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Procedure for analyzing the quality, structure and subjective rating of distorted images by the Full-Reference technique

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Abstract. In the present work, we study the regularities of the influence of the type of distorting algorithm on the result of evaluating the image quality by the Full-Reference method in the presence of subjective quality assessments. As an example, we used the TID2013 database with 3000 images distorted by 24 types of algorithms and subjective mean square scores (MOS) quality ratings. An image quality score based on the Weibull distribution model and the usual PSNR similarity measure is applied. It is shown that the applied distorting algorithms are classified into two types - normal, leading to results consistent with the Human Visual System, and "anomalous", the corresponding quality estimates of which are disordered or chaotic.

Keywords: IMAGE QUALITY, DISTORTED IMAGE, FULL REFERENCE, MEAN OPINION SCORE, SIMILARITY MEASURE, WEIBULL DISTRIBUTION.

1. Introduction

Image quality estimation is an important component in various applications such as pattern recognition, classification, restoration, and other tasks. At the same time, as it is known, the concept of quality does not have an acceptable unambiguous formal definition, therefore, in problems of this type, one has to limit oneself to specific considerations of the concept of image quality and the corresponding methods for its estimation [1, 2].

There are three types of methods for evaluating image quality. The first type assumes that there are two samples of images, one of which is considered to be the original with acceptable quality in this task, and the second is the test one, which is the original somewhat distorted as a result of some actions, and it is required to estimate the degree of quality change as a result of distortion. The degree of distortion is taken as an estimate of the quality of the second image. Here we deliberately indicate not an expected decrease in quality, but only a change, since as a result of the mentioned actions, quality can increase. In most applications, the quality score is identified with the similarity score of the images under consideration. Methods for solving the problem in the situation under consideration are called Full-Reference (FR) methods [3, 4].

The second type of methods is called No-Reference and is characterized by the absence of a sample for comparison, and the decision on quality is made solely on the basis of the analysis of the properties of the tested sample. Here, the structural and other properties of the image are used, which in this problem may contain information about its quality [5, 6].

The third type of methods is called Reduced-Reference because it partially uses information about the original image.

At present, there is extensive scientific and scientific-technical literature devoted to methods for evaluating image quality for all three types described above. These methods, in turn, can be classified into two classes - objective and subjective methods. Objective methods are implemented using formal methods of the theory and image processing techniques, while subjective methods involve the use of expert quality assessments obtained as a result of analysis by the human visual system (HVS) [7, 8]. Subjective quality ratings evaluated by test subjects are then averaged to obtain the mean opinion score (MOS).

It should be noted that the result of quality assessment is formed by the influence of the following, necessarily present, main factors. First of all, this is the initial quality of the tested image. Any image initially has some implicit quality level, characterized by the content of the depicted scene. The second factor is originated by objective actions during image acquisition, its visualization, transmission over communication channels, etc., which introduce distortions into the image content. The third factor operates in the process of image processing, in which, in order to achieve the goal, it is necessary to

change the structure and parameters of the image, using various mathematical or computer methods. This leads to the conclusion that there are no universal methods for assessing the quality of an arbitrary image, therefore, when making quality decisions, it is necessary to take into account the individual properties of the tested objects and the methods used, combining them with the available subjective assessments. Therefore, the tasks of developing new quality criteria, evaluating the similarity of images, methods of analysis and comparison of different approaches remain relevant.

The literature offers a number of image databases with MOS estimates obtained experimentally as a result of interviewing a large number of experts using a special technique. For example, 40 such databases are described and critically analyzed in [8]. There is also an extensive bibliography devoted to the issues of assessing quality by objective and subjective methods. At the same time, an important link in the experiments of the type under consideration is the application to images of various methods of artificial distortions with several levels of influence, making it possible to compare objective and subjective quality assessments.

In this paper, using the example of a specific database and a specific measure of image quality, we study the following issues:

- Classification of types of distortions according to the degree of impact on the similarity with the original and according to MOS values,
- Investigation of the nature of the impact of "bad" and "good" types of distortion on quality estimates and image structure.

2. Research Methodology

Let us briefly describe all the components of the proposed research methodology.

The selected database is TID2013 [9]. This database contains 3000 images obtained from 25 originals, distorted by 24 different types with five levels each. Fig. 1 shows an example image and its distorted version.



Fig. 1. Image i07.bmp from TID2013 base and its distorted version.

The authors of the database conducted an extensive experiment on the visual assessment of the quality of the images of the database according to the point system by a large number of people in different countries. As a result of processing these data, each of the 3000 images is assigned a numerical MOS score.

FR-method of image quality estimation. The article proposes to use the previously developed image gradient magnitude model based on the Weibull distribution, the density of which is given by the formula

$$f(x; \lambda, \eta) = \frac{\eta}{\lambda} \left(\frac{x}{\lambda}\right)^{\eta-1} \exp\left[-\left(\frac{x}{\lambda}\right)^{\eta}\right], x \geq 0, \quad (1)$$

where $\eta > 0$ is the shape parameter, $\lambda > 0$ is the scale parameter. Distribution parameters (1) are estimated from the totality of all gradient magnitudes using the Sobel operator. The similarity (proximity) of two images is estimated by the proximity of the estimates of the parameters of the Weibull distribution by the formula

$$W^2 = \frac{\min(\eta_1, \eta_2) \min(\lambda_1, \lambda_2)}{\max(\eta_1, \eta_2) \max(\lambda_1, \lambda_2)} \quad (2)$$

This measure turned out to be convenient and very useful in a number of processing tasks, including those with large arrays of images. Details of the procedures and the corresponding experimental results with an illustration of the quality factor of the proposed models can be found in [10].

To evaluate the quality of images, we also use PSNR, taking advantage of the fact that all images in the base have the same size. Note that earlier, as a measure of quality, we also used the shape parameter of the Weibull distribution [11] and the Shannon entropy [12].

Methods of analysis and interpretation of the results of quality assessment. Calculations of all necessary quantities are made using the developed software system, and the results are entered into Excel tables. The base data are the results related to the original and five distorted samples of a particular image. For each such data set, the Spearman correlation coefficient ρ (SCC) and PSNR between the quality score under study and MOS are estimated. All cases, where the modulus of the correlation coefficient differs from 1, are considered anomalous and are recorded for further analysis.

3. Experimental results

Experiment 1. Preliminary analysis of the obtained results of calculation revealed a behavior of the correlation of the measure of similarity between each original and the corresponding five distorted samples. There are cases when $SCC=1$ (these cases are detected even visually if the compared series are monotonous) and when $SCC \neq 1$ (SCC of both signs and/or with low values). In the latter case, we consider weak correlations as anomalous, and they manifest themselves in different ways, depending on the type of image and/or type of distortion. The distribution of the number of anomalies by types of distortions in the whole database is given in Table. 1. The data in the table shows that there are types of distortions that lead to anomalies for most images (for example, types with numbers 7, 14-18).

Table 1. Distribution of the number of anomalies by types of distortions.

Dist. type	Anom. Number	Dist. type	Anom. Number	Dist. type	Anom. Number	Dist. type	Anom. Number
1	2	7	21	13	15	19	1
2	9	8	1	14	24	20	7
3	0	9	6	15	25	21	2
4	11	10	13	16	22	22	14
5	6	11	5	17	23	23	3
6	2	12	19	18	23	24	3

Experiment 2. Analysis of the influence of types of distortions on image quality estimates. Two sets of types of distortions are considered based on the manifestation of anomalous correlation values.

First, consider cases with normal correlation values and as an example, choose the i01 image of the TID2013 base and following type distortions

- 3 Spatially correlated noise,
- 8 Gaussian blur,
- 9 Image denoising.

To demonstrate the degree of distortion of the selected image i01.bmp, Table. 2 shows images corresponding to the fifth level of the impact of distortions of the types under consideration. The W^2 and PSNR estimates relative to the original are also given for review.

Table 2. Illustration of corrupted image samples.

					
i01.bmp	i01-03-5.bmp	i01-08-5.bmp	i01-09-5	i01-14-5	
$W^2/PSNR$	>	0.49/24.4	0.22/21.0	0.13/21.4	0.99/23.4

To compare the proximity measures of the original and the distorted sample, W^2 and PSNR were calculated for five levels of distortion. The results of the calculations and the corresponding MOS values are shown in Table 3. The visual analysis allows us to conclude that in all three cases, $SCC=1$. This means that these types of distortions change the structure and content of the image in proportion to the applied level of exposure, as it should be.

Anomalous data obtained, for example, from distortion types, look completely different.

- 14 Non-eccentricity pattern noise,
- 15 Local block-wise distortions of different intensities,
- 18 Change of color saturation.

Table 3. Quality scores of the image i01.bmp for distortion types of 3, 8, and 9.

