# Modelling uncertainty in multisensory systems at conflict measurements with dempster-shafer combinatorial rule

Varbanov Vladimir
Institute of Metal Science, Equipment and Technology with Hydroaerodynamic Center at Bulgarian Academy of Sciences, 67 Shipchenski Prohod Street, 1574 Sofia, Bulgaria
vdvarbanov@abv.bg

***Abstract:*** *Decision making in conflict data is a problem that plague the information fusion. It is usually resolved by the use of probabilistic methods such as Bayes. Still Bayes becomes impractical in strong presence of conflicts or lack of data. In that case Dempster–Shafer (DS) evidence theory is applied, because of its ability to handle conflicting sensor inputs. Because of that, DS-based information fusion is very popular in decision-making applications, although the classic combinatorial rule may produce counter intuitive results, especially when combining evidences with high level of conflict. The paper present a method which can be applied to the decision making process, thus resolving this problem.*
***Keywords****: Dempster-Shafer, Dempster-Shafer evidence distance*

### *1. Introduction*

Information fusion technology is usually used to analyze multisource uncertain information. A multi sensor system has significant advantages over single sensor systems, especially when combined proper algorithm :

- Robustness – the the system provides overlapping information, regarding the observed event, thus mitigating contribution of fault sensor to the accuracy of decisions and measurement.
- Multi dimensionality of measurements – a multisensory system can receive information from sources in different domain which enrich the quality of the data and decrease the chance of interference.

The problem is that this advantages in multisensory systems can easily be turned in to liabilities. For small number of sensors a fault sensor can make the decision process impossible, mainly because the system will be unable to single out the defected device or sensors in case of sensor working in different physical domain a high level of conflict may occur even though each one of the sensors is working properly. In example measuring distance with laser and sound may lead to conflict in case of transparent obstacle although both sensors are working as intended. In order to obtain reliable information from fusing measurements in cases of conflict a suitable mathematical methods are needed and Dempster-Shafer theory of evidence can be applied in such cases. Proposed at first in 1967 by Demster and the developed into its current form in 1976 by Shafer the method suffered from the very same issues mentioned above. Counter intuitive results were generated if the input data was conflicted. However nowadays this method is preferred for such cases. To solve such a problem, two major methodologies are popular. One is to preprocess the bodies of evidence (BOEs) and the other is to modify the combined rule. There are many modifications to the combinatorial rule Dubois and Prade's disjunctive Yager's combination rule and others. All of the most used rules has their pros and cons and careful consideration of athe advantages and disadvantages of each of them is needed in ordro to be achieved stable logic. The methodology of preprocessing bodies of evidences is based on distance from evidence. There are different methods proposed by Sang, Yong and Anne-Laure Jousselme.

### *2. Dempster–Shafer Combinatorial rule*

*2.1 Frame of discern (FOD)*

The frame of discernment contains M mutually exclusive and exhaustive events.

$$\Theta = \{\theta_1, \theta_2, \theta_3 \cdots \theta_M\} \quad (1)$$

The representation of uncertainties in the DS theory is similar to that in conventional probability space Θ, although DS probabilities are defined as masses and are view more as options instead of physical world probabilities. The DS theory also allows this "probabilities" to be assigned to subsets of $\Theta$ - $\{\theta_i, \theta_j, \cdots \theta_n\}$ as well to individual element $\theta_i$. Accordingly, we can derive a power set of DS theory which includes all the combinations of the events of $\Theta$ and consists of $2^M$ elements:

$$2^\Theta = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \dots, \{\theta_1, \theta_2\}, \dots \Theta\}$$
(2)

where $\emptyset$ is the empty set.

2.2. Basic Probability Assignment (BPA)

Mass function (mass) is a function, $m: 2^\Theta \to [0,1]$ that satisfies the following equations:

$$m(\emptyset) = 0$$
(3)

$$\sum m(\theta) \forall \theta \in 2^\Theta = 1$$
(4)

Equation (3) means that the empty does not take part in forming decision according to DS combinatorial rule, while equation means that cumulative mass is normalized. Elements of power set having $m(\theta) > 0$ are called focal elements.

