), its X2 parts of the second type (here X2 is the number standing in the second position), etc. The object (unit, assembly, machine) can have some the SLS. For example, a mechanical gearbox (Fig. 13) has the following SLS: (1,0,0,0) (0,3,0,0) (0,0,1,2).

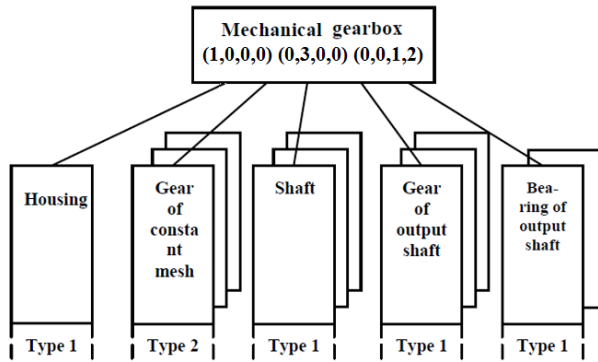


Fig. 13. SLS for the mechanical gearbox (own work).

In the general case, there is a multilevel SLS that reproduces the following levels: 1 = the machine (e.g. a car); 2 = aggregates and systems (e.g. transmission, carrier system); 3 = units and subsystems (e.g. gearbox, drive axle); 4 = parts (e.g. gear), typical component parts (e.g. ball bearing), joints (e.g. splined connection); 5 = constructional elements (e.g. gear teeth); 6 = the simplest components (e.g. local area of the surface layer of the gear teeth). Methods of mechanics are used for calculations of limiting states at levels 6, 5, and 4 (in some cases), and structural ones are used at levels from 4 or 5 to 1.

The complex object life cycle forecasting is based on the Monte Carlo method that has the following features. Every simulation cycle after determining components lifetimes is supplemented by the SLS analysis and lifetime determination for objects of intermediate and highest levels bottom-up. When reproducing processes and states related to mechanical levels, factors and effects leading to dependent behaviors of the mechanical components are realized, for example, the general loading levels of components in the particular transmission unit.

Above provisions and methods are included in the State Standard of the Republic of Belarus [25].

7 Monitoring lifetime and diagnostics of transmission in operation

Individualization of TCI and its information model (Digital twin) from stage “making and assemblage” is presented in Fig. 14. These are the supplied components; used equipment and personnel (the stage of making and assemblage); the operation conditions, the operator(s) of TCI, the maintenance (the stage of operation).

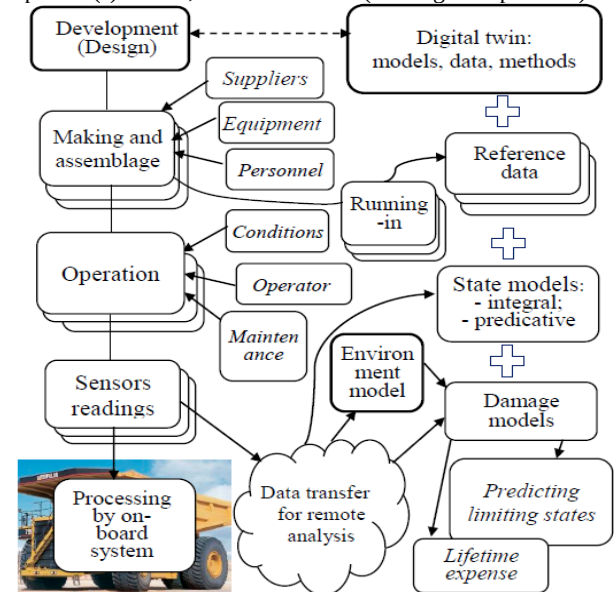


Fig. 14. Individualization of TCI and its information model (Digital twin) from stage “making and assemblage” (own work).

The current level of development of sensors and computer facilities also allows recognizing the state of the environment and transferring data for management systems and forecasting models that use data from the environment [26]. In this case, the environment model also refers to the TCI digital twin.

7.1 Peculiarities of TCI monitoring formation

When installing the sensors, the **principle of minimum multiplicity** should be adhering. Sensors, even in the minimum number, should reflect the force and speed constituents of the unit operation modes. In the ideal case, a one sensor can be used to obtain a load (force) spectrum, by which the load processes of all the limiting components are reconstructed.

Since it is not always possible to install a sensor to generate a signal about the loading the limiting element, the task of **reconstructing local loads** of limiting elements arises. To solve it, models that allow to re-construct the stresses (or other loads) of the limiting elements from the signals of existing sensors, should be developed and used.

It is rational to use the data on the loading in the form of general power flows, and individualize the loads for the limiting components to conduct when considering their limiting states

The feature of the developed diagnostic method is using conceptual modeling the oscillating process for the gear drive and the propagation of vibrations in the transmission. It is advisable to applicate together **integral diagnostic** models and **predictive** ones based on **damage accumulation** (Fig. 14, block “State models”). Such a “two-coordinate” approach (from two points of view) ensures a higher veracity of the individual lifetime forecast.

The main **variable individual component** of the transmission unit is the level of its internal dynamic loading. Changes in this level are due to the peculiarities of the manufacture of the unit and the operating conditions of the machine, which includes the transmission unit. The most sensitive components to changes in the internal dynamic load of the assembly are gears and bearings. The greatest damage to the clutches is associated with short-term transient processes, which requires individual monitoring of the modes of these processes.

7.2 Development of technique and means for transmission diagnostics by example of the gear units of mining dump truck

Transmission systems of vehicles and tractors operate under conditions of varying speeds and loads. In addition to internal factors, external ones (the road surface, vehicle loading, driver skills) influence on the vibration characteristics of transmission units.

Under such conditions, the nature of vibrations is constantly changing. The well-known equipment and techniques are not suitable for vibration monitoring of the technical state of the transmission components.

Wear and pitting change the parameters of the gearing and the areas of the contact pads; fatigue cracks reduce the rigidity of the engagement. This leads to the appearance of a shock pulse during the teeth contacts. As a result of theoretical and experimental studies [27], the vibroimpulse, which is the response of the system to the shock pulse produced by the interaction of the teeth of the gears, was selected as the main diagnostic feature. The chosen approach uses a dynamic model containing all the main components of a real object. This model serves to establish the features of the dynamic process under study and the links between the sources of vibration and the parameters being diagnosed.

Developed approach based on the consideration of a meaningful model of the oscillating process for the tooth gear drive and the propagation of vibrations in the reducer system. This provides advanced capabilities, including analysis of unstable regimes, use of an integral state indicator and calculation of damage accumulation in limiting components.

The main processes for the emergence, transformation and processing of signals in the reducer of a motor-wheel (RMW) and its monitoring system are shown in Fig. 15.

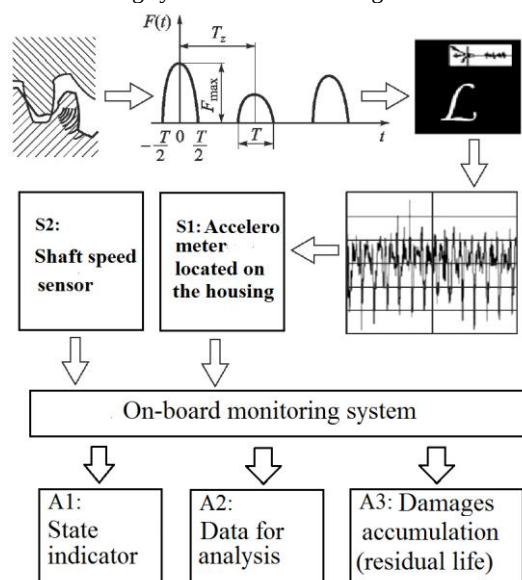


Fig. 15. Processes in the RMW and its monitoring system (own work).

The problem of rational choice of sensors and the task of **reconstructing data** for processes occurring in gears is solved as follows.

Two different types of sensors are used to diagnose RMW. The first sensor S1 is an accelerometer that senses the vibration of the housing. The second sensor S2 transmits data on the speed of the RMW shaft.

The signal S1 is the result of the propagation of a shock pulse along the mechanical transmission paths. It manifests itself in the vibration spectrum of the motor-wheel housing.

Vibrodiagnostics problems associated with the non-uniformity of rotation of the gears, solved by using the developed algorithms for processing the vibration signal and the signal of the rev counter recorded in real time. As a result, the temporal realization of the vibration signal is converted into a realization by the angle of the

shaft rotation for the diagnosed gear. Analysis of this implementation using the method of synchronous accumulation and ordinal analysis, adapted to the solution of a particular task, allows us to estimate the vibration parameters for each gear wheel (Fig. 16).

The spectra clearly show peaks at frequencies of 21, 42, 63, 84 Hz, multiples of the tooth frequency of the sun gear, which has 21 teeth.

The technical state of gears is monitored as follows.

The mean square value and the mean amplitude of the first seven harmonics for the tooth frequency are adopted as indicators of the technical state of the gears. They characterize the aggregate energy of the vibrations of the drive elements at the tooth frequencies.

The operation of the system (Fig. 15, block A1) includes:

- analysis of the parameters for vibroimpulses synchronized with the angle of rotation of the diagnosed tooth gear;

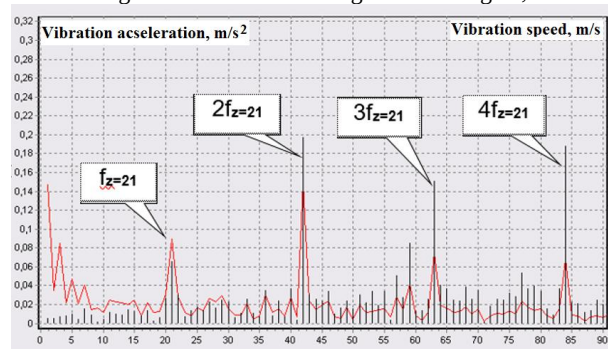


Fig. 16. Harmonic spectra of vibrations generated by the sun gear of the RMW planetary row (own work).

