Mathematical modeling of automated production systems

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Abstract: *The paper theoretically substantiates and practically implements mathematical methods of modeling production systems, proves the adequacy of mathematical models and optimization methods, as well as selected rational optimization methods and created software that implements the selected methods in practice. The analysis of methods of modeling and optimization of production technological systems is performed, the most effective methods of solving these problems for discrete production are selected. The main provisions of infinite-valued logic and ordinal determinants, on which the structural-logical approach to the study of complex systems, which include production systems in instrument making, are considered.*
KEY WORDS*: production system, mathematical modeling, optimization, structural-logical method, the method of branches and boundaries.*

1. Introduction

Development of methods for modeling and optimization of production systems is a promising scientific and technical task, which is of great importance for creating the optimal structure of the production system, which will reduce production costs without investing significantly in re-equipment, development of technological equipment [1–3].

Solving this problem involves various aspects of production, one of the main ones being the manufacturing time of the product. Along with the improvement of technological processes and material and technical base, the optimal loading of the production system equipment allows to significantly reduce the manufacturing time of products by minimizing equipment downtime. Thus, the optimization of the loading of technological equipment is an urgent problem in modern instrumentation. This problem is solved by modeling the operation of production systems [3-5].

To date, there is no single method of modeling production systems in general, i.e. those that would adequately and accurately describe the parameters of production systems of different structures, so to model a particular type of system, or at least several similar types of structures use different mathematical models [6–9].

To model the loading of technological equipment of production systems, it is proposed to apply the structural-logical method, as it can be used in production systems of various structures, namely: serial, parallel, parallel-serial, serial-parallel. In addition, this method is easily formalized and programmatically implemented [10, 11].

2. Fundamentals of structural and parametric optimization of production systems

When designing technological processes and operation of production equipment, the main set of tasks is related to the choice of the best version of the production system on a set of technical and economic indicators. These problems have several varieties, which correspond to three levels of optimization [11]:

- the first level of optimization is to choose the best technical idea or principle of operation of the projected object;
- the second - in search of the best structure or scheme within the chosen principle of action;
- the third - in determining the best values of parameters for the selected structure (scheme).

The division into three levels is conditional, and it is impossible to draw a strict line between them. The expediency of such a division is due to the need to distinguish between simpler and more complex and time-consuming tasks, which belong to different stages of technology design and at the same time differ significantly in the methods of solution.

First-level tasks are specific to the external design stages and are solved using heuristic programming approaches and methods. Modern systems of automated design of technological processes are focused on the stages of internal design. In this case, the problems of the second and third level are typical, which are called structural and parametric optimization problems, respectively.

When solving these problems, the term production technological system means a system whose purpose is to manufacture a certain number of products in a certain time, using a clearly defined amount of equipment. Strictly speaking, in this case we will describe the production system in terms of queuing theory.

Here, the system, which in connection with possible technical, economic and other applications, is considered structural, in contrast to the abstract system, which is the subject of study of general systems theory. An element of the system is an arbitrary indivisible object in this problem.

The indivisibility of an element is conditional and is caused by the desire to simplify the problem. It is possible that the transition from one task to another will require the decomposition of one element or, conversely, the combination of several elements into one. An object that does not decompose in this but may decompose in another problem is a block.

The system is a defined set of blocks (elements), combined by some set of connections to achieve a common goal. In service systems, this goal is to perform a given set of jobs. Each job consists of a number of different operations performed by the respective blocks.

The system as a whole provides a consistent passage of each job through certain blocks, a set of operations in which ensures the performance of this job. We believe that the operation is the smallest, indivisible part of the job.

In this regard, in the study of queuing systems, quantitative characteristics (parameters) of operations must be specified.

These parameters can be set by a matrix:

$$A = \|a_{i,j}\|,$$

where $a_{i,j}$ is the quantitative effect of the i-th operation in the *j*-th job.

Values $a_{i,j}$ – can mean either the cost of time resources, or income and so on. The specific content of these values affects the formulation of problems of studying systems. The parameters $a_{i,j}$ express the quantitative characteristics of the queuing system as a whole.

The functioning of an arbitrary system is a set of rules that determines what the system must do to achieve its goal; these rules do not necessarily use the knowledge of building a system. In general, according to the level of abstraction adopted in the study of the system, its operation can be described by more or less detailed rules.