3			8			9		
MOS	W^2	PSNR	MOS	W^2	PSNR	MOS	W^2	PSNR
4.77	0.9	36.5	5.19	0.83	34.0	6.08	0.82	34.7
4.27	0.84	33.4	4.47	0.71	29.1	5.17	0.69	31.5
3.70	0.75	30.5	3.49	0.53	25.3	4.43	0.47	27.7
3.19	0.63	27.4	3.00	0.35	22.7	2.81	0.32	24.6
2.71	0.49	24.4	2.11	0.22	21.0	1.54	0.13	21.4
SCC	1	1	SCC	1	1	SCC	1	1

The data in Table 4 show that in this case the descending order of the ratings of quality and applied impact level is violated. The W^2 measure shows a very high and almost constant degree of similarity between the distorted samples and the original, which is evidence of the preservation of the image structure with these types of distortions and the degree of their impact. It should be noted that the MOS estimates vary within a narrow range, i.e. HVS also does

not detect strong distortions in the image structure. Moreover, the MOS values may contradict the quality values expected under the experimental conditions (see, for example, the data in Table 3).

Table 4. Image quality scores for image i01.bmp for distortion types 14, 15, and 18.

14			15			18		
MOS	W ²	PSNR	MOS	W ²	PSNR	MOS	W ²	PSNR
5.69	0.99	32.8	2.83	0.94	26.9	4.62	1	52.9
6.05	1	29.6	2.83	0.96	25.9	4.22	1	52.9
5.76	0.99	26.9	3.11	0.97	25.4	3.77	1	53.0
5.56	0.99	25.2	3.58	0.97	24.7	3.47	1	53.3
4.47	0.99	23.4	3.89	0.97	25.8	3.42	1	52.9

Thus, the observed patterns indicate a weak effect of the selected types of distortions on the structure and content of the image and, most likely, are a reflection of the internal properties of the distorting algorithm.

4. Conclusion

In this article the patterns of mutual influence of the type of distorting algorithm, the properties of the applied FR-algorithm, and subjective assessments of image quality are studied. As an example, the TID2013 database was used, which included 3000 images distorted by 24 types of algorithms and subjective MOS quality estimates, the W^2 measure of image similarity based on the Weibull distribution model, and the PSNR similarity measure. It is shown that the applied distorting algorithms are classified practically into two types - normal, leading to results consistent with the HVS, and "anomalous", the corresponding quality estimates of which are disordered or chaotic.

The results obtained show that when developing databases of the type under consideration, it is important to choose such distorting algorithms that really provide a change in the structure and content of the image when they are applied.

The proposed approach to image quality analysis can be useful for developers of databases containing subjective quality assessments, as well as for developers of new, more universal, image quality assessment measures.

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D – optimal plans in the case of a piecewise constant function

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Abstract: It is known that a large number of algorithms of the Monte Carlo method and experiment planning are based on the choice of a certain probability distribution p .

This probability distribution p is given on a measurable space (X, B) . And a given measurable space (X, B) has a density $p(x) = dp/dv$ by some σ -finite measure v on (X, B) .

When choosing a probability distribution p , the problem of solving the problem of finding the optimal density p arises.

As a result of solving the tasks, an explicit form of the Least squares Method of unknown parameters and variance was obtained. The criterion of D -optimality is considered.

The D -optimal plans considered in this article are well known due to an important class of efficiency functions. To compare plans in terms of D -optimality, the effectiveness of an arbitrary plan relative to the optimal plan is determined.

Thus, this article is devoted to the analysis of methods for constructing D -optimal experimental plans, where the basic object is a piecewise constant function.

Keywords: OPTIMAL DENSITY, FUNCTIONAL, OPTIMALITY CRITERION, INTERPOLATION AND DISPERSION MATRIX, OPTIMAL PLAN.

A large number of algorithms for the Monte Carlo method and experiment planning is based on the choice of some probability distribution p given on a measurable space (X, B) with a density $p(x) = dp/dv$ for some σ -finite measure with v on space (X, B) . As well as in modeling the distribution of p and estimating on the basis of the obtained sample some finite or infinite set of linear functional [1].

The effectiveness of these algorithms depends on the choice of p distribution P and will be determined by some functional Φ , given on the set of allowable densities P and being a composition of integral functionals linearly dependent on $1/p$.

Such problems of optimal density construction include, for example, the following problems:

- random interpolation;
- optimal choice of densities for regression estimating from observations at random points;

At the same time, in all problems of finding the optimal density, p consists in finding the density

$$\rho^* = \operatorname{argmin}_{\rho \in P} \Phi[D(\rho)] \quad (1)$$

where

$$D(\rho) = \int [\varphi(x) \varphi^T(x) / \rho(x)] v(dx) - A \quad (2)$$

$$\varphi(x) = \{\varphi_1(x), \dots, \varphi_m(x)\}^T$$

v is a measure on (X, B) , Φ is some functional, $\{\varphi_1(x), \dots, \varphi_m(x)\}^T$ - are piecewise continuous functions from $L^2(x, v)$.

$$v \left\{ x \in \chi \mid \sum_{i=1}^m \varphi_i^2(x) \right\} > 0$$

P is the set of densities p with respect to the measure v on χ such that $p(x) > 0$ for almost all $x \in \chi$.

Formulation of the problem.

Suppose there is a classical linear regression model. Suppose that the measurement result $y(x_j)$ at the point $x_j \in \chi$ is represented as [2]:

$$y_j = y(x_j, \omega_j) = \sum_{i=1}^n \theta_i \varphi_i(x_j) + \varepsilon(x_j, \omega_j), \quad j = 1, \dots, N \quad (3)$$

where, θ_i – are unknown parameters, $\varepsilon(x_j)$ – are random errors, with respect to the distribution of which the conditions are assumed to be fulfilled

$$E\varepsilon(x) = 0, \quad E\varepsilon^2(x) = \sigma^2(x) = \text{const} < \infty, \\ E\varepsilon(x_i) \varepsilon(x_j) = 0, \quad i \neq j \quad (4)$$

$\varphi(x) = \{\varphi_1(x), \dots, \varphi_m(x)\}^T$ – are known measurable basis functions on χ . It is required to evaluate unknown parameters θ and their variances.

The given regression model is written as $(Y, F\theta, \sigma^2 I_n)$ where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad F = \begin{bmatrix} \varphi_1(x_1) & \dots & \varphi_n(x_1) \\ \vdots & & \vdots \\ \varphi_1(x_n) & \dots & \varphi_n(x_n) \end{bmatrix} = \begin{bmatrix} \varphi^T(x_1) \\ \vdots \\ \varphi^T(x_n) \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}.$$

The basic functions $\varphi_1(x), \dots, \varphi_n(x)$ are a system of step functions that are piecewise constant. The system of step functions on the set χ with a given (probabilistic) measure μ is defined as follows: the set χ is divided into n subsets $d(n, i_n)$ of equal measure and it is assumed that

$$\varphi_i(x) = \begin{cases} \sqrt{n}, & x \in d(n, i_n) \\ 0, & x \notin d(n, i_n) \end{cases} \quad (5)$$

For unknown parameters θ_i of the linear model (3), let us construct estimates by the least squares method. For this, let k_j ($j=1, \dots, n$) the number of points belonging to the subset $d(n, j_n)$ of the set χ and

$$\sum_{j=1}^n k_j = N$$

As it is known [2], if the matrix of the system of normal equations $F^T F$ is non-degenerate, then the LSM estimate has the form

$$\hat{\theta} = (F^T F)^{-1} F^T Y \quad (6)$$

variance

$$D\hat{\theta} = \sigma^2 (F^T F)^{-1}$$

where

$$F^T F = \left\| [\varphi_1, \varphi_1] [\varphi_1, \varphi_2] \dots [\varphi_1, \varphi_n] \right\|_1^n$$

$$[\varphi, \psi] = \sum_{j=1}^N \varphi(x_j) \psi(x_j), x_j \in \chi$$

The matrix $F^T F = M$ is an information matrix for the model (3). The matrix $D\hat{\theta} = \sigma^2 M^{-1}$ is a dispersion matrix.

Taking into account the orthogonality of the system of step functions $\{\varphi_i(x)\}_1^n$ on χ , we get the matrix

$$F^T F = \begin{bmatrix} [\varphi_1, \varphi_1] & 0 & \dots & 0 \\ 0 & [\varphi_2, \varphi_2] & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & [\varphi_n, \varphi_n] \end{bmatrix} = \begin{bmatrix} nk_1 & 0 & \dots & 0 \\ 0 & nk_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & nk_n \end{bmatrix} \quad (7)$$

Direct calculation of the determinant of the $F^T F$ matrix gives

$$\Delta^2(Q) = \det[F^T F] = n^n k_1 k_2 \dots k_n \quad (8)$$

and for the non-degeneracy of the matrix $F^T F$, it is necessary that in each subset $d(n, j_n)$ there is at least one point x_j ($j=1, \dots, N$) i.e. the condition must be met

$$k_j \geq 1, j=1, \dots, n$$

LSM estimates of unknown parameters are found for step functions without much difficulty, as the arithmetic mean of y_j observed in this interval.

Therefore, for the LSM estimation of unknown parameters θ of the linear model (3) and their variance, we obtain the following expressions

$$\hat{\theta} = (F^T F)^{-1} F^T Y = \begin{bmatrix} \frac{1}{\sqrt{nk_1}} & \beta_1 \\ \dots & \dots \\ \frac{1}{\sqrt{nk_n}} & \beta_n \end{bmatrix} \quad (9)$$

$$D\hat{\theta} = \sigma^2 (F^T F)^{-1} = \sigma^2 \begin{bmatrix} \frac{1}{nk_1} & 0 & \dots & 0 \\ 0 & \frac{1}{nk_2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{1}{nk_n} \end{bmatrix} \quad (10)$$

where

$$\beta_i = \sum_{j=k_1+\dots+k_{i-1}+1}^{k_1+\dots+k_i} y(x_j), i=1, 2, \dots, n, k_0 = 0 \quad (11)$$

Task 1.