- identification of vibroimpulse harmonic components, which multiple of the tooth frequency and locate in the region of resonance frequencies of the mechanism as well as excite the most intense oscillations in the system;

- analyzing the changing the parameters of vibroimpulses.

The system periodically interrogates the sensors, processes the diagnostic information, evaluates the technical state of the reducer, comparing the received root mean square (RMS) values of vibration acceleration with the maximum permissible values for each of the states of the reducer. The system constantly informs the driver by means of an appropriate light signal on the instrument panel in the cab of the truck.

Actions, marked in Fig. 15 as blocks A2 and A3, are carried out by the service department with the help of special software. They include the following operations:

- transfer of stored data files from the on-board system memory to a stationary computer;
- obtaining harmonic spectra of vibrations of each gear of the reducer by means of synchronous accumulation algorithms and ordinal analysis;
- analysis of spectra and identification of gears generating high levels of vibration;
- for the teeth of gears with high vibration levels, the operating load and contact stresses are determined using the parameters of the vibroimpulses.

Then, taking into account the loading cycles, the residual life is calculated according to the developed technique. After that, a decision is made to continue the operation of the reducer or opening it to eliminate the causes of increased vibration (adjustment, tightening of connections, topping up oil, etc.). Based on the results of the opening, a decision is made to repair or further operating the truck.

At the same time, lifetime forecasting is performed based on a **predictive model**.

The method of forecasting the lifetime is based on the determination of the shock pulse in a meshing according to the results of vibration monitoring. From these data, the actual

circumferential force and contact stresses in the meshing are calculated [27].

The measure of fatigue damage to the gear is determined for each fixed i th interval of the running time by the formula

$$\Delta Q_{Hi} = \sigma_{Hi}^{q_{Hi}} N_i \quad (4)$$

where σ_{Hi} = contact stress; q_{Hi} = the exponent of the fatigue curve under calculating the gear for the contact endurance; N_i = the number of loading cycles of the gear tooth.

An example of the application of the technique for estimating the state of the gearing “sun gear / satellite” of the second planetary row of RMW is presented below. When the technical state is evaluated via integral assessment, the change in root mean square values of the vibration acceleration is monitored. The change in this indicator is shown in Fig. 17. When the dump truck mileage is less 200,000 km, this value remains practically constant. Further it increases, at the same time the peak value of vibration acceleration begins to increase.

When lifetime is calculated using the diagnostics current data, it is taken into account that a linear relationship exists between the amplitude of the shock pulse and the peak value of the vibration acceleration. The growth of peak values means an increase in the dynamic factor K_{Hv} , which is used in the calculation of contact stresses σ_H .

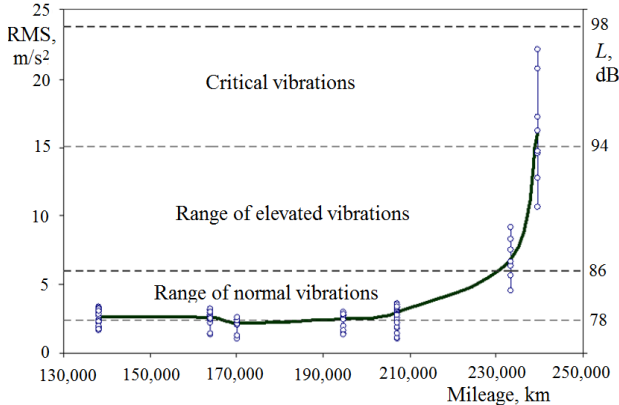


Fig. 17. Dependence of root mean square (RMS, m/s^2) values of vibration acceleration in RMW on the mileage of a dump truck (own work).

The calculation performed for the most probable value of the load-carrying ability of the considered gears gave a result similar to the integral assessment. Residual life is equal to zero under the general dump truck mileage equal to 235,000 km (taking into account the descent movement in the opencast mine and the transportation mode).

The developed technique of dynamic analysis of gear units which based on the evaluation of vibroimpulses in gearing is universal. Unlike other known approaches, it can be used in steady-state and transient modes of transmission operation.

8 Lifetime expense

The concept of the term “consumption of lifetime” for the component is obvious, as well as the understanding of its additional term “residual life”. The assessment of the state of life for a complex object, consisting of several components and having several different SLS, is ambiguous. And besides, some components can be replaced or repaired.

To eliminate this uncertainty and evaluate this property, the term “lifetime expense” of a complex object and a new indicator are introduced. This indicator has the value of the life potential of the object and is determined as follows

$$K_A = \sum_{i=1}^n \xi_i K_{Pi} \quad (5)$$

where K_{Pi} = lifetime expense of the i th main part of the TCI (presented as relative value); ξ_i = the weight factor that determines the contribution of the main part to the total lifetime expense of the TCI. It is proposed to consider ξ_i as the relative mass of the main

part (the fraction of this part mass in the total mass of n parts that determine the TCI lifetime).

The next fundamental point is lifetime expense of the main part:

$$K_p = 1 - (1 - K_L)(1 - K_T) \quad (6)$$

where K_L = lifetime expense by lifelength (mileage, operating time); K_T = lifetime expense by time (age); K_L relates to damage processes under loads during working; K_T relates to damage processes under time (ageing processes).

The values of K_L and K_T are determined at the time of monitoring / evaluating the technical condition of the object. Usually, the processes responsible for K_L and K_T can be considered as independent. Then, K_L can be interpreted as the probability of failure in the running hours (under the influence of loads in the duty cycle), and K_T as the probability of failure in age; and K_p is the probability of failure of the main part of the object under the combined action of loads and age at the considered time.

Fig. 23 shows the graphs of $K_L = K_L$, $K_T = K_T$, and $K_p = K_p$. It is estimated that 100% lifetime expense by age occurs over 30 years. The same period corresponds to 100% lifetime expense by lifelength (mileage). This case is illustrative, because in real practice, the full expenditure of the lifetime in mileage and age, as a rule, is not achieved simultaneously.

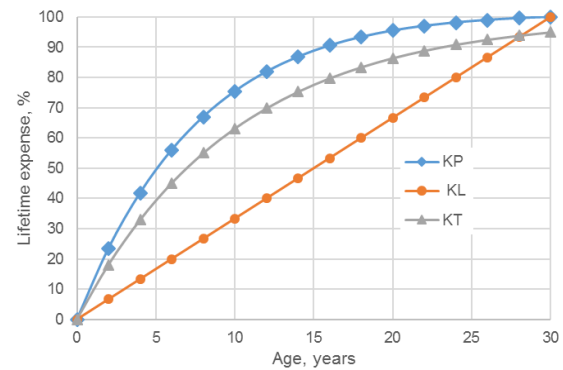


Fig. 23. Lifetime expense K_p for linear law K_L and exponential K_T (own work).

9 Conclusions

The features of creating an information model of technically complicated items (as an analog and the predecessor of Digital Twin) by the example of the power transmission is presented. This model covers and coordinates a set of systemic representations, as well as models and methods for calculation and diagnostics of gear transmissions.

System representations of transmission in the area of mechanics include regular mechanical system as well as kinematic and dynamic schemes. The latter contain symbolic representations of various components with the same mathematical descriptions. This makes it possible to create universal methods and software for solving kinematic and dynamic problems based on a structural-distribution matrix that reflects the structural relationships of transmission components.

In the field of dependability, the scheme of limiting states is entered that provides a simple way to calculate reliability indicators for objects with complex logic of their limiting states.

Individualization of the transmission condition during operation provides for using a system of diagnostics based on monitoring of vibroimpulses generated by gears. This provides information for a comprehensive assessment of the transmission state, as well as internal dynamic load data for predicting the lifetime calculations of limiting components.

Individualization of the transmission condition is also based on a new concept of lifetime expense which provides a comprehensive assessment of a specific object, taking into account the lifetime expenses of its main parts from mileage and age. The weighting of each part contribution is also taken into account.

The presented models and methods are assumed as the basis for information support and calculation of the life cycle of transmissions that includes using data from diagnostic system for individual synchronization of the state of a particular transmission.

A set of representations and methods developed for transmissions can be considered as typical in a methodical sense for mechanical objects and combined objects, the basis of which are mechanical systems. Many of them are universal and can be applied to a wide range of technical items. They are implemented in practice.

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Model of hydraulic turbine taking into account the impact of efficiency

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Abstract: Research, optimization and practical implementation of a Small Hydropower Plants as a source of clean electricity are one of the actual tasks in the current energetics, which is virtually impossible to solve without powerful computer support due to the strongly nonlinear nature of such systems. The article presents an overview of the most common simulation model schemes of Small Hydropower Plants, whereas explores the sub-models of its individual subsystems. The hydraulic turbine is considered as a core of the hydropower plant, whose efficiency calculation is analytically demanding and dependent on parameters that are often obtained only by theoretical estimation. In the article a fuzzy system was used to create its model based on measured operating data on the turbine flow rate and the height of its water column without the need to know the turbine parameters. Such a model is applicable in practice for the design of an energy-optimized hydro-turbine control as well as to the effective determination of the deterioration rate. The correctness of the results was verified by simulation measurements in the MATLAB program.