Thus, the operation of queuing systems is usually described at the level of system structure. Under the structure of an arbitrary system we understand the set of a set of blocks (elements) and a set of connections between them. The structure of the service system is usually represented by a digraph, in which the vertices denote the blocks (elements) of the system, and the arcs - the directions of movement of works from block to block in the process of their execution.

To quantify the degree of achievement by an arbitrary system of the goal set before it, various characteristics of its functioning are introduced. Each individual characteristic evaluates one side of the system, and only together they allow to assess how the system has achieved its goals.

Production systems form structures that can be serial, parallel, parallel-serial, serial-parallel, where □ is the block of the production system, i.e. the machine [10].

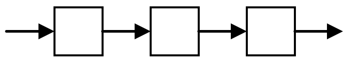

Fig. 1. An example of a serial structure of the system.

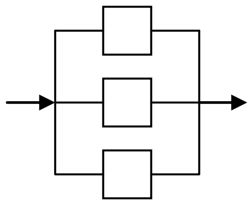

Fig. 2. An example of a parallel system structure.

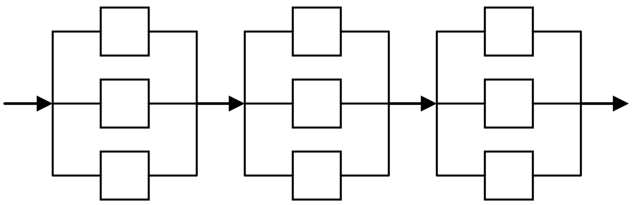

Fig. 3. An example of a parallel-serial structure of the system.

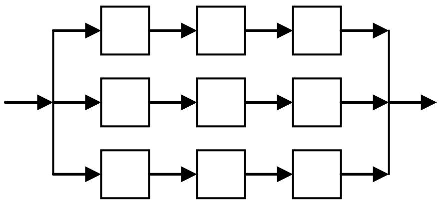

Fig. 4- Example of a serial-parallel structure of the system.

The quality of operation of any system can be assessed using certain characteristics. In this case, each performance characteristic, as a rule, evaluates a separate component of the quality of system operation, although generalizing characteristics are possible. Each class of systems has its own types of performance characteristics, although there are universal characteristics that apply to all classes of systems. Below is a short list of types of characteristics with an indication of the components of the quality of systems functioning evaluated by them. Some of these characteristics are universal, others belong to separate classes of systems.

The performance characteristics of the systems are universal. They evaluate the speed of operation of the systems. As such characteristics in service systems usually use the total execution time of the system of a fixed set of works or the number of works performed per unit time (speed of operation of the system). In other systems, performance is assessed similarly.

The boot characteristics of the systems are also universal. They assess the degree of employment of individual units and the set of units in achieving the goal of the system. As such characteristics in service systems apply a fraction of time during which the block is occupied with performance of works (average loading of the block); average for all blocks the share of employment time (average system load), etc. In other systems, the load is estimated similarly. The load is related to the performance of the systems as follows: the higher the load, the (other things being equal) the higher the performance.

The stability characteristics of the systems quantify the ability of any system to continue to function properly (albeit with poorer performance, load, etc.) in the event of block failures. To assess the stability usually use the maximum number of blocks, with the simultaneous failure of which still retains the correct operation of the system. Stability, as well as speed and load, is a universal characteristic of the system.

The cost characteristics in the systems show the generalized costs (financial, material, energy, etc.) that are necessary for the normal functioning of the system. For service systems, such characteristics may be the total cost of the system to perform a fixed set of works; specific costs per work performed or per unit of system operation time, etc. Costs are not a universal characteristic of system operation. They relate mainly to technical, organizational, economic and production systems.

Characteristics of profit in systems estimate the generalized profit (financial, material, etc.) which appears as a result of system functioning. In service systems, these characteristics may be the total income that arises as a result of performing a fixed set of works; specific income per work performed or per unit of system operation time, etc. Profit is not a universal characteristic of systems. It applies only to organizational and economic systems.

The characteristics of the efficiency of systems are generalizing characteristics that bring together the assessments of all the above individual components of the quality of systems. The efficiency of the system shows the degree of achievement of the goal of the system. The characteristic of efficiency is often taken as the probability of achieving the goal of the system. Efficiency is a universal characteristic of the functioning of systems.