It is required to find a discrete optimal design of the experiment for N measurements, i.e. plan,

$$\xi_N^* = \left\{ \begin{matrix} x_1^*, \dots, x_N^* \\ \frac{1}{N}, \dots, \frac{1}{N} \end{matrix} \right\} \quad (12)$$

optimal according to some given criterion Φ in the set of plans Ξ_N of the form

$$\xi_N = \left\{ \begin{matrix} x_1, \dots, x_n \\ \frac{1}{N}, \dots, \frac{1}{N} \end{matrix} \right\} \quad (13)$$

$$\xi_N^* = \operatorname{argmin}_{\xi_N \in \Xi_N} \Phi[D(\xi_N)] \quad (14)$$

In the future, when constructing optimal experimental plans, it is more convenient to work not with information matrices $F^T F$, but with normalized information matrices [2].

The normalized information has the form

$$M(\xi_N) = \frac{1}{N} F^T F = \frac{1}{N} \sum_{j=1}^N \varphi(x_j) \varphi^T(x_j) \quad (15)$$

for plan (13) or

$$M(\xi_N) = \sum_{j=1}^N p_j \varphi(x_j) \varphi^T(x_j), p_j = \frac{r_j}{N}$$

where r_j number of repetitions x_j .

Let's return to finding the exact experimental plan that is optimal in the sense of the location of a finite number of N points x_j ($j=1, \dots, N$).

As a criterion for the optimality of the plan, consider the criterion standard for the classical theory of experimental planning, $D -$ optimality:

$$\Phi[M(\xi)] = -\ln \det |M(\xi)| \quad (16)$$

From the formula [2] it follows that

$$\xi_N^* = \operatorname{argmax}_{\xi_N \in \Xi} \det M(\hat{\theta}, \xi) = \operatorname{argmax}_{\xi_N \in \Xi} \det M(\hat{\theta}_1, \xi_N), \quad (2.55)$$

where M is the information matrix. Since the determinant of the information matrix in the case of step functions is easily calculated and is equal to.

$$\det M(\hat{\theta}_{(1)}, \xi) = n^n k_1 k_2 \dots k_n, \quad (17)$$

then it is enough to maximize this determinant.

Let $N = \ln$.

Theorem 1.

The determinant of the information matrix (17) in the case of step functions reaches its maximum value when $k_1 = k_2 = \dots = k_n$.

Proof:

$$\text{Let } k_p = \max \{k_1, k_2, \dots, k_n\}, \text{ then } k_1 k_2 \dots k_n \leq k_p^n$$

Equality is achieved when all k_j , $j=1, \dots, n$ are equal to each other. What needed to be proved.

Now let the number of points N not be a multiple of the partition number n , i.e. $N \neq \ln$. Let's determine at which distribution of points x_j the determinant of the information matrix $M(\xi)$ takes the maximum value.

Theorem 2.

The determinant of the information matrix (17) in the case of step functions reaches its maximum value when all k_i differ from each other by no more than one [3].

Indeed, for this we will order the values $k_1, k_2, \dots, k_n$ and we get a variational series,

$$k_{(1)} \leq k_{(2)} \leq \dots \leq k_{(n)},$$

where

$$k_{(1)} \cdot k_{(2)} \dots k_{(n)} = k_1 k_2 \dots k_n = \frac{1}{n^n} \det M(\xi)$$

let

$$k_{(n)} - k_{(1)} > 1 \quad (18)$$

By choosing a new plan in which all $k_{(i)}$, except for $k_{(n)}$ and $k_{(i)}$ remain the same, $k_{(n)}$ replace with $k_{(1)}+1$. For the new plan, the determinant is $(k_{(1)}+1)\dots(k_{(n)}-1) > k_{(1)}\dots k_{(n)}$ i.e. the original plan is not optimal, so assumption (18) is wrong.

Thus, in the optimal plan, all $k_{(i)}$ differ from each other by no more than one. The number of such plans is C_N^S . One of these plans is

$$\xi = \left(\frac{x_1}{N}, \frac{x_2}{N}, \dots, \frac{x_s}{N}, \frac{x_{s+1}}{N}, \dots, \frac{x_n}{N} \right)$$

where the number N is represented as $N = l + S$, $0 \leq S < n$ and

$$k_1 = k_2 = \dots = k_s = l + 1,$$

$$k_{s+1} = k_{s+2} = \dots = k_n = l.$$

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Classification of Digital Images using topological signatures – A Case Study

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Abstract: Topological Data Analysis (TDA) is relatively new field of Applied Mathematics that emerged rapidly last years. The main tool of Topological Data Analysis is Persistent Homology. Persistent Homology provides some topological characteristics of the datasets. In this paper we will discuss classification of digital images using their topological signatures computed with Persistent Homology. We will experiment on the Fashion-MNIST dataset. Using Topological Data Analysis, the classification was improved.

Keywords: TOPOLOGICAL DATA ANALYSIS, PERSISTENT HOMOLOGY, MACHINE LEARNING, COMPUTATIONAL TOPOLOGY

1. Introduction

Image classification has been one of the most trending topics in both research areas machine learning and computer vision. Daily usage of high-quality cameras and social media platforms have provided many pictures that can be used in machine learning. There are machine learning models, especially deep learning models, that have shown state of art performance in image classification. In this paper, we investigate how persistent homology can improve image classification using neural networks with simple architectures.

Homology is a mathematical concept which associates sequences of algebraic objects with topological spaces. One way to study a topological space is to find and compute its homology groups. The motivation behind defining homology groups was that two shapes can be distinguished by examining their holes. For example, a disk is different from a circle, or a disk is not a circle, because the disk is solid while the circle has a hole through it. Homology groups are set of invariants of a topological space. Homology groups characterize the topological space. The main idea of Persistent Homology is to track topological characteristics of a reconstructed from a dataset.

The advantages of using Persistent Homology are the robustness and invariance of the topological signatures which can be traced by Persistent Homology. Topological signatures are global and more resistant to local deformations and computations of these signatures do not depend on the scale of data. The Persistent Homology has found applications in gene expression, cancer detection, chemoinformatics, natural language processing, sensor networks, complex networks, noise detection, signal processing, bioinformatics and many other research areas [1-5].

In this paper we will discuss how persistent homology may improve image classification that using topological signatures combined with original digital images as a dataset for classification task.

2. Persistent Homology of digital image

In this work it is applied Persistent Homology on digital images. CW-complexes are generalization of simplicial complexes that allow cells that are not necessarily simplices, homeomorphic to balls or open discs [6,7]. For example, cubes instead of tetrahedra. In this work it is used CW-complexes that are regular and in this paper for such complexes we will use the term cell complex or simple complex. We will use cell complexes instead of simplicial complexes because digital images are matrices. By the nature they are some kind of cell, grid structures.

Definition 1. A filtered cell complex is (X, F) is a cell complex X together with a monotonic function $f: X \rightarrow \mathbb{R}$. A linear ordering $\sigma_0, \sigma_1, \sigma_2, \dots, \sigma_n$ of the cells in X , such that $\sigma_i \leq \sigma_j$ implies $i \leq j$, is compatible with the function f when

$$f(\sigma_0) \leq f(\sigma_1) \leq f(\sigma_2) \leq \dots \leq f(\sigma_n)$$

We will construct cell complexes from the images and then for the complexes, we will compute Persistent Homology to detect topological signatures of the images.

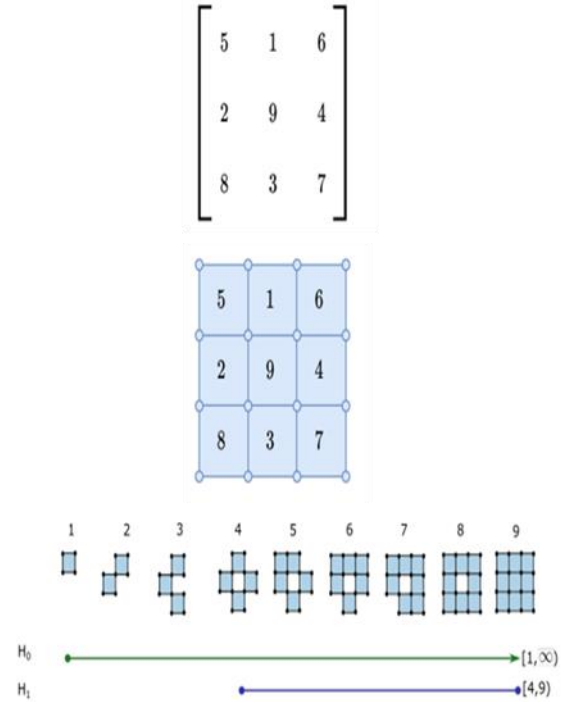


Figure 1. An example of constructing cell complex from two dimensional image together with persistent homology.

In the Figure 1, there is an example of construction of cell complex from a digital image together with persistent homology. More information about construction of cell complexes from digital images and computing Persistent Homology of cell complexes can be found in [7].