Keywords: HYDRAULIC TURBINE, SMALL HYDROPOWER PLANT, FUZZY SYSTEM, EFFICIENCY

1. Introduction

Hydroelectric power generation complies with ecological criteria aimed at environment protection. There is a lot of renewable energy accumulated in the surrounding water resources (lakes, rivers, artificial water reservoirs, etc.). This hydropower can first be converted into rotational mechanical power using hydraulic turbines, then, using synchronous and asynchronous power generators, it can be converted into electrical energy, and finally this whole energy conversion process can be controlled taking into account the requirements of the electrical energy end user.

In the energy conversion process, an essential role is played by the efficiency of the hydraulic turbine, both in terms of the overall efficiency of the conversion, and in terms of wear and possible maintenance or replacement.

2. Hydraulic Turbine Modeling

The model of a hydraulic turbine is a relatively complex non-linear dynamic system, some of the parameters of which are difficult to compute. However, correct and accurate modelling of the turbine is crucial in terms of accurate diagnosis and specification of the response of the entire hydropower plant to external influences, as well as in terms of correct tuning of its control.

Several approaches to hydraulic turbine modelling have been listed in the literature, differing in the manner they take into account the efficiency of the turbine or its impact on the final mechanical output of the turbine P_m [1-2]

2.1. Model Taking into Account Turbine Efficiency Through Turbine Flow Value Q_n at Zero Load

The development of a hydraulic turbine model is based mainly on theory and on the mathematical description given in expert publications [3] and [4]. The resulting block diagram of the hydraulic turbine structure is shown in Fig.1.

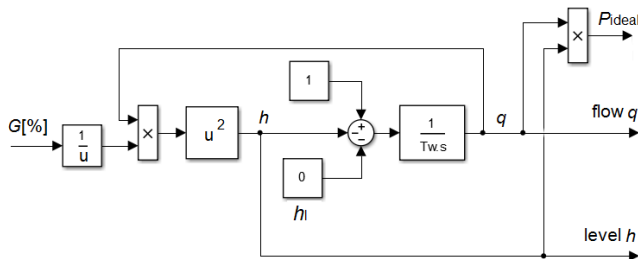


Fig. 1 Hydraulic turbine model

The penstock has been modelled on the assumption that the fluid is incompressible and that the penstock is sufficiently rigid, of L length and A diameter. The level loss due to the influence of the

penstock has been disregarded here. We therefore start from the equation:

$$\frac{d\bar{q}}{dt} = (\bar{h}_0 - \bar{h} - \bar{h}_l)g \frac{A}{L} \quad (1)$$

where:

$\bar{q}$ = standardized turbine flow	[m ³ /s]
A = penstock cross-section area	[m ²]
L = penstock length	[m]
g = acceleration due to gravity	[m/s ²]
$\bar{h}_0$ = static height of level	[m]
$\bar{h}$ = level at point of entry into turbine	[m]
$\bar{h}_l$ = level loss due to friction in the penstock	[m]

The above equation can be rewritten into the standardised form:

$$\frac{dq}{dt} = \frac{(1-h-h_l)}{T_w} \quad (2)$$

where parameter h denotes the level in the turbine area and parameter h_l accounts for the level losses that are being disregarded. The equation also includes a previously unused time constant T_w , referred to in the literature as the water time constant or the water start time. Its value can be obtained using the relationship (according to 1)

$$T_w = \left(\frac{L}{A}\right) \frac{q_{base}}{h_{base} g} \quad (3)$$

where q_{base} is the turbine flow at maximum opening of the inlet valve ($G=1$) and h_{base} is the value of the total available static height of level.

By introducing values h_{base} and q_{base} we are able to express, by means of the turbine characteristic, the value of water flow at $G=1$ as

$$q = G\sqrt{h}, \text{ and thus } q_{base} = 1\sqrt{h_{base}} \quad (4)$$

The turbine flow q is therefore the function of the position of servo valve G (inlet valve position) and the level h .

The mechanical output of the turbine would in an ideal case be characterised as turbine flow multiplied by the level.

2.2 Hydraulic Turbine Model Taking into Account Penstock Losses

In real operation, a turbine does not operate at 100% efficiency and hence the requirement to consider this fact in the process of hydraulic turbine modelling. There are several ways of

incorporating the impact of turbine efficiency into a simulation model [5].

One of the simplest ways is to use the so-called zero-load turbine flow. This flow, referred to as q_{nl} , is a constant that characterizes the value of the turbine's constant power losses, and in the formula this constant is subtracted from the actual turbine flow. It is then this final flow value that is multiplied by the level h as in

$$P_m = A_t h(q - q_{nl}) - DG\Delta\omega \quad (5)$$

Another factor present during the operation of the hydro power plant is the turbine damping effect, which is a function of the differences in speeds, servo position and water supply valve position. The $\Delta\omega$ variable represents the change in mechanical shaft speed, D is the penstock diameter (also as the turbine damping coefficient), and G is the servo position as a percentage.

The A_t parameter is considered to be the hydro turbine amplification. Its quantification differs in various scientific publications, e.g. according to [3] it can be calculated as follows:

$$A_t = \frac{\text{Turbine Output [MW]}}{(\text{Generator Output [MVA]} \cdot h_r(q_r - q_{nl}))} \quad (6)$$

where the quantities with the subscript r can be considered as rated. This means that q_r is the turbine flow at rated load, and h_r is the level required for achieving the mentioned rated flow of q_r . These parameters are usually part of the technical documentation of the particular systems in a hydro power plant. The model for computing hydraulic turbine losses is illustrated in Fig. 2.

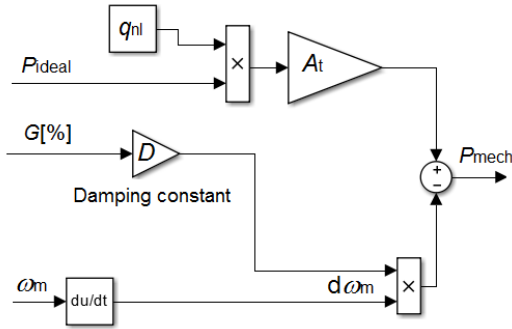


Fig. 2 Model for computing hydraulic turbine losses

2.3. Model Taking into Account Turbine Efficiency by Means of Its Efficiency Computation

This option of developing a simulation structure of a hydraulic turbine is based on the basic relationship describing the computation of its output mechanical power P_m

$$P_m = \eta \rho q g h \quad (7)$$

where η represents the hydraulic turbine efficiency value; ρ is the water density (value of 1000 kg/m^3 is used mostly); q is the actual turbine flow; g is acceleration due to gravity (9.81 m/s^2); and h is the actual level of the hydro power plant. Standardised quantities were applied in the model. For the expression of the efficiency of the hydraulic turbine in this case we applied the computation quoted in 5. The efficiency computation presents a separate simulation block as it defines efficiency η as a function of λ and turbine flow q , i.e. $\eta = f(\lambda, q)$.

Turbine efficiency computation $\eta_t(\lambda, q)$

As already mentioned, there are several ways of expressing hydraulic turbine efficiency. In this case we present the computation quoted in [5], which requires knowledge of turbine parameters such as the turbine blade radius - R_{blades} [m] and the size of the area swept by the blades - A_{blades} [m²]. Other data required for the computation

includes the actual values of turbine flow q and turbine mechanical angular velocity ω_m

$$\eta_t(\lambda, q) = \left[\frac{1}{2} \left(\frac{90}{\lambda_i} + q + 0.78 \right) \exp \left(\frac{-50}{\lambda_i} \right) \right] (3.33q) \quad (8)$$

where λ_i is

$$\lambda_i = \left[\frac{1}{(\lambda + 0.089)} - 0.0035 \right]^{-1} \quad (9)$$

and λ is defined as

$$\lambda = \frac{R_{blades} A_{blades} \omega_m}{q} \quad (10)$$

In this case the input into the computation of hydraulic turbine efficiency would be the actual turbine flow q and also the mechanical angular velocity of the turbine ω_m . The mathematical description is represented in the block diagram in Fig. 3.

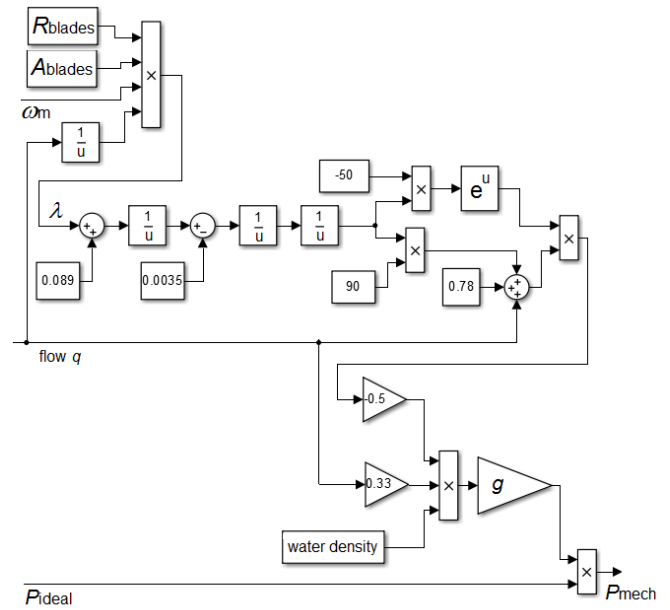


Fig. 3 Hydraulic turbine efficiency computation model

The block scheme of hydraulic turbine overall model taking into account losses and taking into account efficiency is shown in Fig. 4.