There are a number of limitations to the structure of the input data, namely:

- completely fixed structures of systems of the four above-mentioned types are taken into consideration: serial, parallel, parallel-serial and serial-parallel;
- the system has a completely fixed number of blocks, in addition, usually the number of blocks is determined by an integer;
- the system receives a clearly defined number of works, the number of which is also an integer;
- each block of the system performs a certain job for a certain time, then the matrix of time of work is the input data for calculating the parameters of the system.

The following assumptions should also be taken into account, namely:

- absolute reliability of all blocks of the service system;
- for each work is set many components of its operations, the order of execution of which is proposed by some given order ratio, and each operation is not more than one previous and one subsequent operation;
- no more than one operation can be performed simultaneously in any work;
-_at any time, each block can perform no more than one operation;
- the operation started by any block is performed to the end.

To model the load of production systems it is necessary to solve a number of problems, namely: system representation, system calculation, system analysis and synthesis.

Representation of the system is the presentation of all given information about the system in a compact form, which facilitates the description of the system, as well as the formulation and solution of problems of its calculation, analysis and synthesis.

The calculation of the system is to determine the expressions or numerical values of various characteristics of the system according to the given structure of the system, the mode of its operation, the order of work entering it, and the numerical value of the parameters of its blocks.

The analysis of the system is to determine the type of dependence of various characteristics of the system on its structure, the mode of its operation, the order of execution of works in it, the values of the parameters of its blocks. The analysis allows to establish the degree of influence of these factors on the characteristics of the system. Due to this, it is possible to calculate the accuracy (stability) of the system when changing its parameters and structure.

The synthesis of the system is to find its structure and / or the order of execution of works in it on a given set of works to be performed, the values of the parameters of the blocks forming the structure, and the required values of the various characteristics of the system.

As an adequate mathematical apparatus for describing systems, infinite-valued logic is chosen - the theory of functions, the domain of which is a set of real numbers, and the main operations - disjunction, ie maximum, and conjunction, ie minimum, which are similar in properties to binary disjunctions and conjunctions. Using the structural representation of the studied system, it is possible to express any of its characteristics through similar characteristics of the blocks, using these logical operations.

This opens the way for the uniform solution of problems of calculation, analysis and synthesis. As an adequate mathematical apparatus for a compact representation of a given information about a complex system selected logical determinants - a kind of extreme numerical characteristics of rectangular matrices, expressed through the elements of the matrix by operations of infinite logic and their properties resemble ordinary determinants of square matrices.

To optimize the loading of technological equipment of the production system (ie to find the optimal order of launching parts into the system) it is advisable to use the method of branches and networks, because it is the most flexible. This method includes standard elements, namely: branching, estimation and clipping, combined into a typical optimization algorithm, consisting of a single first step and repeated the second step. The specificity for each type of system is manifested only in the form of an evaluation function.

The study of any real system is carried out on a model that is a simpler system than the original, while maintaining its essential features. Models are divided into physical and abstract, which in turn are divided into conceptual and mathematical.

Conceptual models characterize only causal relationships that are essential to describe the functioning of the system; while leaving aside the quantitative and qualitative aspects of this description.

The mathematical model is based on the conceptual and characterizes the functioning of the system in quantitative and qualitative terms.

The mathematical model of the production or technological system itself is described as follows.

There are some set of products B that need to be manufactured, and some set of units of technological equipment (machines) M, as well as workers for the manufacture of these products. Each product is characterized by its technological route, ie the sequence of passage of a certain subset M, which leads to the finished product.

The system in this case is a set of sets B and M, and its purpose – the manufacture of all products from the set B. The machines are blocks of the system, and various products - robots to be performed in this system. The technological route of the product is an ordered set of operations, which consists of the manufacturing process of this product. Each operation is performed by a separate machine (block). The effect of the operation is measured by the time required, costs, and so on.

All machines with connections between them form the structure of the system. The directional connection between the two machines shows that after performing the operation in one of them, the operation can be performed in the other.

The functioning of the system is to lay routes in its structure, in accordance with the specified technological routes of individual products, and the agreed passage of certain routes of all products until their full manufacture.

The task of structural modeling and optimization is to investigate the mathematical apparatus, which would adequately describe the structure of the production system and would allow to algorithmize the procedures for calculating the most important parameters of this system.

Usually in detailed consideration, this problem is divided into several subtasks:

a) analyze the work of production equipment for different structures and explore the features of modeling each of them;

b) choose a method of modeling the equipment of the production system, which would describe as fully as possible the parameters of the system and would allow to obtain its adequate model;

c) choose a method of optimizing the operation of equipment that would meet the given constraints and would have the best possible convergence;

The classification of systems aims, if possible, to unify the quantitative characteristics of systems, the tasks of their study and methods of solving these problems.