3. Combining Topological Signatures with Original Images

The main idea in this work is to combine original images with topological signatures of the images and do a classification task. We start with a dataset consisted of digital images. In the experiments for this paper grey-scale digital images are used. Every image in the dataset is represented with two-dimensional matrix. Firstly, every image of the dataset is preprocessed. After preprocessing, two different datasets are constructed. The first dataset is constructed in a way that every image is represented as a sequence of three two dimensional matrices, we will call them three channels, and every matrix in this sequence is the matrix that represent original image. In other words, on every image of the new constructed dataset three channel matrix and on every channel there is the original matrix of the grey-scale image.

Persistent Homology can be represented with persistence diagrams [9]. In Figure 2, upper right picture, it is shown synthetic generated dataset of points in a two dimensional plane which represent two noisy circles. For this dataset Persistent Homology is computed and it is constructed persistent diagram. The diagram is shown on the left upper picture in Figure 2. The black circles in the persistent diagrams are signatures with dimension 0 and the red triangles are the signatures with dimension 1. More for persistence diagram, as a tool in TDA, can be found in [9]. Then from the persistent diagrams we compute persistent image for dimension 0 and persistent image for dimension 1 that are shown on the two down pictures in Figure 2. Persistent images are stable representation of persistence diagrams. More about persistent images can be found in [10].

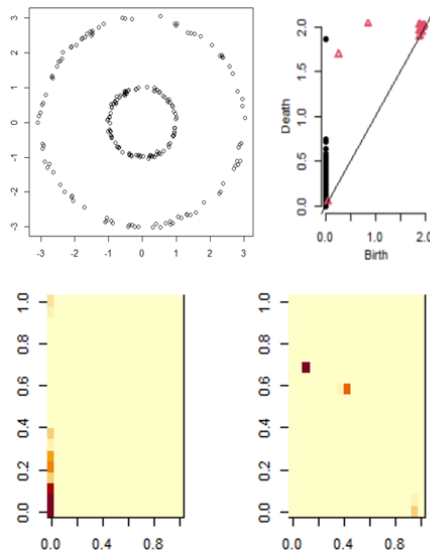


Figure 2 A noisy circles dataset with persistence diagram and persistent images for the dataset

For the construction of the second dataset, there is one additional step. In this step, the persistent homology of the images is computed to track the topological signatures of the images. For each image in the dataset persistent homology is computed and represented with a persistence diagram. After this, persistent image for dimension 0 and persistent image for dimension 1 are computed from the persistence diagram. The dimension of persistent image is the same as the dimension of the matrix of the original images. After these computations, we create a sequence of three matrices. At first place, there is the original image matrix. At second place there is the matrix which represents the persistent image for dimension 0 of the original matrix and, as a third element in this sequence, there is the persistent image for dimension 1 of the original matrix. Finally, the second dataset is constructed of such three element sequences for every original image.

In the experiment we will classify both dataset, the first dataset which is without topological signatures, and the second which is with topological signatures.

4. Experiments

Dataset

For the purpose of this paper, the Fashion-MNIST dataset [11] is used. Fashion-MNIST is well known dataset and is a good starting point to evaluate new model for classification. It has 70000 grey-scale digital images which represent article clothing. The dataset is divided in train set consisted of 60000 images and test set of 10000 images. Each image is 28 pixels in height and 28 pixels in width, for a total of 784 pixels in total. Each pixel has a single pixel-value associated with it, indicating the lightness or darkness of that pixel, with higher numbers meaning darker. This pixel-value is an integer between 0 and 255. The training and test data sets have 785 columns.

The first column consists of the class labels, and represents the article of clothing. Class labels are 0:T-shirt, 1:Trousers, 2: Pullover, 3: Dress, 4: Coat, 5: Sandal, 6: Shirt, 7: Sneaker and 9: Ankle boot, see Figure 3. The rest of the columns contain the pixel-values of the associated image.

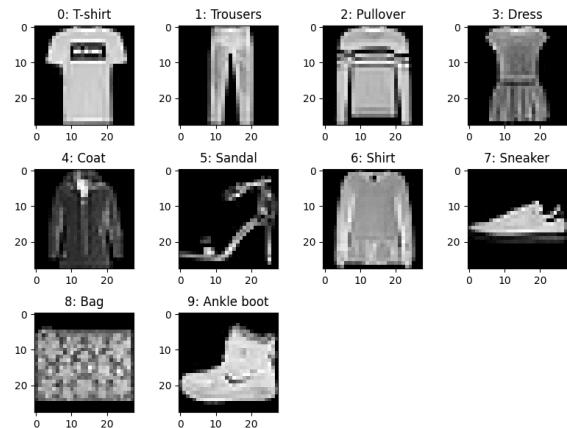


Figure 3 Images from the Fashion-MNIST with their classes

In Figure 3 ten pictures from Fashion-MNIST. Each of them represents one class of the dataset.

Preprocessing of data

In the preprocessing of data, the original dataset's digital images were scale such that the value of the pixels will be in the range from 0 to 1. This process was done using R.

Constructing the persistent images

For each image in the dataset, persistent image for dimension 0 and persistent image for dimension 1 were constructed. The dimension of the persistent images was 28x28. It is the same dimension as the dimension of original images. Then the concatenating these persistent images with the original images, as it is explained in previous section of this paper, was done. And finally the second dataset consisted of original images and topological signatures was created.

Classification

Simple neural network is used as a classifier. First layer transforms the input from three matrices to a row of 2352 elements. The neural network has two hidden layers which are fully connected. The last layer, the output of the network has 10 neurons. "Softmax" is used as an activation function and „adam“ as an optimizer. "Sparse_categorical_crossentropy" is used as a loss function. The accuracy converges after 30th epoch in the training process in both tasks without topological signatures and with topological signatures. The coding is done in R for computing topological signatures, using RCPPL library, and for the classification are used Keras models.

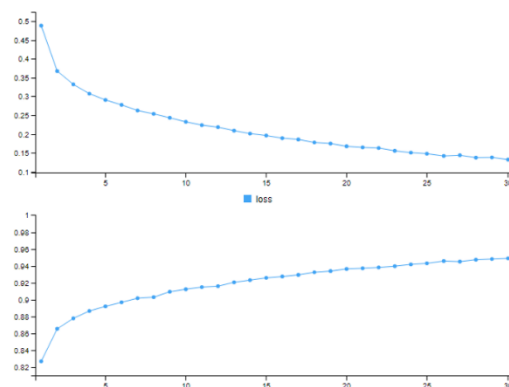


Figure 4. Loss and accuracy of training process of the model without topological signatures.

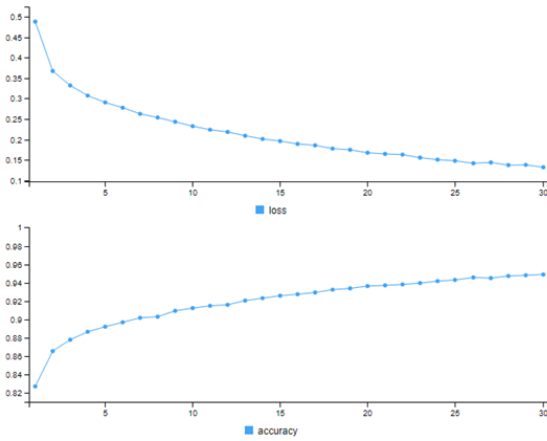


Figure 5. Loss and accuracy of training process of the model with topological signatures.

The loss and accuracy of training process of the model without topological signatures and the model with topological signatures are given in Figure 4 and Figure 5, respectively.

Hardware and time consumption

As it is mentioned, for the classification task it is used neural network with simple architecture. So, we used cheap hardware uncharacteristic for training neural networks. The training process for both the dataset without topological signatures and the dataset with topological signatures ended in less than an hour. Before that the process of computing topological signatures ended in 1 hour. So experiments ended in less than 3 hours.

5. Results

In this section, the results of two models, one without topological signatures and second with topological signatures, are given. For evaluation of this models on the test dataset we used four metrics: accuracy, precision and f1-score. The accuracy for the model without topological signatures is 0.879 and the loss is 0.573. The values of other metrics are given in Table 1.

Table 1. Metrics of testing of the model without topological signatures.

CLASS	PRECISION	RECALL	F1-SCORE
0	0.8534031	0.815	0.83376
1	0.9928058	0.966	0.97922
2	0.8658854	0.665	0.752262
3	0.8957055	0.876	0.885743
4	0.7150974	0.881	0.789427
5	0.9835391	0.956	0.969574
6	0.6666667	0.746	0.704106
7	0.9412341	0.961	0.951014
8	0.9876923	0.963	0.97519
9	0.9543198	0.961	0.957648
AVERAGE	0.8856349	0.879	0.879794

The accuracy for the model without topological signatures is 0.889 and the loss is 0.4. The values of other metrics are given in Table 2.

Table 2. Metrics of testing of the model with topological signatures.