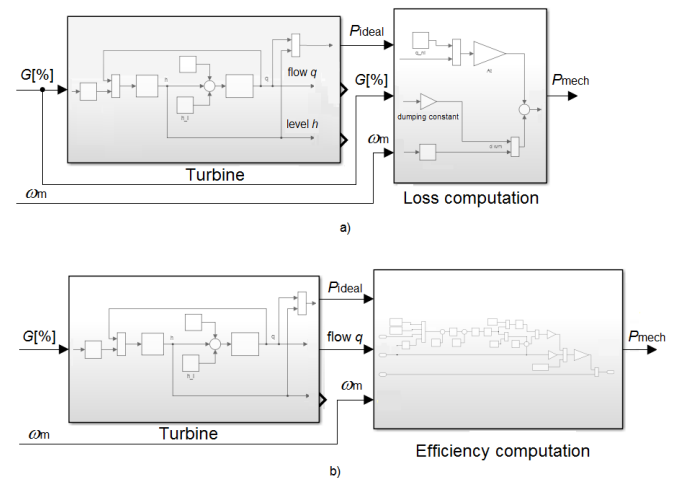


Fig. 4 Overall model of hydraulic turbine a) taking into account losses b) taking into account efficiency

3. Hydraulic turbine efficiency fuzzy model

As it follows from the previous sections, an exact determination of the efficiency of a particular turbine is rather complicated. For example, according to equation (10), this efficiency depends on the parameters of the blades of a particular turbine, which can change significantly throughout operation. Furthermore, this dependence is strongly non-linear (see equations (8-10) and is dependent on several only approximately determined parameters. For example, the efficiencies at various turbine flows and mechanical angular velocities for the blade radius $R_{blades} = 1.9$ m and the area swept by the blades $A_{blades} = 9$ m², are illustrated in Fig. 5.

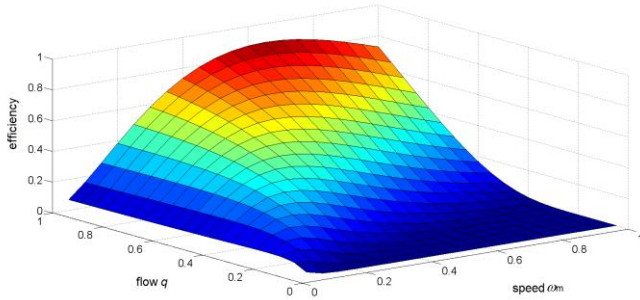


Fig. 5 Turbine efficiency for concrete blade conditions $R_{blades} = 1.9$ m and $A_{blades} = 9$ m²

When the circumfluence area of the blades changes (e.g. as a result of wear) to $A_{blades} = 8$ m², this turbine “efficiency image” will change, as shown in Fig. 6. The maximum efficiency of the turbine in this case is approx. 77%, while in the previous case it was approx. 88%.

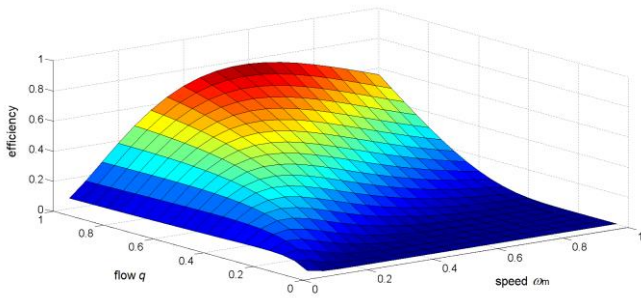


Fig. 6 Turbine efficiency for concrete blade conditions $R_{blades} = 1.9$ m and $A_{blades} = 8$ m²

An image of turbine efficiency can also be generated by measuring flow q , speed ω_m , and turbine efficiency in steady state operation, e.g. according to Tab. 1.

Table 1. Measured turbine efficiencies in steady state operation

Measured turbine efficiency η_t		ω_m				
		0,2	0,4	0,6	0,8	1
q	0,2	12,77%	30,36%	36,23%	37,83%	37,97%
	0,4	2,74%	26,40%	49,60%	64,41%	73,24%
	0,6	0,35%	13,64%	40,90%	66,92%	87,22%
	0,8	0,04%	5,68%	27,20%	56,26%	84,31%
	1	0,00%	2,11%	16,07%	42,03%	72,49%

The data from Table 1 can be used in searching for the fuzzy inference structure (FIS) of the modelled efficiency describing the measured relations between $[w_m, q] \rightarrow h_t$.

The fuzzy model can be designed based on this database using standard cluster analysis tools with adaptive approaches resulting in improved quality of modelling and reduced development time. In terms of the adaptive approach, a hybrid arrangement based on a fuzzy inference tool in connection with a neural network was used

[6]. From the large number of methods for adaptive fuzzy system development [7] we chose the adaptive neuro-fuzzy inference system (ANFIS). The basic characteristics of cluster analysis are the reduction of the fuzzy rules number and good initial rule parameters setting. We used subtractive clustering [8] as a fast and robust data analysis method, with the following parameters: Range of influence = 0.5, Squash factor = 1.1, Accept ratio = 0.45, Reject ratio = 0.005.

As a result, we obtained a static Sugeno type fuzzy system with twelve rules, as shown in Fig. 7.

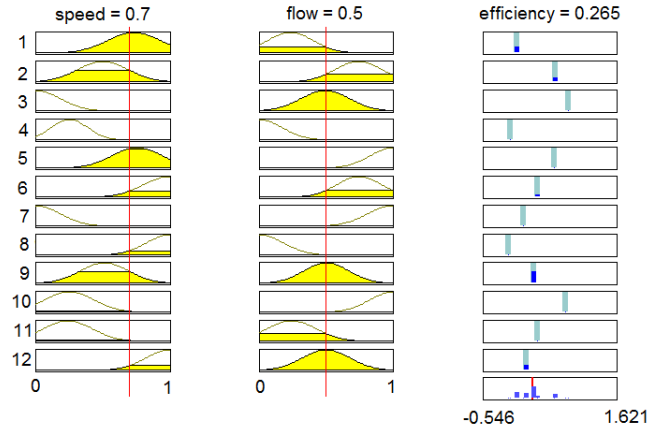


Fig. 7 Efficiency fuzzy model – Sugeno type with 12 rules

From the data in Table 1, using the Anfisedit tool of MATLAB, we compiled a static fuzzy system which approximates turbine efficiency, without the need to know its specific parameters, as illustrated in Figure 8.

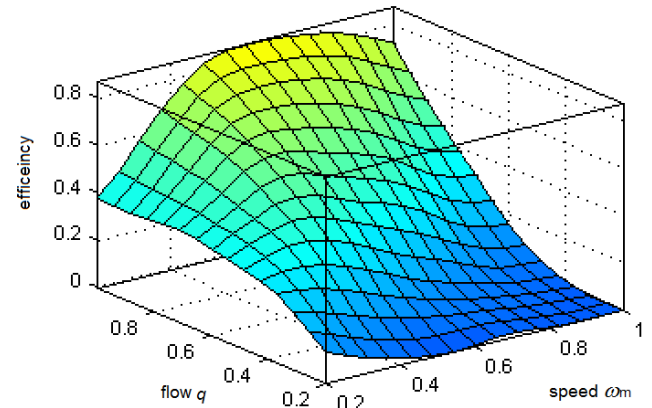


Fig. 8 Fuzzy image of turbine efficiency based on measured data

4. Discussion and conclusion

Computer models of power systems make it possible to significantly simplify and speed up their design, as well as save costs in their implementation. The computation of hydro turbine efficiency is analytically demanding and dependent on parameters which are often obtained only by theoretical estimation. Knowledge of the “efficiency image” of a particular hydro turbine is essential for its optimal power control.

The paper describes a fuzzy model of a hydraulic turbine that takes into account its efficiency. The model can be obtained from measured data on turbine flow and head. It is shown that the model obtained in this way practically equals the analytical models obtained by analytical methods. The model is applicable in practice in the design of optimal hydro turbine power control as well as in the effective determination of its wear rate.

Acknowledgement

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Development of a simulation model of an information-measuring system of electrical characteristics of the functional coatings of electronic devices

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Abstract: The article develops a simulation model (based on the MatLab Simulink mathematical processor) of an information-measuring system of electrical characteristics (residual surface electric charge, surface layer capacity, etc.) of functional coatings of electronic devices. The main purpose of the simulation model was to determine the rational parameters of measurement and control of the information-measuring system developed by the authors, which would simplify the process of setting up such a system, as well as explore the dynamic modes of its operation. The main advantage of the developed simulation model is the ability to conduct an interactive study of the operation of the information-measuring system under various, including limiting, modes. Tests of the simulation model of the information-measuring system made it possible to study its operation under various conditions and modes of the measurement experiment, as well as to virtually determine the rational operating parameters of such a measurement and control system. A satisfactory discrepancy of 8-11,5% was established for the experimental results compared with the data obtained analytically, which proves the correctness and adequacy of the compiled model.

Keywords: ELECTRONIC DEVICE, FUNCTIONAL COATING, INFORMATION-MEASURING SYSTEM, ELECTRICAL CHARACTERISTIC, SIMULATION MODEL

1. Introduction

Microelectromechanical devices and systems (MEMS), which have found wide application in many fields of human activity, are talking about their widespread introduction and further development [1-3]. However, further miniaturization of these devices, enhancement of their functionality and reliability, necessitates a more compact arrangement of the microcomponents of such MEMS in their working volumes. This, in turn, leads to the appearance of various electrical (yes, parallel conductors create a harmful capacity, whereas any closed circuit with current - forms an induction element on the surface of the device) and mechanical (tribological, dynamic) interference in such devices [4]. Such obstacles should be taken into account at the MEMS design stage. However, in most cases, such effects manifest themselves at the stages of pilot testing, or in general, the operation of products.