The classification of systems can be based on various features, so the paper considers only some of them, namely - on the structures of deterministic queuing systems.

Production systems, depending on the conditions of a particular production can have a series, parallel, parallel-series or series-parallel structure, and in the case of the intended return of the finished product to correct the defect - the structure of the feedback system. Thus, production systems can have a diverse structure.

According to the mode of operation of the units, all systems are divided into: systems with single-program and multi-program mode; static and dynamic; without downtime and with downtime of blocks. Systems with single-program (multi-program) mode of operation include those in which only one action (several actions) is performed simultaneously (for service systems - one job (several works)).

Static are systems in which during the execution of the purpose of the system (in service systems - during the execution of a given set of works) each block performs its operation not more than once, dynamic - those in which there are blocks that perform their operations for a specified time at least twice.

According to the mode of submission of works, service systems are divided into: systems with waiting and without waiting; systems with free and forced on time of receipt of works; conveyor and non-conveyor; systems with stationary and non-stationary workflow; systems with one and with several works.

Waiting systems (without waiting) include those in which the work after the previous operation is waiting for the release of the unit performing the next operation (immediately transferred to the free unit performing the next operation).

In the system with free time of work receipt all works are available already at the initial moment $t = 0$ of start of system therefore the moments of actual occurrence of works are not specified. The works are entered into the system for their execution in accordance with the presence of free blocks in it.

In systems with forced time of work, the latter appear at specified points in time, in the general case later than the moment $t = 0$ of the system startup. Here works are submitted to the system only after its appearance and in the presence of free blocks in it. Conveyor type includes systems in which the order of work is the same for all operations, and non-conveyor - other systems.

In systems with a stationary flow of work, the flow parameters do not change over time, and in systems with a non-stationary flow - change.

According to the discipline of service of works waiting in blocks, it is possible to allocate systems of service of types: 1) "came first - served first", 2) "came first - served last", 3) "came first - served average in order". 4) in ascending order of some indicator (usually the time required to complete this work in the system), 5) random service.

The scheme of the system in general is very important for its correct modeling, because often the methods of mathematical modeling are able to adequately describe only a certain, one structure and are not acceptable for modeling others.

Structural optimization of the technological systems created on the basis of the production equipment is connected with variation of their components: types and options of manufacturing of products, types, sequence and options of concentration of technological operations.

Each variant of the system is formed of elements $w_g, g = \overline{1,G}$, indicators of which $f_{\omega g}$ have different meanings for implementation: $w_g = \overline{1,W_g}$.

The set of elements w_g and the relationship between them characterize the structure of the technological system, and the task of optimization is to choose a set of combinations of elements w_g of variant s^*, which is optimal in terms of the specified technical and economic requirements: ${F_i}^*, i = \overline{1,I}$.

Possibilities of change of the listed components are defined by a variety of two sets and communications between them:

- production objects

$$R \subset\times \{r_j : j \in J\},$$

where $\subset$ – sign of relations;

$\times$ – sign of a Cartesian multiplication;

$r_j = \overline{1,R_j}$ – sets of numbers of production objects j-th name;

J – set of indexes for names of production objects;

– technological operations

$$V \subset\times \{v_t : t \in T\},$$

where $v_t = \overline{1,V_t}$ – set of numbers of technological operations t-th name;

T – set of name indexes for technological operations.

Thus, the structure of the technological system is a set of elements $r_i \in R, v_t \in V$ and the relationships between them.

In this case, it is convenient to consider the technological system in the form of a relation on a nonempty set of production objects and technological operations:

$$S \subset R \times V. \quad (1)$$

The elements of the set (1) are pairs of vectors r, v

$$s_l = (r, v), s_l \in S, l = \overline{1,L}.$$

Here

$$r = (r_1, \dots, r_j, \dots, r_l), j = \overline{1,J}, r_j = \overline{1,R_j};$$
$$v = (v_1, \dots, v_t, \dots, v_T), t = \overline{1,T}, v(t) = \overline{1,V_t}.$$

Denote the common pair of elements $(r_j, v_t) = w_g, g = \overline{1,G}$.

Then

$$s_l = (w_1, \dots, w_g, \rightleftarrows \cdots, w_G), l = \overline{1,L}, \quad (2)$$

Where $w_g = \overline{1,W_g}, g = \overline{1,G}$ – elements of the technological system;

L – the total number of variants of the technological system.