CLASS	PRECISION	RECALL	F1-SCORE
0	0.844898	0.828	0.836364
1	0.98491	0.979	0.981946
2	0.811695	0.819	0.815331
3	0.865275	0.912	0.888023
4	0.822497	0.797	0.809548
5	0.988566	0.951	0.969419
6	0.714141	0.707	0.710553
7	0.949744	0.926	0.937722
8	0.975928	0.973	0.974462
9	0.914019	0.978	0.944928
AVERAGE	0.887167	0.887	0.886829

We can say that two models have good results on this dataset. The accuracy of the model with topological signature is improved. We should accent that we have less loss in the model with topological signatures than in the model without topological signatures

6. Discussion and further work

In this paper, classification model with topological signatures and without topological signatures are evaluated on digital image dataset. From the results of the experiment, two of the models are good. Also, the model with topological signatures has improved results for the metrics. Maybe, the improvements are not significant because this dataset is classified as “easy” for classification task. The results of the model with topological signatures are comparative with state of art models, which are using complex deep network architectures combined with transfer learning.

As, a further work we will define loss function based on the topological signatures of a dataset. Also, we will construct more complex neural network combining convolutional networks, which are concentrating on the local features of the images and topological signatures, which give insight for global features of the images, to investigate the improvements of classification task.

7. Acknowledgement

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A mathematical pathway to achieve sustainable design and friendly management of combined biodiesel/dairy supply chain

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Abstract: This study proposes a mathematical pathway to achieve sustainable design and resource-friendly management of a biodiesel supply chain produced from dairy waste scum. In the work, the features of a combined biodiesel/dairy supply chain are outlined and a set of design and management tools is developed, which is formulated in MILP terms. The optimization criterion is defined in terms of economic sustainability and environmental assessment data is applied as part of it. The purpose of the defined mathematical model is to obtain optimal operating conditions of the considered combined supply chain. A test example based on a case study from Bulgaria was considered. The results of the toolbox implementation can be used as a decision-making tool.

Keywords: MILP, OPTIMIZATION, SUSTAINABLE, SUPPLY CHAIN, BIODIESEL, DAIRY WASTE SCUM

1. Introduction

Energy saving is pointed out as one of the world's problems to solve in the modern world. This is dictated by the gradual but sure exhaustion of natural resources and the complex economic, political and social situation in different parts of the world. Along with this, the need arises to overcome climate change, the degradation (deterioration) of lands and ecosystems. These facts provoke the scientific community to face global challenges, such as finding alternative sources of energy and developing approaches for sustainable use of energy carriers. In the last few decades, biodiesel has been introduced as an analogue of traditional energy carriers and in particular promising alternative source of energy in transport sector [1], [2]. Biodiesel is defined as ecologically clean fuel obtained from practically inexhaustible raw material (biomass).

The first document to pay significant attention to the consumption of biofuels and set the stage for the development of a policy on biofuels was the 1997 White Paper "Energy for the Future: Renewable Energy Sources" [3]. The negatives in the use of conventional fuels, such as environmental pollution, disrupted supplies around the world, energy security [4] reinforce the role of biofuels as a possible substitute for fossil energy carriers. Currently, the role of biofuels could be defined as the "Gold Rush" of industry in the 21st century. Biodiesel is classified as a monoalkyl ester obtained by the transesterification of methanol with higher fatty acids, which may be of vegetable or animal origin [5], [6]. Main raw materials are soybean oil, rapeseed oil, sunflower oil, waste oil from cooking, animal fats and other edible or inedible fats [7]. The replacement of traditional diesel fuels with biodiesel leads to minimization of emissions of greenhouse gases and dust particles in the air [7]. The cost of biodiesel is higher than that of petrodiesel and it's a major difficulty in its commercialization [8]. The cost of raw materials constitutes 75–85% of the production cost of biodiesel [9]. However, what is the real price to pay for biodiesel to be a sustainable alternative to fossil fuels? The problem comes from the fact that a large part of the planet's population is starving or undernourished. In counterpoint to this, the cultivation of oil crops used as raw material for the production of biodiesel takes up agricultural land, thereby reducing the area sown with food crops. Advanced second-, third- and fourth-generation feedstock's such as cellular biomass, microbial oils, algae and others are attracting increasing interest because they have a shorter life cycle, do not require arable land, and can use organic waste as a carbon source [10], [11], [12]. These feedstock has opened new avenue for less expensive and competitive biodiesel production [13]. The food industry and in particular milk processing and the production of dairy products (drinking milk, cheese, yogurt, milk powder, ice cream, melted butter, breeding and others) also generate large amounts of waste biomass with a high fat concentration, which can be used as a potential feedstock in biodiesel production. A large dairy that processes 500,000 liters of milk per day generates about 250-300 kg dairy waste scum per day. A significant amount, which makes it difficult to dispose of it [14]. The dairy waste scum (DWS)

is a non-solid, dense, floating substance, white in texture and color because it is usually formed by a mixture of fats, proteins, lipids and others. DWS is generated during equipment cleaning, processing, storage, packaging and transportation of milk and milk products. A large proportion of dairies dispose of this waste in solid waste storage facilities or incinerate it [15]. This is economically unprofitable and could be seen as a "dissipation" of hidden energy carriers. Waste from the dairy industry causes direct and indirect obstacles to wastewater treatment, and can lead to eutrophication in water bodies. There are mainly four different methods used for production of biodiesel, viz. blending, micro emulsification, pyrolysis, and transesterification. The transesterification is the most convenient and established methods. The reaction of alcohol and triglycerides in presence or absence of catalyst to produce alkyl ester is known as transesterification. The triglyceride molecules having long and branched chains are converted to monoester, known as biodiesel. The high proportions of saturated and mono saturated fatty acids in dairy waste scum oil are considered optimal for fuel production [8].

Apart from the type of feedstock used, the quality and price of the biodiesel produced depends on transport logistics, production and storage technologies, as well as the location of the biorefineries because of which the commercialization of biodiesel is related to the design and management of efficient and sustainable supply chains [16]. The sustainable design and friendly management of supply chain is one possible way to increase the economic and environmental benefits of biodiesel production through optimize all activities across the network from feedstock's, through the production itself, to the customer. This could be achieved through the development and implementation of special mathematical approaches to designing sustainable supply chains

The present study proposes a MILP approach for sustainable design of combined biodiesel/dairy supply chain. Dairy waste scum is considered as a potential raw material for the production of biodiesel satisfying economic and environmental criteria. In the work, the features of a combined biodiesel/dairy supply chain are outlined. Optimization problem have been formulated and solved either at in terms of economic sustainability and environmental assessment data is applied as part of it. The purpose of the defined mathematical model is to obtain optimal operating conditions of the considered combined supply chain. The proposed approach has been implemented on real case study, in which the territory of the Republic of Bulgaria has been considered with its 27 administrative regions. The results of the toolbox implementation can be used as a decision-making tool.

2. Supply chain optimization problem formulation

This study presents a combined biodiesel/dairy supply chain represented in Fig. 1. It comprises suppliers of dairy farms, dairy plants, customers of milk products and bio refinery plants.

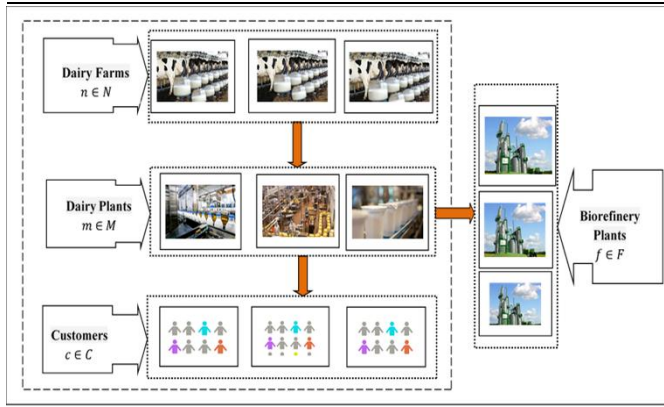


Fig. 1 A combined biodiesel/dairy supply chain

The problem considered in this paper can be formulated as follows Table 1:

Table 1: Problem statement

Given are:	Determine the optimal parameters of the supply chain as:
the location of farms and their production capacity;	dairy farms portfolios;
the selling price of milk;	dairy plants portfolios;
the locations of dairy plants and their capacities;	size, capacity and location of biorefineries that should be built;
the technologies for production of dairy products and the technologies for biodiesel production from dairy waste scum;	optimal performance of biorefineries;
the dairy products and biodiesel demands;	transportation connections.
type of transportation for their delivery;	
greenhouse gas emissions for each stage of the life cycle;	
transportation costs for each transport connection and transportation mode.	

MILP optimization model includes the definition of constant parameters and decision variables; the optimization criterion is defined in terms of economic sustainability and environmental assessment data are implemented as part of it. An objective function and constraints are included. To model the set of time intervals of the planning horizon $\tau = \{1, 2, \dots, T\}$ is introduced. The ecological, economic constants and continuous and binary variables are taken from [17].

The optimization problem can be solved using the GAMS application software and it is possible to use it to make comprehensive smart decisions. After changing the necessary data, the proposed plan can be adapted to different territories.

2.1. Environmental performance of the combined biodiesel/dairy supply chain [18]

The environmental impact of the combined dairy and biodiesel supply chain is represented as the total greenhouse gas emissions generated over the whole live cycle of the production of dairy and biodiesel products. They are converted into carbon credits multiplied by the price of carbon emissions on the market.