The main electrical characteristics that have a significant effect on the performance of functional coatings are surface electric charge and the electrical capacity of the surface layer.

Therefore, in order to quickly and efficiently determine the electric charge and residual electrical capacity in the surface layer of the dielectric, which is the basis of most MEMS, it is proposed to develop and use information-measuring systems (IMS), electrical characteristics of functional coatings of electronic devices. To create such an IMS, it is necessary to carry out its simulation in order to establish the operating parameters and modes of determining the electrical characteristics and eliminate the subjective errors that may occur at the stage of IMS design.

A number of scientists are engaged in the development of similar simulation models (however, in most cases for the determination of surface and bulk electric charge in dielectrics), namely: P.A.Taragaev, S.E.Kuznetsov, E.Sabat, A.Kuffel, and others [5-8]. However, in the works of these scientists the results of research of residual electric charges arising on the surfaces of dielectric materials under the products of microelectronics and microcircuitry (silicon, glass, sital, etc.) are insufficiently presented.

Therefore, the development of a simulation model of IMS electrical characteristics of functional coatings of electrical devices, which simplifies the process of tuning such IMS and the study of the dynamic modes of its operation is an issue that is still unresolved.

The purpose of this work is to develop a simulation model of IMS of electrical characteristics (residual electric charge, surface layer electric capacity, etc.), which is developed on the basis of a mathematical processor at Matlab Simulink and allows to conduct interactive studies of the functional coatings of electronic devices by this IMS at various (including boundary and boundary) critical) modes.

2. Mathematical description of IMS

Measurement of surface electric charge is carried out according to the scheme shown in Fig. 1.

The surface charge in the dielectric under the detector ring is described by the capacitance C_e . The capacitance C_b , which is connected in series with the capacitance C_e , is characterized by a dielectric region bounded by a detector ring. The C_a capacity is characterized by the part of the dielectric that is outside the action of the electric charge detector. The basic equations for calculating these capacities are as follows:

$$C_a = \frac{\varepsilon_{rd} \cdot \varepsilon_0 \cdot (S - S_q)}{d}, \quad (1)$$

$$C_b = \frac{(\varepsilon_{rd} + \varepsilon_{ra}) \cdot \varepsilon_0 \cdot S}{d}, \quad (2)$$

$$C_e = \frac{\varepsilon_{ra} \cdot \varepsilon_0 \cdot S_q}{d_n}, \quad (3)$$

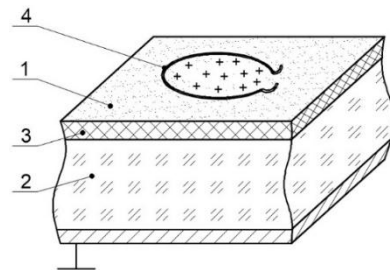


Fig. 1 Schematic diagram of surface electric charge measurement:
1 – functional coating (dielectric); 2 – dielectric base; 3 – an electrode;
4 – electric charge detector

where S is the surface area of the dielectric sample bounded by the detector ring; S_q is the surface area of the dielectric below the ring of the electric charge detector; d is the thickness of the dielectric; d_n is the thickness of the surface layer of the dielectric; ε_0 is the dielectric constant ($\varepsilon_0 = 8,8 \cdot 10^{-12}$); ε_{ra} - air dielectric constant ($\varepsilon_{ra} = 1$); ε_{rd} is the dielectric constant of a dielectric (for example, for glass, $\varepsilon_{rd} = 2,4$).

The maximum permissible voltage U_c and the total charge q measured on the dielectric surface are determined by the following formulas:

$$U_c = \frac{U_a \cdot C_b}{(C_a + C_b)}, \quad (4)$$

$$q = C_e \cdot U_c, \quad (5)$$

where U_a is the critical voltage of the dielectric breakdown (for glass 1 mm thick, $U_a = 5$ kV).

By formulas (1) – (3), the capacity of each dielectric section is calculated. Using the formulas (4) and (5), the values of the voltage under the detector on the dielectric and the charge generated on it

were determined. The results obtained at critical stress and different surface areas S_q (for dielectrics with parameters: $S = 0,05\pi$; $0,5\pi$; $\pi \text{ mm}^2$; $d = 1,2 \text{ mm}$; $d_n = 0,1 \text{ mm}$) are presented in Table 1.

Table 1: The parameters of the IMS measurement scheme for the critical breakdown voltage are calculated $U_a = 5$ kV

Sample number	The surface area of the sample bounded by the detector ring S [mm ²]	Capacity C_a [F]	Capacity C_b [F]	Capacity C_c [F]	Allowable voltage U_c [V]	Total charge q [Coul]
1	$0,05\pi \approx 1,57$	$2,5 \cdot 10^{-15}$	$3,9 \cdot 10^{-15}$	$1,2 \cdot 10^{-15}$	3043,6	$3,8 \cdot 10^{-12}$
2	$0,5\pi \approx 3,14$	$26,9 \cdot 10^{-15}$	$39,1 \cdot 10^{-15}$	$3,9 \cdot 10^{-15}$	2965,7	$11,6 \cdot 10^{-12}$
3	$\pi \approx 6,28$	$54,2 \cdot 10^{-15}$	$78,3 \cdot 10^{-15}$	$5,5 \cdot 10^{-15}$	2955,5	$16,3 \cdot 10^{-12}$

3. The procedure for developing a simulation model

Based on the obtained capacitance values (Table 1) characterizing the surface layer on the dielectric material, simulation of the distribution of the generated electric charge was simulated.

Compilation of the simulation model was carried out in order to determine the rational parameters of measurement and control,

developed by the authors of the information-measuring system, which would allow to simplify the process of setting up such a system, as well as to study the dynamic modes of its operation. For modeling, the modern simulation tool “MATLab Simulink” was used, which makes it possible to significantly simplify the analysis of modern IMS with different algorithms of their operation [9].

A simulation model of IMS created in the MATLAB Simulink environment is presented at Fig.2.

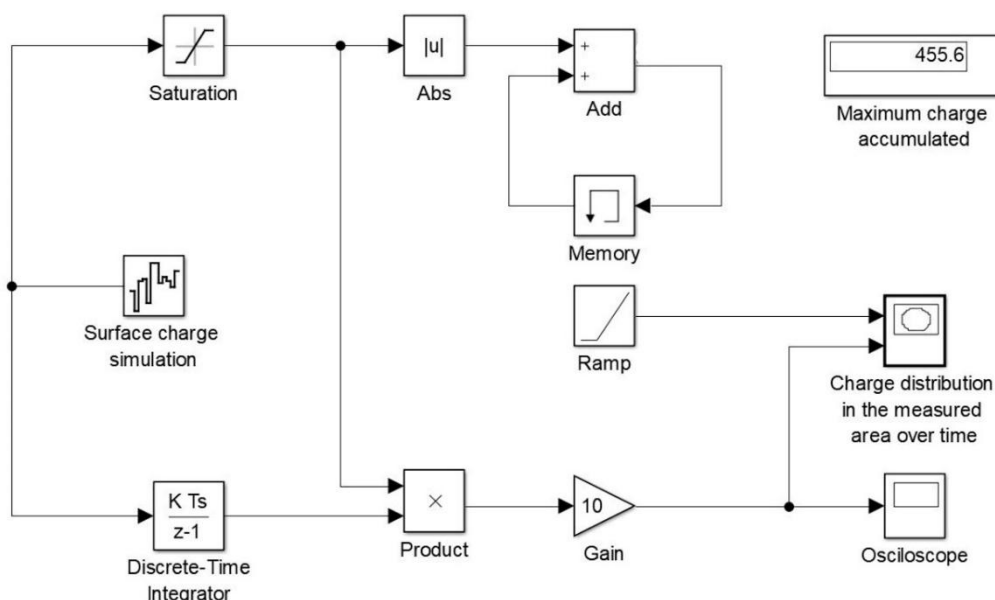


Fig. 2 IMS simulation model of electrical characteristics of functional coatings (“MATLab Simulink”)

The developed simulation model represents a set of blocks that model the operation of the IMS electrical characteristics of functional coatings. This model contains the following blocks: Surface charge simulation, which changes by stochastic law every 0,05 seconds, and whose main purpose is to simulate surface charge arising on the dielectric surface and to be measured; Discrete-time integrator simulates the change and distribution of electric charge over the surface over time; saturation unit allows to determine the limiting cumulative capacity of the measuring sensor; blocks of execution of mathematical operations (Abs, Product, Add) are intended for realization of simpler mathematical operations for calculation of measured electric charge and its distribution; The memory unit (Memory) accumulates and stores measured charge data; amplifier (Gain) enhances the measured value of the charge to a level sufficient to display it on information devices; a linear source (Ramp) signal source designed to form on the information devices a correct pattern of electric charge distribution over time; assignments of a number of information devices (Oscilloscope and others) - intended for information display of information on the maximum charge accumulated on the measuring section, its amplitude and frequency characteristics, as well as distribution over the measuring section over time [10].

The principle of operation of such IMS can be explained by the example of a simulation model developed and it is as follows.

At the initial stage, when there is no residual surface charge on the measuring sensor, the Abs and Product blocks are not signaled, respectively, the entire electrical circuit of the device is at a "pause" - it has a zero output level.