Universal methods of structural optimization take into account only the combinatorial nature of the formation of options. Therefore, only partial problems of choosing the optimal structure of technological systems can be solved on their basis.

One way to solve them is to go over and over. It involves a preliminary synthesis of options that can be obtained by forming all possible combinations of elements: $w_g = \overline{1,W_g}$, $g = \overline{1,G}$.

Then, for each implementation of the system, the values of the indicators are calculated $F_i, i = \overline{1,I}$, and the choice of the optimal option is carried out.

The positive side of a full search is the review of all permitted combinations, which ensures high reliability of the optimal decision.

This approach has shortcomings that impose limitations on its application.

First, the lack of mathematical models that take into account the system connections of production facilities and technological operations requires the formalization of the allowed combinations for each partial problem, which significantly increases the cost of preliminary preparation of the automated solution.

Secondly, the total number of variants of the technological system is quite significant, and the machine time required to generate them becomes unacceptable.

Third, in the case of multiple technical and economic requirements, the search for the optimal system leads to additional costs for re-search and analysis in the field of trade-offs.

An approach aimed at limiting the number of pre-synthesized variants is abbreviated random search.

However, due to the lack of mathematical models of structural optimization of the technological system, it is impossible to build targeted search procedures that predict the position of the system in the space of indicators, which reduces the reliability of incomplete search. In addition, it is not always possible to justify the condition of stopping an uncontrolled random search process.

A formalized approach to taking into account the combinatorial nature of structural optimization is achieved on the basis of discrete programming methods, because the components w_g variations are given on a discrete set $\overline{1,W_g}$.

Technological systems used in the manufacture of parts of devices and their assembly are discrete systems, so to model and optimize their work should use appropriate methods [11].

3. *Method choice for modeling of production systems*

The production system in General is a system with a fixed number of equipment (system units) that process a fixed number of parts (perform certain work).

Unfortunately, today there is no single method of modeling production systems in general, ie those that would adequately and with sufficient accuracy would describe the parameters of production systems of different structures, so to model a particular type of system, or at least several closely related types of various mathematical models. They are somehow able to model the production system, but usually each of them has its limits of application.

Models obtained by linear integer programming can only be used if a large number of constraints are imposed on the problem. In addition, to apply these methods, it is necessary to obtain the appropriate dependencies, which could be used to calculate the parameters of the system. Therefore, mathematical programming methods can be used as optimization methods in the presence of well-formalized system models [11].

Models of queuing systems are acceptable only for workflows with a random number and time of passage, because they use the apparatus of probability theory. You can usually consider degenerate cases where a random stream is replaced by a stream with predefined parameters. Therefore, we will consider deterministic queuing models that describe very well processes similar to those that need to be modeled. But such models are very different from each other depending on the type of system structure.

Models based on the provisions of graph theory differ from others by the simplicity and clarity of representation of both input and output data, and the image of the structure of the system itself.

Methods of classical modeling and optimization are well studied, widely used in theoretical problems, but have several significant shortcomings, in particular in terms of modeling systems in their application it is difficult to develop an adequate objective function that would satisfy the limitations of this method. would be able to interpret the results of further calculations, in particular the results of model optimization.

Statistical methods have been used for a long time and allow to determine the factors that most affect the characteristics of the system. However, they require a large amount of experimental data and therefore are more likely to be useful for solving problems of optimization of empirical processes, ie where it is impossible to establish unambiguous relationships between system parameters.

Structural-logical method of modeling is acceptable from the point of view of clarity of representation of system and calculations of its parameters, quite well algorithmized, has good convergence at application of the corresponding optimization methods, and also is suitable for modeling of systems with various types of structures. clarity of presentation of the system.

Based on the analysis of methods of modeling production systems in order to optimize their work, it is established that the most interesting are the structural-logical method of research of technological systems and methods based on graphical representation of production structures using Petri nets. These methods are discussed in this article.

4. Structural and logical method of modeling and optimization of production systems

There are two main directions in the methods of research of production systems: general (abstract) systems theory, which studies the behavior of the system out of connection with its structure, as the ratio of inputs and outputs, and structural systems theory, which studies the functioning of the system depending on its structure .