The environmental impact criteria include the overall environmental impact of the operation of the combined dairy and biodiesel supply chain, by means of the generated greenhouse gas

emissions for each time interval $t \in T$. The latter includes greenhouse gas emissions generated at each stage of the life cycle of the products. They are determined for each time interval $t \in T$:

$$EIJ_t = EPI_t + EPB_t + EWS_t + ETF_t, \quad \forall t \quad (1)$$

Where:

EIJ_t - Total environmental impact of the operation of the combined dairy and biodiesel/diesel supply chain for the whole life cycle, $[kgCO_{2eq}/d]$;

EPI_t - Environmental impact in the production of dairy products in dairy plants, $[kgCO_{2eq}/d]$;

$$EPI_t = \sum_{i \in I} \sum_{m \in M} \sum_{k \in K} (EIM_{im} CI_{imkt} PIB_{imkt}), \quad \forall t \quad (2)$$

EPB_t - Emissions from biodiesel production using dairy waste scum as feedstock, $[kgCO_{2eq}/d]$;

$$EPB_t = \sum_{j \in J} \sum_{f \in F} \sum_{p \in P} (EFB_{jfp} PBFP_{jft}), \quad \forall t \quad (3)$$

EWS_t - Emissions from unused dairy waste scum for biodiesel production, $[kgCO_{2eq}/d]$;

$$EWS_t = \sum_{j \in J} \sum_{m \in M} \sum_{k \in K} \left(\sum_{i \in I} (\beta_{ijmkt} CI_{ijmkt} CI_{imkt} PIB_{imkt}) - \sum_{l \in L} \sum_{f \in F} (QJ_{jmfl3t}) \right) EDWS_j, \quad \forall t \quad (4)$$

ETF_t - Emissions released during transport of raw materials, products and dairy waste scum to the respective facilities, $[kgCO_{2eq}/d]$.

$$ETF_t = ETNM_t + ETMC_t + ETMF_t, \quad \forall t \quad (5)$$

2.2. Economic performance of the combined biodiesel/dairy supply chain [18]

For definition of the economic impact assessment of the supply chain operation the annual operating costs are used. The economic criterion includes the total investment costs for the biodiesel plant, costs for building the biodiesel plants and the operation of the combined dairy and biodiesel supply chain for the planned time period. They define for each time interval $t \in T$ as:

$$TDC_t = TIC_t + TPC_t + TDWS_t + TDKN_t + TPI_t + \quad (6)$$

$$TPW_t + TTC_t + TCO2_t - TB_t - TL_t, \quad \forall t$$

TDC_t - the total costs of the combined supply chain per year, $(\$/y)$;

TIC_t - the total investment costs for the production capacity of the combined supply chain according to the operation period and the purchase of the biodiesel plant per year, $(\$/y)$;

$$TIC_t = \varepsilon_t \sum_{f \in F} \sum_{p \in P} (Cost_{pft}^F Z_{pft}), \quad \forall t \quad (7)$$

TPC_t - the production costs for the total quantity of biodiesel per year, $(\$/y)$;

$$TPC_t = \alpha_t \sum_{f \in F} \sum_{j \in J} \sum_{p \in P} \sum_{l \in L} \sum_{m \in M} (UPB_{jpft} QJ_{jmfl3t}), \quad \forall t \quad (8)$$

$TDWS_t$ - the dairy waste scum costs for the biodiesel production per year, $(\$/y)$;

$$TDWS_t = \alpha_t \sum_{f \in F} \sum_{j \in J} \sum_{p \in P} \sum_{l \in L} \sum_{m \in M} \sum_{i \in I} \sum_{k \in K} (PJ_{jimkt} CI_{ijmkt} CI_{imkt} QJ_{jmfl3t}), \quad \forall t \quad (9)$$

$TDKN_t$ - the raw materials (milk) costs for the production of dairy products per year, $(\$/y)$;

$$TDKN_t = \alpha_t \sum_{n \in N} \sum_{m \in M} \sum_{k \in K} \sum_{l \in L} (UNMK_{nmkt} CK_{knt} QKNM_{knl1t}), \quad \forall t \quad (10)$$

TPI_t – the production costs for dairy products per year, (\$/y);

$$TPI_t = \alpha_t \sum_{i \in I} \sum_{m \in M} \sum_{k \in K} (UIMK_{imkt} PIB_{imkt}), \quad \forall t \quad (11)$$

TPW_t – the production costs for utilization of the unused dairy waste scum remaining from biodiesel production per year, (\$/y);

$$TPW_t = \alpha_t \left(\sum_{j \in J} \left(\sum_{m \in M} \sum_{k \in K} \left(\beta_{jmkt} CI_{jmkt} PIB_{imkt} \right) - \sum_{l \in L} \sum_{f \in F} (QJ_{jmf3t}) \right) \right) UPW_{jt}, \quad \forall t \quad (12)$$

TTC_t – the total transportation costs for the combined supply chain per year, (\$/y);

$$TTC_t = TNM_t + TMC_t + TMF_t, \quad \forall t \quad (13)$$

TTC_t are the total transportation costs for transportation of raw materials (milk), dairy products and dairy waste scum per year (\$/y);

TNM_t are the total transportation costs for transportation of raw materials (milk) per year (\$/y);

TMC_t are the total transportation costs for transportation of dairy products per year (\$/y);

TMF_t are the total transportation costs for transportation of dairy waste scum per year (\$/y).

$TCO2_t$ – the carbon tax charged according to the total amount CO_2 released during the operation of the combined supply chain per year, (\$/y);

$$TCO2_t = \alpha_t EIJ_t CCO2_t, \quad \forall t \quad (14)$$

TB_t – the revenue from the sale of biodiesel produced per year, (\$/y);

$$TB_t = \alpha_t PB_t \sum_{j \in J} \sum_{p \in P} \sum_{f \in F} (PBFP_{jfp}), \quad \forall t \quad (15)$$

TL_t – the government incentives for biodiesel production and building per year, (\$/y).

$$TL_t = \alpha_t \sum_{j \in J} \sum_{p \in P} \sum_{f \in F} (INS_{ft} PBFP_{jfp}), \quad \forall t \quad (16)$$

2.3. Constraints

The constraints are linear functions with respect to:

- ✓ the production capacity of biorefineries;
- ✓ providing admissibility of flows;
- ✓ providing that the needs of all regions of dairy products will be met;
- ✓ providing the necessary amounts of milk for the production of dairy products;
- ✓ logical and transportation constraints;
- ✓ providing that produced dairy waste scum from dairy plants will be used for the biodiesel production;
- ✓ providing satisfaction of the material balance of combined supply chain;
- ✓ providing satisfaction of the annual demand of the dairy plants;
- ✓ providing maximum annual production capacity of farm $n \in N$ for the production of milk of type $k \in K$;
- ✓ providing admissibility of flows.

2.4. Objective function

The optimization problem for determining the optimal location of the plants in the regions and their parameters is formulated as:

Find: X_t [Decision variables]

MINIMIZE { $COST(T_t)$ } s. t.: {Constraint s}

$$COST = \sum_{t \in T} (LT_t TDC_t) \quad ()$$

where

LT_t - duration of time intervals $t \in T$, (y). The objective function and all constraints are linear functions of all decision variables.

3. Results and discussion

The formulated above optimization problem have been solved in terms of economic sustainability, and environmental assessment data are implemented as part of it. For that purpose, GAMS® optimization software-CPLEX solver has been used. The prepositional mathematical model is implemented on an example study from Republic of Bulgaria. The obtained results are demonstrated in Fig. 2 and Fig 3.

Fig.2 presents the distribution of GHG emissions generated at each stage of the supply chain operation and shows that the largest amount of GHG emissions are generated during the production of milk products in dairy plants, as well as during disposal of unused waste biomass for biodiesel production.

Fig. 3 presents the cost structure of the supply chain and shows that the largest cost is the carbon tax, calculated according to the total amount of CO_2 generated during the operation of the supply chain and the production costs of the utilization of the unused waste biomass to produce biodiesel

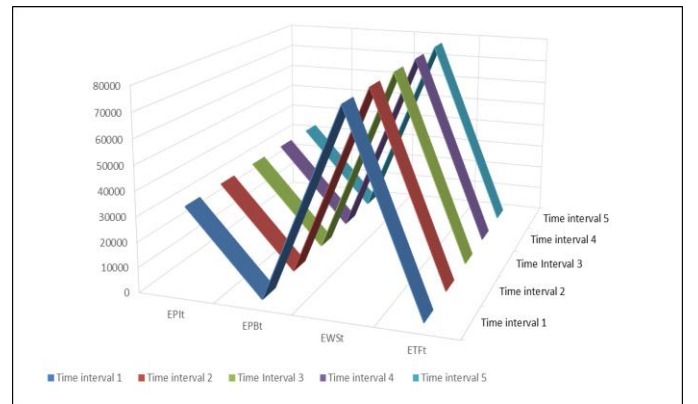


Fig. 2 Greenhouse gas emissions generated at each stage of operation of the combined supply chain

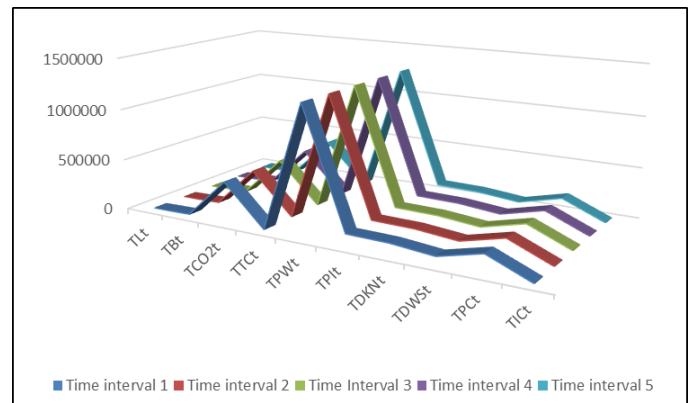


Fig. 3 The costs for operation of combined dairy and biodiesel supply chain.