When raising the measuring sensor at a sufficient distance (determined by the amount of surface charge absorbed by the sensor necessary to ensure the minimum current level in the Saturation block) to the dielectric surface of the area to be measured, a minimum signal level appears on the corresponding blocks of mathematical operations (Abs Product) leading to a signal on the following blocks of the circuit. The Saturation unit is required to protect the developed IMS from accumulating too high an electric charge, which can lead to an electrical breakdown and, consequently, to the failure of the entire IMS. The values of the parameters of this block cannot be changed. On the other hand, the minimum value of the output parameter is the IMS sensitivity limit.

After the Saturation block, the output is output to the Add adder, which is connected in parallel with the Memory block.

In parallel to the link leading to the adder, the output signal is fed to the Product block, where it is multiplied by the signal received from the Discrete-time integrator block. The main purpose

of the Product unit is to approximate as closely as possible the simulation model of electric charge distribution over the selected area of the studied surface by scaling the stochastic electric charge generated in the Surface charge simulation in the time range specified by the Discrete-time integrator. After further amplification in the Gain block, the charge signal simultaneously arrives at Oscilloscope and the unit visualizes the amplitude-frequency characteristics of the received signal in time (Charge distribution in the measured area over time).

Oscilloscope block allows to obtain the distribution of electrical signal on the area of the study surface in terms of amplitude-frequency characteristics over time.

In order to obtain an energy distribution of the electrical signal, in addition to the amplified signal, a steady signal from the Ramp unit is sent to the Charge distribution in the measured area over time unit, which allows considering the distribution of the output signal in the visualization unit as a non-decaying signal that does not decrease over time due to electrical and energy costs for

the IMS elements. This minimizes the signal flow in the circuit of the measuring system and provides the approximate approximation of the simulated electric charge distribution to its true distribution over the surface.

Caution should also be exercised when setting coefficients (especially in the Saturation, Gain, and Discrete-time integrator blocks), as incorrectly set them can cause the system under test to become unstable, which is unacceptable.

4. Discussion of simulation results

The results of the simulation model of the electrical characteristics of the functional coatings allowed us to determine and investigate the change in the energy distribution of the electric charge at the test site of materials with different conductivity (Fig.3), as well as the change in the value of such charge over the time during which the study was conducted (Fig.4) .

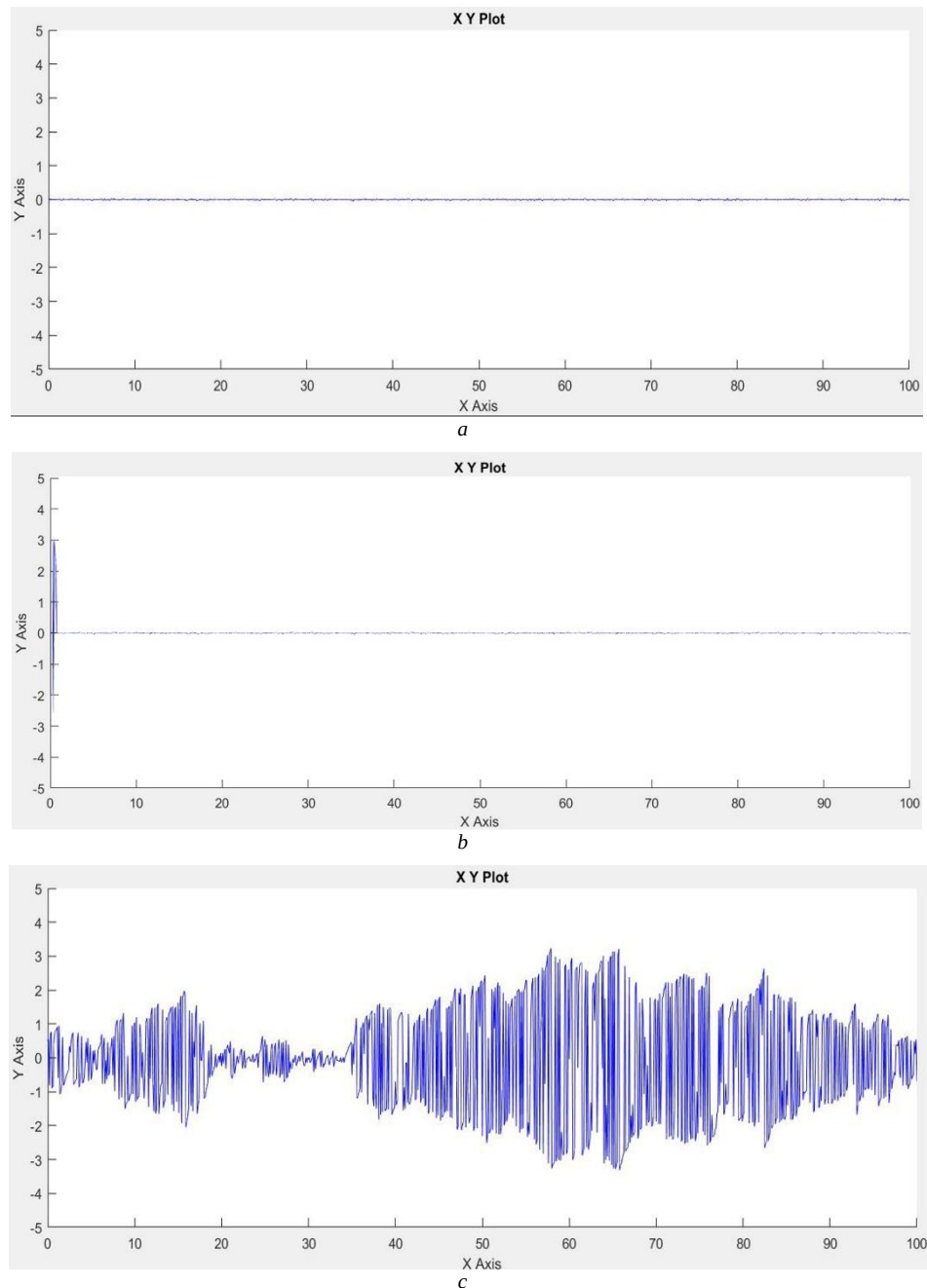


Fig.3 Changing the energy distribution of the electric charge at the test site of materials during the measurement time (10 s): a – on the conductor (grounded solid silver coating Ag-999 on a dielectric basis); b – on unpolarized dielectric (K-8 optical glass); c – on polarized dielectric (PZT-8 piezoelectric ceramics)

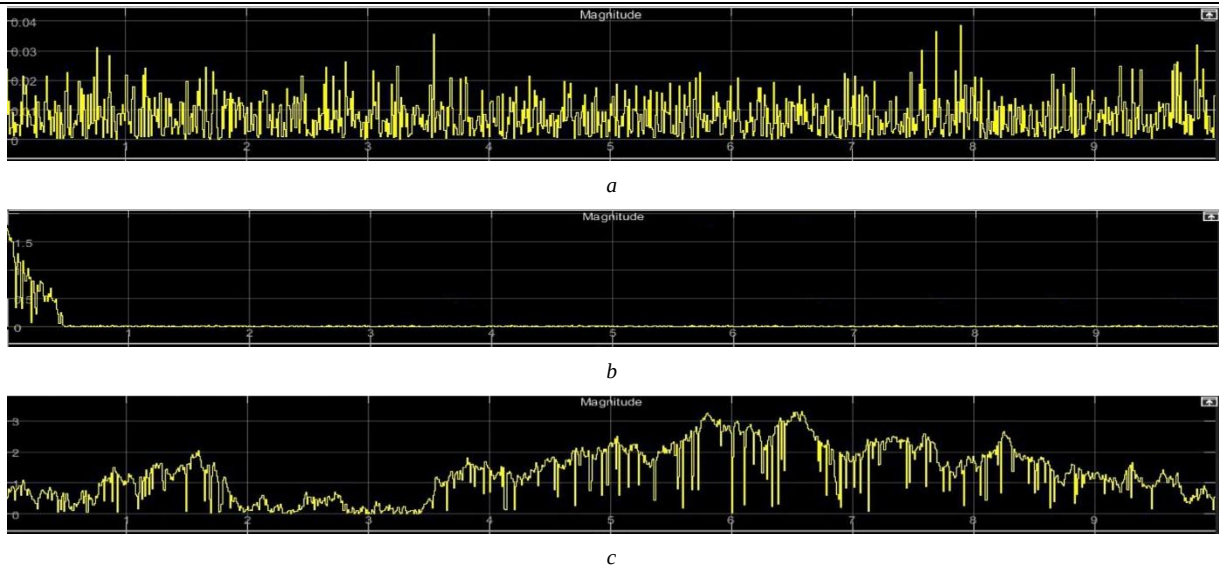


Fig.4 Changes in the value of the electric charge on the test section of the materials during the measurement time (10 s): a – on the conductor (grounded solid silver coating Ag-999 on a dielectric basis); b – on unpolarized dielectric (K-8 optical glass); c – on polarized dielectric (PZT-8 piezoelectric ceramics)

In the process of analysis of the change in the energy distribution of the residual electric charge in the studied area (Fig.3):

- in the case of a grounded conductor (Fig.3.a), there is no residual electrical signal (small disturbances of the distribution curve determine the noises caused by the electrical and magnetic action of the external fields, as well as interference with the electronic measurement circuit itself);

- in the case of unpolarized dielectric research (Fig.3.b), a sharp one-off perturbation of an electrical signal is observed, which immediately fades. Such perturbation corresponds to the residual surface charge accumulated on the surface of the unpolarized dielectric due to electrical and mechanical external action;

- in the case of a polarized dielectric (Fig.3.c), the charge and magnitude distribution of the electric charge are observed. However, as you can see, this distribution is periodic with a period $T \approx 0,6 - 1,5$ s (Fig.3.c, Fig.4.c) (this period corresponds to the range of operating parameters of the Discrete-time integrator), which can be explained by the accumulation and maintenance of residual charge on the surface of the material due to the electric field energy accumulated in the material, as well as the dynamically variable volumetric electrical anisotropy, which is a feature of polarized dielectric. Such a dynamic change in the electric field inside the dielectric also explains the different distribution of the electrical signal (which mimics the instantaneous charge on the test surface at a certain point in time) over time.