The possibility of constructing a deterministic theory of service systems on the basis of the mathematical apparatus of infinite logic and logical determinants was first shown in, where the problem of synthesis of one class of static systems was solved. Subsequently, the possibility of applying this apparatus to the synthesis of other classes of systems, as well as to their analysis was shown.

The advantages of the apparatus of infinite logic and logical determinants are realized within the structural-logical approach to the study of service systems, which involves structural representation of the studied system and expression of its characteristics through the characteristics of subsystems using infinite logic and logical determinants.

Structural-logical method does not have most of the shortcomings of other methods of modeling and optimization of production systems, ie it is acceptable in terms of clarity of the system and calculations of its parameters, quite easy to formalize, has good convergence in the application of appropriate optimization methods, and that most importantly, suitable for modeling systems with different types of structures, namely: serial, parallel, parallel-serial and serial-parallel. This approach makes it possible on the basis of a small number of such algorithms to calculate, analyze and synthesize the optimal structure of the production system.

5. Construction of mathematical models of production systems

The tasks of studying systems are very diverse and are largely determined by the class of the studied system. However, there are universal types of problems that are essential for many classes of systems: representation, calculation, analysis and synthesis.

Representation of the system is a representation of all given information about the system in a compact form, which facilitates the description of the system, as well as the formulation and solution of problems of its calculation, analysis and synthesis.

The calculation of the system is to determine the numerical values or expressions of various characteristics of the system according to the given structure of the system, the mode of its operation (for the service system - also the order of work coming to it) and the numerical value of its blocks.

The problem of system calculation is important for all classes of systems. It reveals the possibilities of existing natural (biological, social) and underlies the design of artificial (technical, organizational and economic) systems.

The analysis of the system consists in determining the type of dependence of various characteristics of the system functioning on its structure, the mode of its functioning (for the service system - also the order of execution of works in it), the values of its blocks.

The analysis allows to establish the degree of influence of these three factors on the characteristics of the system. Due to this, it is possible to perform calculations of accuracy (stability) of the system at different values of its parameters and structure and calculations in the design of systems in order to achieve the desired values of system characteristics by some change in their parameters and structure.

The synthesis of an arbitrary system consists in determining its structure and mode of operation for a given purpose of the system, the valuesof the parameters of its blocks and the required values of the characteristics of the system.

The synthesis of the service system is to find its structure and the order of execution of works from a given set of works to be performed, the values of the parameters of the blocks that form the structure of this system, and the required values of various characteristics of the system.

The task of system representation is important only for complex systems, while the tasks of calculation, analysis and synthesis are essential for any system - both simple and complex. In our case, service systems are considered, for which the problems of representation, calculation, analysis, and synthesis problems are solved in the form of the problem of finding the optimal order of execution of a given set of works in the system, which provides optimal values of certain system characteristics. The structure of the system and the parameters of its blocks are considered specified.

The paper considers deterministic problems of representation, calculation, analysis and synthesis of service systems with a typical structure (serial, parallel, parallel-serial, serial-parallel), with different modes of operation of units and submission of works, with the possibility of imposing restrictions on the operation of units.

Here are the mathematical models for each type of system, according to the problems to be solved. The tasks of calculation and analysis of the service system are respectively:

a) calculation of its various characteristics (performance, load, etc.) in accordance with the specified similar characteristics of its units, the scheme of their connection, the number of works entering the system, and the mode of operation of the system;

b) determining the nature and degree of influence of all given factors on the characteristics of the system.

The solution of the first problem must precede the solution of the second.

The complexity of solving both problems significantly depends on the scheme of connection of blocks to the system (serial, parallel, etc.), the mode of operation of the system (single-program or multi-program, waiting for work or without waiting, etc.), the number of jobs entering the system. A single-work system can only be without waiting and with a single-program execution mode.

Analysis of such systems is not a problem and is not considered here. Of the other systems, serial systems are the most convenient for calculation and analysis. Consider a system in the form of m series-connected blocks, designed to sequentially perform m different operations.

The system receives n different works, each of which breaks down into m of the above operations. Opening times a_{ij} i $(i = \overline{1,m})$ for jobs j $(j = \overline{1,n})$ given by the matrix: $A = \|a_{i,j}\|$,

Execution of works in the system can be carried out in two modes: single-program and multi-program.

In the first mode, first all the operations of the first work are performed sequentially, then in the same order all the operations of the second work, and so on.

In the second mode of operation are started in the system and pass its successive blocks 1,2, ..., m in the same order $P_n = (j_1, j_2, \ldots, j_n)$; here j_k is the number of the work that follows the k-th in order.