4. Conclusion

The study proposes a MILP model for sustainable design and friendly management of combined biodiesel/dairy supply chain. The proposed approach has been based on example case study, in which the territory of the Republic of Bulgaria with its 27 administrative regions is considered. The optimization criterion is defined in terms of economic sustainability and environmental assessment data is applied as part of it. The proposed task is solved using the application software GAMS and it can be used as a decision-making tool.

Acknowledgments

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Appendix A

- Sets, subsets and indices
- K Set of types of milk used as a raw material in dairy plants,
- k
- I Set of produced dairy products in dairy plants generating dairy waste scum, i
- J Set of generated types of dairy waste scum in dairy plants,
- j
- P Sets of the capacities of used biorefineries for biodiesel production, $p=(1, N_p)$
- GF Sets of the administrative regions into which the territory of Republic of Bulgaria is divided, gf
- L Set of vehicles used for transportation of raw materials and products, l
- T Sets of time intervals, t
- Subsets/indices
- F Sets of regions for biodiesel production. It is a subset of GF ($F \subset GF$), f
- M Set of regions where dairy plants are built. It is subset of GF ($M \subset GF$), m
- N Set of dairy farms which produce milk used as a raw material in dairy plants. It is subset of GF ($N \subset GF$), n
- C Set of regions where dairy products are used. It is subset of GF ($C \subset GF$), c
- L1 Set of vehicles used for transportation of raw materials (milk), l1. It is subset of L ($L1 \subset L$)
- L2 Set of vehicles used for transportation of dairy products, l2. It is subset of L ($L2 \subset L$)
- L3 Set of vehicles used for transportation of the types of dairy waste scum, l3; It is subset of L ($L3 \subset L$)

Irrigation Scheduling for Some Crops Under Arid Environment of Rosso, Mauritania

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Abstract: The study was performed to identify irrigation scheduling of some common crops such as seed-maize, tomato, carrot, rice, alfalfa, and tomato plants at arid Rosso province, Mauritania. In that purpose, crop water requirement, ET_c , for those crops were calculated by Cropwat software computer program. In results, ET_c values for Rice, Alfalfa, Carrot, Maize (grain), and Tomato crops were calculated as about 1378 mm, 1542 mm, 814 mm, 705 mm, and 956 mm respectively. The irrigation water requirements, I , for the examined crops were found as 1677 mm, 1535 mm, 811 mm, 699 mm, and 950 mm, respectively. In accordance with our findings, the following suggestions can be considered; crop pattern should be designated in accordance with current water resources; water delivery systems should be converted to modernized systems; deficit irrigation can be applied for some crops; farmers should be educated about correct water management, and there should be strong relationships between landowners and water managers.

Keywords: IRRIGATION SCHEDULING, CROP PATTERN, CROPWAT MODEL, EVAPOTRANSPIRATION

1. Introduction

One of the great interests in arid and semi-arid environment is productive utilization of limited water resources in agriculture. The main target of irrigation is to enhance water savings [1]. Water is a vitally important element in agriculture, as well as for all living creatures and agriculture is the highest water consumer with 70% in the world in general, 90% in developing countries [2], and around 80% in Central Anatolian Region, Türkiye. Therefore, water saving must be started in irrigation processes, and even little amount of water saving in irrigation will lead to notable contributions of water in both the industrial and residential sectors.

Experience of landowners in irrigation system managements, is possibly one of the most important factors in achieving desired irrigation performance. In that context, improving farmer's qualifications particularly in water management at farm levels results in improving irrigation success [3].

Adequate water management is necessarily prerequisites for maximum crop returns particularly in water-starved regions like Rosso province of Mauritania. In addition, proper agricultural water management is more important than selection of the irrigation techniques. In Rosso, province, main income of nationals is obtained from the farming activities. Grain-maize, sunflower, wheat, carrot, potato and tomato are main crops in region. Irrigation is necessarily prerequisites for increasing the crop yield as well as quality in the region since annual rainfall is less than 200 mm in general. The distribution of such rainfall is not uniform among the months. In other word, each drop of rainfall as well as irrigation water is just like a pearl for farmers. Therefore, the maximum yield from unit-applied water is the main objective of irrigation managers in such environments.

Productive use of current water resources is key issue for sustainable irrigated agriculture especially in such kind of arid and semi-arid environments [4].

One of the practical ways for calculation of crop water requirement is application of Cropwat computer software program. This model is very beneficial for water resources allocation works, irrigation water planning and accomplishing greater efficiency in agro-productivity [5]. In other word, Cropwat is very practical way for agro-meteorologists, agronomists, and irrigation managers to perform standard calculations of consumptive water use and crop water requirement studies and more specifically design and management of the irrigation systems. It facilitates the practical suggestions for productive irrigation practices, planning of the irrigation programs under various water supply conditions, and comparatively evaluation of agro-production under rain fed and deficit irrigation conditions [6]. Well-management of crop patterns is also one of the most important strategies in irrigation scheduling. The yield response factor, ky , is a good indicator for the

interpretation of crops to different water stress conditions. The ky values for maize, alfalfa and tomato were calculated as 0.96, 1.00, and 0.88 in semi-arid Qazvin province of Iran [7]. Those values were determined as equal or lower than 1.00 so those crops were accepted as water stress tolerant crops, and they well response to the deficit irrigation.

Crop water requirements depend highly on location, vegetation cycles, seasons, soil, and moisture availability in rooting depth, crop varieties, and irrigation techniques. Water stress conditions within the crop rooting systems have negative effect on crops leading to reductions in stoma conductance, transpiration rate which affects crop yield performance [8]. In one study [9] irrigation water for wheat, grain-maize, and barley crops were determined as about 336 mm, 344 mm, and 308 mm, respectively in Al-Qassim province of Saudi Arabia. The ET_c values of such crops were found as 462 mm, 465 mm, and 423 mm, respectively in that research.

The grain-yield of maize crop in Giza province of Egypt was reported as an average of two-year around 7.82 t/ha for fully and 4.65 t/ha for marginal water stress conditions. ET_c for those irrigation levels were found as 603 mm, and 537 mm, respectively. It is very practical way in irrigation water management in farm level to apply irrigation water by using 1.0 pan evaporation coefficient to save more water as well as greater water productivity [10].

In dry lands, deficit irrigation is feasible solution for obtaining satisfactory water savings in irrigation processes. Maize is one of the best-suited field crops to the certain water stress conditions. In that regard, Attia [11] found no significant yield reduction in maize crop under 80% of ET_c by comparison to 100% of ET_c . Similarly Demelash [12] stated that based on Cropwat irrigation program of 2-days interval during whole vegetation cycles, 75% of full irrigation could be implemented for well tuber performance of potato crop as well as notable water savings which is vitally important for scarce water ecologies. The author declared the marketable tuber yield of 26.83 t/ha for full irrigation and 25.68 t/ha for 75% of full irrigation.

The grain yield of corn for full, deficit, and rain-fed conditions were reported as 11.8 t/ha, 10.1 t/ha, 5.6 t/ha, respectively [13].

In the literatures, almost none study was performed for determination of irrigation program of common crops in Rosso region of Mauritania so the aim of the current study therefore, is to design irrigation scheduling for widely planting crops in accordance with Cropwat modelling, and to propose some applicable recommendations for successful agricultural water management particularly at poor water ecologies.

2. Materials and Methods

The study site, Rosso province of Mauritania Northwest Africa nearby Atlantic Ocean, is located at 16.50 °N and 15.81 °W with

altitude of 6m (Fig. 1).



Fig. 1 Position of Rosso province in Mauritania

The region is characterized as arid environment due to the long-year average annual precipitation of 180 mm. The main source of income of such province is based on agricultural production. Irrigation water obtained from the surface water supply, which is on tail of the Senegal River and crops are irrigated by surface irrigation methods. The soil is Clay-Loam (CL) in general at research location.

In this study, irrigation scheduling was done using Cropwat software program for carrot, tomato, alfalfa, maize (grain), and rice crops for Rosso province, Mauritania. The crop patterns of those crops in research site 12.5 %, 12.5%, 25%, 25%, and 25%, respectively. The climate data obtained from Meteorological station of Rosso mostly relevant to 2000-2020 were used in calculation of evapotranspiration. The CROPWAT software (8.0) was used to determine radiation MJ/m²/day and reference evapotranspiration, ETo, as mm/day [6].