A similar situation for different materials according to the electrical properties of materials is observed in the study of changes in the actual value of the electric charge in the studied area (Fig. 4). The only difference in obtaining such dependencies is that when studying conductive material or unpolarized dielectric, there is no zero level of the output signal generated by the source of the additional Ramp signal.

In the process of research, analysis and subsequent comparison of the results of the simulation model with the data obtained experimentally for certain modes and time points of measurement, it was found that these results have a discrepancy of 8 – 11,5%, which proves the correctness and adequacy of the compiled simulation model.

5. Conclusion

Thus, the developed and tested simulation model of the information-measuring system of electrical characteristics (residual surface electric charge, electrical capacity of the surface layer, etc.) of functional coatings of electronic devices allows to study its operation, as well as to virtually determine the rational operating parameters of this system.

The tests of the simulation model of the information-measuring system made it possible to investigate its operation under different conditions and modes of conducting the measurement experiment, as well as to virtually determine the rational operating parameters of the measurement and control system.

A satisfactory discrepancy of 8 – 11,5% for the experimental results compared to the data obtained by the simulation modeling method was established, which proves the correctness and adequacy of the developed simulation model.

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Harmonic regression based on the output signals of vibration processes

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Abstract: The article deals with probabilistic and statistical models of vibration processes. The covariance moment is considered. A dimensionless normalized covariance is introduced. To evaluate these theoretical characteristics, we used a statistical set of observational data. Observations that are cleared of measurement errors are combined into a sample of a specific volume. Random processes in vibration analysis are considered in detail. The characteristics of random processes are given: attenuation with a random beginning, a linear process with a random slope, and a harmonic process with a random amplitude. A statistical model of a linear system based on observations of input and output vibration processes is considered in detail.

Keywords: VIBRATION, COVARIANCE, MODELS, SIGNALS

INTRODUCTION

Vibration processes in natural and technical systems are complex and depend on a variety of different causes, so it is reasonable to consider them as random processes that are not clearly predictable, but have certain averaged characteristics. The reasons for this approach to vibration processes are: first, it is the broadband frequency of these processes, and secondly, it is difficult and impossible to take into account all the factors in the dynamic analysis of vibration devices. The probabilistic approach allows us to take into account the influence of secondary factors, which are quite rough, but quite capable of assessing their impact. The deterministic input and output processes of devices are superimposed on purely random processes-interference with certain characteristics.

Preconditions and means for resolving the problem

Events in observations are random and unpredictable. If events in experience occur massively and under the same conditions, then the statistical probability of these events is characterized by the relative frequency of their repetition.

$$p_i \approx v_i; \quad v_i = n_i / n; \quad n = \sum_i n_i,$$

where n_i is the number i -events in a series of n observations. The set of all possible events in the experience is united by the concept of a random variable. Random variables can be discrete or continuous, but the vibrational and vibrational phenomena discussed below are continuous and, therefore, we consider only continuous quantities. To describe them, you must:

$$\Omega_x = (a, b), \quad -\infty < a < b < +\infty,$$

$$F_x(x) = P(X < x) \text{ and } f_x(x) = \frac{dF_x}{dx} \geq 0.$$

Here a, b is the boundary of a simply connected finite or infinite continuous domain, $FX(x)$ is the random variable distribution function that determines the probability that the random variable takes values smaller than its argument x , and $fX(x)$ is the probability distribution density function of the random variable. Statistical dependence of random variables occurs when the distribution of one random variable depends on what values another value takes.

Experimental stand

Let there be two random variables X_1 and x_2 with their distributions, but they are statistically dependent. Then their distribution can be described using the function of the joint distribution or conditional distribution function. $f_{x_1}(x_1|x_2), f_{x_2}(x_2|x_1)$. The correlation moment is introduced as a numerical characteristic of the degree of dependence of random variables:

$$K_{12} = \text{cov}(X_1, X_2) = M[\tilde{X}_1 \cdot \tilde{X}_2] = M[X_1 \cdot X_2] - M[X_1] \cdot M[X_2] \quad \text{To}$$

calculate it, you need to know the joint distribution of SV, but it is known that the covariance moment has the properties:

- if X_1 and X_2 are independent SV, then

$$\text{cov}(X_1, X_2) = 0$$

- if X_1 and X_2 are linearly dependent SV, then

$$\text{cov}(X_1, X_2) = \sigma_{x_1} \cdot \sigma_{x_2}$$

$$\text{cov}(X_1, X_1) = D_{x_1}, \quad \text{cov}(X_2, X_2) = D_{x_2},$$

$$M[X_1 \cdot X_2] = m_{x_2} \cdot m_{x_1} + \text{cov}(X_2, X_1)$$

$$D[X_1 + X_2] = D_{x_1} + D_{x_2} + 2\sigma_{x_1}\sigma_{x_2}\text{cov}(X_2, X_1)$$

A dimensionless normalized covariance (or correlation coefficient) is introduced:

$$R_{12} = \frac{\text{cov}(X_1, X_2)}{\sigma_{x_1} \cdot \sigma_{x_2}}$$

moreover, it is always performed for it $-1 \leq R_{12} \leq 1$. If several random variables $X_1, X_2, \dots, X_n$ are observed simultaneously in the experiment, then symmetric matrices of covariance moments K_{ij} or correlation coefficients R_{ij} are introduced :

:

$$K_{ij} = \begin{pmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & K_{2n} \\ \dots & \dots & \dots & \dots \\ K_{n1} & K_{n2} & \dots & K_{nn} \end{pmatrix}; \quad R_{ij} = \begin{pmatrix} R_{11} & R_{12} & \dots & R_{1n} \\ R_{21} & R_{22} & \dots & R_{2n} \\ \dots & \dots & \dots & \dots \\ R_{n1} & R_{n2} & \dots & R_{nn} \end{pmatrix}$$

To evaluate these theoretical characteristics, you can use a statistical set of observational data for this value or several values. These observations, cleared of measurement errors, are combined into a sample of a certain volume n :

$$x_v = \{x_1, x_2, \dots, x_n\} = \{x_i; i = \overline{1, n}\}.$$

You can use the sample to build its main characteristics (moments), such as:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad \text{- the sample mean,}$$

$$\bar{D}_x = \overline{(x_i - \bar{x})^2} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \quad \text{- is the sample variance,}$$

$$\bar{\sigma}_x = \sqrt{\bar{D}_x} \quad \text{- is the sample standard deviation.}$$

When two or more random variables are observed simultaneously

$$x_v = \{x_{i,j}; j = \overline{1, m}; i = \overline{1, n}\}, \quad \text{multi-factor samples are}$$

obtained, which can be used to construct sample covariances $\bar{K}_{jk}$

and correlation coefficients $\bar{R}_{jk}$:

$$\bar{K}_{jk} = \overline{x_j \cdot x_k} - \bar{x}_j \cdot \bar{x}_k; \quad \bar{R}_{jk} = \frac{\overline{x_j \cdot x_k} - \bar{x}_j \cdot \bar{x}_k}{\bar{\sigma}_{x_j} \cdot \bar{\sigma}_{x_k}}, \quad \text{where}$$

$$\bar{x}_j = \frac{1}{n} \sum_{i=1}^n x_{ji}, \quad \overline{x_j \cdot x_k} = \frac{1}{n} \sum_{i=1}^n x_{ji} \cdot x_{ki}.$$

The constructed sample characteristics are random variables, since they are obtained from the result of a random selection of observations.

In addition to the introduced functions, the random process is characterized by a correlation function that shows the correlation between the time sections of the random process

$$K_x(t, t') = \text{cov}(X(t), X(t')) = M[\tilde{X}(t) \cdot \tilde{X}(t')],$$

$$R_x(t, t') = K_x(t, t') / \sigma_x(t) \cdot \sigma_x(t')$$

Here is a $\tilde{X}(t) = X(t) - m_x(t)$ centered random process with zero mathematical expectation, a $R(t, t')$ centered random process with zero mathematical expectation, and a normalized correlation function. Correlation functions have the following important properties:

$$K_x(t, t) = D_x(t), \quad K_x(t, t') = K_x(t', t), \quad R_x(t, t) = 1,$$

$$-1 \leq R_x(t, t') \leq 1$$

The meaning of the correlation function is illustrated in figure 1, which shows processes with high correlation and low correlation. We see that with approximately the same mathematical expectation and approximately equal variance, the nature of random processes differs sharply.

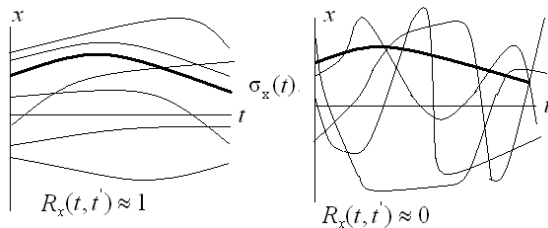


Fig. 1. Illustration of random processes with various correlation functions.