Thus moments of receipt of works in system are not specified (free on time of receipt of works), the block 1 is loaded and starts to carry out the operation in the next work j_k immediately after release from the previous work j_{k-1} (ie in the block there are no downtimes); loading of the first work j_1 occurs at the initial moment $t = 0$.

However, block 2 begins to perform its second operation in the next job j_k not immediately after the release of the previous job, but only after the end in block 1 of the first operation of work j_k.

Similar time relations between the operation of units 2 and 3, 3 and 4, ..., *m-1*, *m*. Thus, in blocks 2, 3, ..., *m* downtime is possible. But it is possible that the work that comes to the unit, finds him busy with previous work.

In this case, the unit works without downtime, but the work coming to it is waiting, forming a queue served by the unit as a rule: first come - first served. So, a multi-program system is a standby system.

For a *serial system*, the total time *To (m, n)* of the passage of all *n* works through all m blocks of the system with a single-program mode of execution of works is equal to the summing logic determinant from the matrix $A = \|a_{i,j}\|$ times of operations, and for the case of multiprogram mode *Tm (m, n)* – disjunctive logic determinant from the matrix $A = \|a_{i,j}\|$.

In the case of a **parallel system**, it is represented in the form of m parallel connected blocks 1, 2, ..., *m*, which are designed for parallel performance of the same type of work. The blocks are functionally equivalent, ie they can perform each of the works.

However, the speed of the blocks in the general case is different - the execution time in the work block is a_{ij}. Times a_{ij} form a matrix of working times $A = \|a_{i,j}\|$ sized $m \times n$.

The system works in multi-program dynamic mode as follows. Works 1, 2, ..., *n* are fed into the system in a given order $Pn = (j_1, j_2, ..., j_n)$, where $j_k \in \{1, ..., n\}$, but without specifying the moments of receipt, ie free time.

In this case, at the initial moment $t = 0$ at the same time work j_1 is loaded into block 1 (which immediately begins to perform its operation), similarly work j_2 – in block 2 and so on – work j_m – in block *m*.

After that, all blocks of the system are loaded by performing the first *m* works $j_1,\dots,j_m$. Therefore, the next job j_{m+1}will have to wait until the first $j^{(1)}$ of the loaded jobs $j_1,\dots,j_m$ is performed, and then take its place in the vacated block.

The next job j_{m+2} will have to wait for its loading into the system until the moment when the first of the new set of jobs $\{j_1,\dots j_m, j_{m+1}\}$ that are currently in the system and so on will be performed. It follows that the considered system works without downtime (ie with a load equal to 1) in the standby mode of the start of its execution.

Simultaneously with the tasks of calculation and analysis, the problem of system representation is solved, using the apparatus of ordinal logical determinants.

Matrix $T(m,n) = \overset{k=1}{\underset{n}{\vee}} t_k$ is the basic ratio for calculating the system speed. It reduces the definition of the performance characteristic of the system *T (m, n)* to the calculation of the vector of moments of completion of work in the system.

In the case of using **parallel-serial and serial-parallel systems**, they are represented in the form of *M* series-connected stages (groups, blocks), designed to perform work consisting of *M* different operations.

Let the i degree ($i = \overline{1,M}$) serves to perform the i-th operation and is a subsystem in the form of mi parallel connected blocks 1, 2, ..., m_i. All blocks of one stage are functionally equivalent, ie configured to perform one operation, and the *i*-th stage can perform simultaneously m_i such operations of the same type (according to the number of blocks in it).

We believe that all blocks of one degree have the same speed (ie degrees are homogeneous in blocks). The system receives a set of *n* different works, each of which consists of *M* of the above operations. Time a_{ij} execution of the *i*-th operation ($i = \overline{1,M}$) for job *j* ($j = \overline{1,n}$) are given and form a matrix $A = \|a_{i,j}\|$.

Execution of jobs in the system is possible in two modes: single-program and multi-program.

In the first mode, first all operations of work 1 are performed sequentially, then in the same order all operations of work 2 and so on.

In the second mode, the system operates as follows. The order $P_{1n} = (j_{11}, j_{12}, ..., j_{1n})$ of start of works on the first operation (the first stage of system) where j_{1k} –number of the work started *k*-th on an order is specified. At the same time the moments of receipt of separate works are set, that is we have system with free on time receipt of works.