*2.3. Dempster–Shafer Rule of Combination*

The data fusion summarizes the information, obtained from multiple independent sources and as such DS combination rule emphasizes the agreement between multiple sources and ignores all the conflicting evidences through normalization. Any two mass functions B and C over the same FOD with at least one focal element in common can be combined into a new mass function[1,2]. The weight of the mass stands for the strength of our belief in the proposition that is represented to us. In other word the combination perform orthogonal sum ⊕ on the input evidences. The combination rule for two evidences $m_1$ and $m_2$ is written with the following expression :

$$m_{1,2}(A) = \sum B \cap C = \frac{\{\{m_1(B)\}.\{m_2(C)\}\}}{1-K}$$
(5)

when $A \neq \emptyset$ and $m(\emptyset) = 0$.
Where $K$ is the degree of conflict in the two evidences:
$$K = \sum B \cap C = \{m_1(B) m_2(C)\}$$
(6)
when $B \cap C = \emptyset$

2.4. Belief and Plausibility Function
Belief function for element $A$, otput of which is defined as:
$$Bel(A) = \sum_{B \subset A} m(B)$$
(7)

Where focal element $A$ can be set, i.e. $A = \{\theta_1, \theta_2\}$ or singleton $A = \{\theta_1\}$ and represents a statement/opinion and $Bel(A)$ measures the belief that the this statement $A$ is fully supported by $B$. $m(A)$ measures the amount of belief that one commits exactly to $A$ alone(not supported by other evidences).

Plausibility is define as

$$Pl(A) = \sum_{B \cap A} m(B) \quad (7)$$

Plausibility measures the hesitation of correctness of statement $A$, because $A$ is just partially supported by $B$. Of $Pl(A)$ can be taught as the total belief mass that might be moved into $A$, whereas Bel(A) measures the total belief mass that is constrained to $A$.

Example 1.
Given $m(A) = 0.51$, $m(B) = 0.23$, $m(C) = 0.06$ and $m(A, B, C) = 0.20$

$$Bel(A, B) = m(A) + m(B) = 0.51 + 0.23 = 74 \quad (8)$$

$$Pl(A, B) = m(A) + m(B) + m(A, C) + m(B, C) + m(A, B) + m(A, B, C) = 0.94 \quad (9)$$

### *3. Paradoxes in DS combination rule*

*3.1 Veto paradox*

For multi sensorial datafusion according to Dempster-Shafer rule if one sensor of the sensors has completely contradict the others it invalidates the evidences of all the other, no matter that they show similar data.

Example 2. Let have four sensors and four evidences in the frame of discern $\Theta = \{A, B, C\}$ and every one of them ) f rom Fig.1 every object can be put in one of

Sensor 1: $m_1(A) = 0.6$, $m_1(B) = 0.2$, $m_1(C) = 0.2$
Sensor 2: $m_2(A) = 0.0$, $m_2(B) = 0.9$, $m_2(C) = 0.1$
Sensor 3: $m_3(A) = 0.7.$, $m_3(B) = 0.15$, $m_3(C) = 0.15$
Sensor 4: $m_4(A) = 0.7.$, $m_4(B) = 0.15$, $m_4(C) = 0.15$

It can be seen that sensor 2 is faulty, as its mass for $A$ is equal to two. Applying equation (5) will produce output $m_{1,2}(A) = 0$, next when we apply same equation and fuse the result with the third sensor we will obtain same result for $m_{1,2,3}(A) = 0$ and again will have same result for $m_{1,2,3,4}(A) = 0$. One defected sensor which is clearly an outlier can compromise the whole fusion process[5,4].

*3.2 Total trust paradox*

This paradox occurs if two or more sensors have strong, conflicting evidences, and weak overlapping evidences.

Example 3.
$\Theta = \{A, B, C\}$
Sensor 1: $m_1(A) = 0.95$, $m_1(B) = 0.05$, $m_1(C) = 0$
Sensor 2: $m_2(A) = 0$, $m_2(B) = 0.1$, $m_2(C) = 0.9$

Applying DS combination rule(equation (5)), we get $m_{1,2}(A) = 0$, $m_{1,2}(B) = 1$, and $m_{1,2}(C) = 0, K = 99$. Here, both masses for evidences $A$ and $B$ are nullified and because of normalization $m_{1,2}(B) = 1$ , which is counterintuitive.

### *4. Elimination of the paradoxes*

Widely used in DS theory in strong cases of conflict is the Yager rule. There the mass associated with conflict is directly added to the subset consisting all singletons, which can be interpreted as ignorance, he also omits normalization and allow the ground probability mass assignment of the null set to be greater than $m(\emptyset) \geq 0$, thus adding spreading the values of the fusion into more non zero elements. This work around fixes the nullifying effect in the veto and total trust paradox[6].

### *5. Conclusion*

Dempster-Shafer is often viewed as generalized Bayes. Biggest advantages of which is the pore realx definition of the term probability which allows to formalize decision making process similar to the probabilistic without knowing the conditional probabilities. The method is prone to high conflicting input data but so is Bayes. However these shortcomings are fixed be use of modified combinatorial rules. This is why the Dempster-Shafer theorem is widely used in Neural network, machine learning, autonomous driving cars as well as computer vision[3,4].

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