Consider several examples of random processes whose parameters are the normal random variable $X=N(m, \sigma)$ and the constants $m, \sigma, \alpha, \omega, \dots$. Let's calculate the characteristics of these processes using the properties of expectation and variance

Example 1. ESP $Y(t) = X \cdot e^{-\alpha t}$ - attenuation with a random start..

$$m_y = M[Y(t)] = M[X] \cdot e^{-\alpha t} = m \cdot e^{-\alpha t},$$

$$D_y = D[Y(t)] = D_x \cdot (e^{-\alpha t})^2 = \sigma^2 e^{-2\alpha t},$$

$$K_y(t, t') = M[\tilde{Y}(t) \cdot \tilde{Y}(t')] = M[\tilde{X} e^{-\alpha t} \cdot \tilde{X} e^{-\alpha t'}] = \sigma^2 e^{-\alpha(t+t')},$$

$$R_y(t, t') = K_y(t, t') / (\sigma(t) \sigma(t')) = 1.$$

Example 2. ESP $Y(t) = X \cdot t + \alpha$ linear process with random slope

$$m_y = M[Y(t)] = M[X] \cdot t + \alpha = m \cdot t + \alpha,$$

$$D_y = D[Y(t)] = D_x \cdot t^2 = \sigma^2 t^2,$$

$$\tilde{Y}(t) = Y(t) - m_y(t) = (X - m_x) t = \tilde{X} t,$$

$$K_y(t, t') = M[\tilde{Y}(t) \cdot \tilde{Y}(t')] = M[\tilde{X} t \cdot \tilde{X} t'] = \sigma^2 t \cdot t', \quad R_y(t, t') = 1.$$

Example 3. ESP $Y(t) = X \cdot \sin(\omega t + \alpha)$ is a harmonic process with a random amplitude.

$$m_y = M[Y(t)] = m \cdot \sin(\omega t + \alpha),$$

$$D_y = D[Y(t)] = \sigma^2 \cdot \sin^2(\omega t + \alpha) =$$

$$\tilde{Y}(t) = Y(t) - m_y(t) = (X - m_x) \sin(\omega t + \alpha) = \tilde{X} \sin(\omega t + \alpha)$$

$$K_y(t, t') = M[\tilde{Y}(t) \cdot \tilde{Y}(t')] = M[\tilde{X} t \cdot \tilde{X} t'] = \sigma^2 \sin(\omega t + \alpha) \sin(\omega t' + \alpha),$$

$$R_y(t, t') = 1$$

The task of statistical analysis of the data obtained by the measuring stand is to analyze and establish dependencies between the measured data and the parameters of the device under study (hydraulic support), which is usually a complex multiphase and multiresonance system. The main task of statistical analysis is to establish the relationship between input X and output Y measured values of harmonics amplitudes and determine the form of this relationship in the form of regression functions:

$$Y_{\text{шина}}^t = \Phi(f_0, X_{\text{шина}}^t), \quad Y_{\text{канал}}^f = \varphi(f_0, X_{\text{канал}}^f)$$

The task of correlation analysis is to establish the relationship between the measurements in the channels and in the tires at the entrance and exit of the vibrostand, to determine their rigidity and significance.

For the purposes of correlation analysis, the correlation matrix between the tires is

$$\text{calculated: } R_{ij}^f = \frac{\overline{x_i^f \cdot y_j^f} - \overline{x_i^f} \cdot \overline{y_j^f}}{\sqrt{D_{xi}^f D_{yj}^f}},$$

Here is the $\overline{x_i^f \cdot y_j^f} = \frac{1}{L} \sum_{k=1}^L x_{ki} \cdot y_{kj}$ - average product of tires.

Each element of this matrix is a pair correlation coefficient between the input i-bus and the output j-bus. To determine the significance of each coefficient, the student's observed and critical statistics are calculated.

$$St_{ij}^f = R_{ij}^f \sqrt{\frac{n-L-1}{1-(R_{ij}^f)^2}},$$

$$St_{\text{крит}}^f = \text{СТБЮДРАСПОБР}(\alpha, n-L-1)$$

Where α is the significance level of the tested hypotheses about the absence of a connection between the tires (the acceptable risk of rejecting the actually existing tire connection). The student's criterion for the significance level sets a threshold $St_{\text{крит}}^f$, which is exceeded by the observed statistics indicates the significance of the correlation between the tires. The rigidity of the connection is the relation

$$Sz_{ij}^f = |St_{ij}^f| / St_{\text{крит}}^f,$$

for example, the greater the unit, the more significant the connection between the tires.

To calculate the significance and rigidity of correlations, it is necessary that the value of degrees of freedom .

$$\nu = n - L - 1 \geq 3.$$

The task of regression analysis is to establish the form of communication between correlating channels and buses. The analysis is based on a linear regression model:

$$\bar{y}_j^f = b_{0j}^f + b_{1j}^f \bar{x}_1^f + b_{2j}^f \bar{x}_2^f + \dots + b_{nj}^f \bar{x}_n^f + \varepsilon_j^f$$

Here is the $\bar{y}_j^f = (y_{1j}, y_{2j}, \dots, y_{sj}, \dots, y_{24j})^T$ measurement vector of the output j-bus,

$\bar{x}_i^f = (x_{1i}, x_{2i}, \dots, x_{ki}, \dots, y_{24i})^T$ vector of input bus measurements,

$\bar{b}_j^f = (b_{0j}^f, b_{1j}^f, \dots, b_{ij}^f, \dots, y_{nj}^f)^T$ - vector of linear regression

coefficients for the j-bus $\bar{\varepsilon}_j^f = (\varepsilon_{1j}^f, \varepsilon_{2j}^f, \dots, \varepsilon_{sj}^f, \dots, \varepsilon_{24j}^f)^T$ - vector of errors (residuals) of the regression.

Linear regression is constructed using the least squares method, according to which the average value of the error vector $\bar{\varepsilon}_j^f = 0$, and its variance $D(\bar{\varepsilon}_j^f)$ is minimal among all linear dependencies on $\bar{y}_j^f$ or $\bar{x}_i^f$. According to the method of the regression coefficients are calculated $\bar{b}_j^f = (X^N \cdot X)^{-1} \cdot (X^T \cdot \bar{y}_j^f)$, as the vector $\hat{y}_j^f = (\bar{x}_i^f)^T \cdot \bar{b}_i^f$ - is explained in part based on the output j-

bus from the input bus, and the regression $\bar{y}_j^f = \hat{y}_j^f + \varepsilon_j^f$ is represented as the sum of the explained and unexplained part based on an output bus from the input.

The share of the variance of the explained part in the regression in the total variance of the observed output is called the regression determination coefficients:

$$R_j^2 = D(\hat{y}_j^f) / D(\bar{y}_j^f),$$

the root of it is the $R_j = \sqrt{R_j^2}$ - multiple correlation coefficient, a $\bar{R} = \sum_{j=1}^m R_j$ and the average multiple correlation

coefficient. To check the significance of the determination coefficients, the observed Fisher statistics vector and its critical value are calculated

$$F_j = \frac{R_j^2 \cdot (n - L - 1)}{1 - R_j^2}, F_{jspum} = FOBP(\alpha, n, n - L - 1).$$

If the observed Fisher statistics exceed the critical threshold, it indicates the significance of the constructed regression level for the output j-bus. Built significant regression model in fact explains the linear part of the transformation of the spectrum of the input signal vibrocore in the spectrum of the output signal, which is characteristic of the work supports. The model can be used to predict the conversion of a given arbitrary input bus to an output bus $\bar{X}_p^f$:

$$\hat{y}_p^f = (\bar{x}_p^f)^T \cdot \bar{b}^f.$$

Here is an example of a statistical analysis of tests in the 6 Hz range. The spectra of input and output signals obtained in measurements when the hydraulic support is excited on a vibration stand.

The calculations for analyzing dependencies between measurements in tires are shown below. A bus correlation matrix and linear regression coefficients between input and output buses are constructed.

A straight line of averaged regression is given against the background of the correlation field of measurements. These calculations show that there is a fairly tight correlation between the measured input and output buses in terms of the significance level $\alpha=0.05$. The regression coefficients averaged $\bar{b}_i^f$ over the measurements allow us to build a significant linear trend, as shown in Fig.2 the Trend has a positive slope with coefficients of 0.9, which indicates a significant average damping of input vibration excitations across all frequency channels.

Conclusion

These calculations show:

1. the Harmonic components of the spectrum in the 50 Hz range represent a weakly correlated array with a multiple correlation coefficient of 0.5. Above 50 Hz, the correlation Coefficient reaches a level of the order of 0.8 – 0.9.

2. the Regression dependence of the output signals on the input harmonic components above 50 Hz can be approximated by a linear dependence. At lower harmonics, it is not possible to approximate the correlation between the signals.

3. the damping Effect is observed only on harmonics above 50 Hz, on average by 10 decibels. On the lower harmonics is not observed.

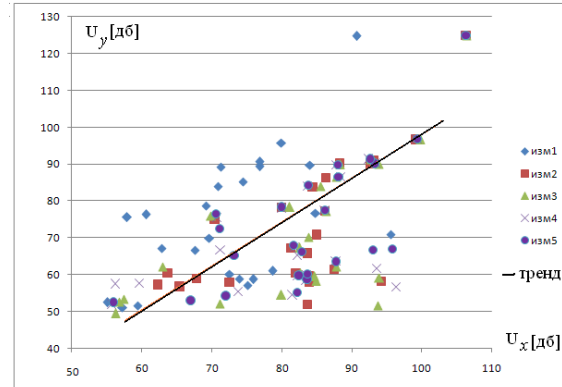


Fig. 2. Correlation field of 5 measurements in 24 channels when the vibration stand is excited at a frequency of 6 Hz

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