Execution of the first operation in the next work j_{1k} begins as soon as among the blocks of the first stage engaged in execution of this operation in the previous works $j_{11},\dots,j_{1,k-1}$ any block is released.

Execution of any *i*-th ($i = \overline{1,M}$) the operation in the next work j_{1k} begins as soon as the execution of the previous (*i-1*)-th operation in this work ends and among the blocks of the *i*-th degree, occupied with the execution of the *i*-th operation in the incoming j_k works, any block will be released.

The mathematical apparatus of infinite logic can serve as an adequate means of describing deterministic service systems. This adequacy is manifested in the fact that for an arbitrary system, any characteristic of functioning can usually be expressed through the parameters of the system using logical operations of infinite-valued logic (sometimes supplemented by ordinary algebraic addition).

In this case, formally, the difference between the characteristics comes down to the fact that some of them receive simple expressions (containing few of these operations), and others - complex (because they contain many operations). Using the rules of equivalent logical transformations developed in infinite logic, the received expression of the characteristic of functioning can always be led to this or that convenient kind. This type depends on the system problem to be solved.

Thus, to solve the calculation problem, the most convenient is the minimum expression of the characteristic that contains the smallest possible number of logical and other operations; to solve the problem of analysis - the expression of a characteristic with a selected parameter, the impact of which on the system is analyzed; to solve the problem of synthesis - the expression of characteristics in a form that is sensitive to changes in the order of work in the system.

This possibility of flexible representation of different characteristics of systems functioning in terms of operations of infinite logic opens the fundamental possibility of the same, based on the rules of equivalent logical transformations of infinite logic, solving the whole set of problems of systems research - calculation, analysis, synthesis.

This possibility of the same solution of problems of calculation, analysis and synthesis of service systems using the apparatus of infinite logic is directly realized only for fairly simple systems, where the number of blocks is small, and the links between them are not developed. In complex systems due to the presence of a large number of blocks with developed links between them on the way to the implementation of this possibility there are additional difficulties associated with the need to solve the problem of system representation.

In overcoming these difficulties the main role is played by the mathematical apparatus of logical determinants. Since the logical determinant is a parameter of the enlarged description of the studied system, they allow to present all given information about the system in a compact, accessible form, which makes the system "simple" and opens the way to its study by the above methods of studying simple systems based on the apparatus of infinite logic. In addition, the apparatus of logical determinants allows to make in the process of studying complex service systems and a number of other significant simplifications.

The method of modeling and optimization of loading of production technological systems on the basis of structural-logical approach to the study of complex systems is tested on real examples.

The use of structural-logical approach to modeling and optimization of production systems has the following advantages:

a) unification with the help of logical determinants of computational procedures in the study of systems leads to the fact that all tasks of calculation and analysis, as well as the synthesis of complex systems are reduced to the calculation of certain logical determinants, ie are the same from a computational and mathematical point of view;

b) obtaining analytical conditions for the optimality of the order of work in the system and on their basis - the procedures for the synthesis of some classes of complex systems have a reduced complexity of calculations;

c) modification of the procedure of branches and boundaries, which is used in the synthesis of complex systems to improve its accuracy and speed, allows to intensify the clipping of unpromising options by additional use of information contained in optimal conditions, and obtaining a more accurate estimates that in addition, it has reduced computational complexity.

The above advantages of using infinite logic and logical determinant are realized within the structural-logical approach to the study of production systems, which involves structural representation of the studied system and expression of its characteristics through the characteristics of subsystems using infinite logical logic and logical determinant.

6. Conclusions

The article theoretically substantiates and implements mathematical methods of modeling production systems, which allow to choose rational conditions for their work.

It is shown that increasing the efficiency of instrument-making production can be achieved by modeling and optimizing the equipment load of production systems.

The analysis of modeling methods for production systems has shown that at the decision of this problem it is expedient to apply modern methods of mathematical and simulation modeling based on use of the structural-logical approach to mathematical modeling of systems.

Methods of infinite logic and logical determinants allow to describe the mathematical model of the production system and its characteristics, and the method of branches and boundaries - to optimize its work. The basic provisions of infinite logic and ordinal determinants, on which the structural-logical approach to the study of complex systems is based, which include production systems in instrument making, allow to create a mathematical model of the system of arbitrary structure.

The actual application of this method of modeling production systems can reduce non-production losses of time, thereby saving production resources without the use of any other measures of administrative or technical nature.

7. Literature

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