In calculation of ETo as mm/day, mean monthly min. temperature (°C), mean monthly max. temperature (°C), mean monthly relative humidity (%), mean monthly wind speed (m/s), mean monthly sunshine (hour) were used in Cropwat program. The effective rainfall was determined in accordance with mean monthly rainfall (mm). FAO Penman-Monteith Formula was used to estimate ETo [14];

$$ETo = \left(\frac{0.408 \Delta (R_n - G) + \gamma \frac{900}{T + 273} u_2 (e_s - e_a)}{\Delta + \gamma (1 + 0.34 u_2)} \right)$$

where; ETo = reference evapotranspiration (mm day⁻¹),

Rn = net radiation at the crop surface (MJ m⁻² day⁻¹),

G = soil heat flux density (MJ m⁻² day⁻¹),

T = mean daily air temperature at 2 m height (°C),

u-2 = wind speed at 2 m height (m s⁻¹),

es = saturation vapor pressure (kPa),

ea = actual vapor pressure (kPa),

es - ea = saturation vapor pressure deficit (kPa),

Δ = slope of the saturated vapor pressure curve (kPa °C⁻¹),

γ = psychrometric constant (kPa °C⁻¹).

The mean rainfall for 20-year period was determined and the data with monthly basis was used in CROPWAT for calculating effective rainfall. The ratio of rainfall that can be productively used by crops was referred to effective rainfall. Although there are some methods for calculating effective rainfall, USDA soil conservation service methodology was used in current study.

Crop water use (ETc) was then calculated by multiplying the crop coefficient (Kc) with ETo. Irrigation requirement was calculated by using following formula [6];

$$\text{Irrigation requirement, I (mm),} = ETc - Reff$$

where; Reff - effective rainfall (mm).

3. Results and Discussions

3.1- Analysis of Rainfall Pattern

There is big fluctuation in rainfall amount among the months for periods between 1990-2021. In accordance with average 32-year, the lowest rainfall recorded at November as 0.17 mm, December as 2 mm, January as 3.82 mm, and February as 1.01 mm. The greatest precipitations as an average were observed in August as 102.3 mm, and in September as 82.2 mm. In examining yearly rainfall amount, the highest value was found in 2010 as about 478 mm, and the lowest one was in 2014 as 77.3 mm (Fig. 2). As seen in fig. 2, rainfall distribution is none uniform in both the months and years. It was very low in some years, so correct agricultural water management is very important for sustainable irrigation in our study region of Rosso province.

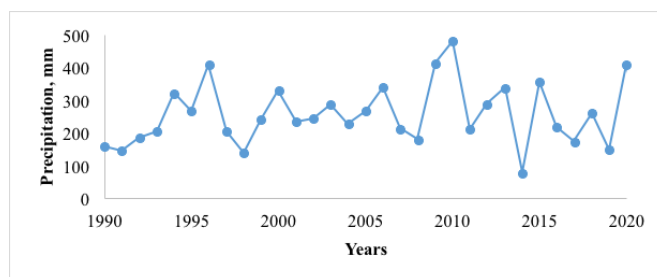


Fig. 2 Rainfall regimes during periods 1990-2020

3.2- Analysis of Humidity Trends

In general, maximum relative humidity, RH, were recorded at July, August and September. The maximum and minimum RH were found as 55%, and 10% in August 2005, and March 2000, respectively. In examining the average of the last 20-year, the RH were less than 20% in December, January, February, March, April, and May. There is very nice relationship between RH and precipitation values. As expected, increase in RH has resulted in an increment in precipitation amount as well. In accordance with RH assessment, RH values were low in most periods so the study region is almost dry environment. The low RH has led to high evaporation and transpiration as a consequence greater evapotranspiration. Therefore, it is impossible to get economical benefits from agricultural activities without irrigation practices. Each drop of water is very useful so whole precipitation and applied water must be stored within the crop rooting depth with minimal losses.

3.3- Evapotranspiration (ETc) and Irrigation water requirement (I)

There are different methodologies such as water balance, soil moisture monitoring, and mathematical equations for the calculation of evapotranspiration. The water balance / budget, and monitoring soil moisture have some difficulties and it is time consuming. Therefore, using computer software program is highly practical in irrigation scheduling studies. In current study, as mentioned in methodology section ETc was computed using well-known

computer program namely Cropwat model which was derived by FAO so as to arrive at a better result in short time. The sample ETc and irrigation water requirement, I, calculation for grain-corn is shown in fig 3.

ETc highly depends on geographical position of study site, meteorological conditions such as temperature, humidity, wind speed, sunshine hours, and crop properties such as crop variety rooting depth, leaf characteristics, growing stages.

The ETc for Rice, Alfalfa, Carrot, Maize (grain), and Tomato plants were calculated as 1377.6 mm, 1541.7 mm, 814.2 mm, 704.5 mm, and 955.6 mm, respectively (Fig. 4). The highest ETc was found in Alfalfa (1541.7 mm), and the lowest one was obtained from grain-maize crop (704.5 mm). The ETc values highly varied from the crop growth stages. For example, the vegetation period of maize crop was about 130-day starting from late December and ending early January. The ETc was found to be high between Late-October and End-January, and ETc was found 346.6 mm (49.2% of whole ETc) in those stages. So water requirement of such crop should be met properly particularly in those growth cycles. Carrot plant used 79.2% (644.7 mm) of whole crop water consumption in periods Early-March-Early-May (Fig. 5).

The irrigation water requirements, I, for those crops were found as 1677.3 mm, 1535.0 mm, 810.9 mm, 698.8 mm, and 949.8 mm, respectively (Fig. 4). The ETc values of grain-maize were reported as 570 mm in Giza region of Egypt [10], and 465 mm in Al-Qassim province of Saudi Arabia [9]. Evapotranspiration of Alfalfa, maize and tomato were calculated as 1541.1 mm, 814.2 mm, and 955.6 mm in semi-arid Qazvin province of Iran [7]. The result of current study is higher than those findings [10] and [9], and is almost in conformity with the finding of Shirshahi et al. [7]. The reason behind could be the differences in evapotranspiration determination methodologies, crop cultivars, ecological conditions, and managerial processes of cultural practices in study regions.

Crop Water Requirements

ETc stationROSSO

CropMAIZE (Grain)

Rain stationROSSO

Planting date26/10

Month	Decade	Stage	Kc coeff	ETc mm/day	ETc mm/dec	Eff rain mm/dec	Irr. Req. mm/dec
Oct	3	Init	0.30	1.92	11.5	0.9	10.7
Nov	1	Init	0.30	1.88	18.8	0.1	18.7
Nov	2	Deve	0.36	2.22	22.2	0.0	22.2
Nov	3	Deve	0.62	3.77	37.7	0.0	37.7
Dec	1	Deve	0.90	5.31	53.1	0.2	52.9
Dec	2	Mid	1.17	6.78	67.8	0.3	67.4
Dec	3	Mid	1.27	7.73	85.0	0.4	84.5
Jan	1	Mid	1.27	8.14	81.4	0.6	80.8
Jan	2	Mid	1.27	8.47	84.7	0.7	84.0
Jan	3	Late	1.26	8.64	95.0	0.7	94.3
Feb	1	Late	1.01	7.18	71.8	0.7	71.1
Feb	2	Late	0.70	5.16	51.6	0.7	50.8
Feb	3	Late	0.44	3.42	24.0	0.4	23.5
					704.5	5.7	698.8

Fig. 3 ETc and I calculations for grain-maize with Cropwat computer program

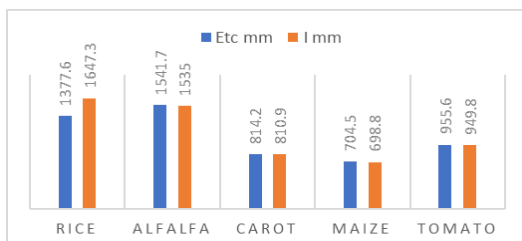


Fig. 4 Evapotranspiration and irrigation water requirement of study crops

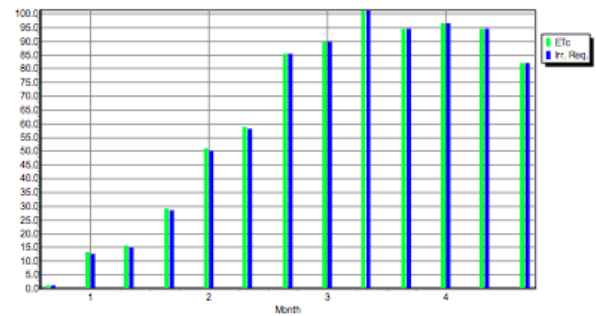


Fig. 5 ETc and irrigation water requirement of carrot plant

4. Conclusion

Considering the current study, water saving in irrigation is one of the most important interests in water shortage or dry environments. The main target of irrigation water managers is to obtain maximal crop production with unit-applied water. In that context, the following solutions could be recommended for sustainable irrigation in such kind of climates: 1- correct irrigation program and its application on irrigated agriculture. Well management of limited water resources results in rising water productivity; 2- Determination of evapotranspiration with computer program, such as Cropwat, is highly feasible and time saving in comparison to the other methods like soil moisture monitoring. It is possible to say that Cropwat model yielded good results, subsequently could be recommended for future studies; 3- if possible shifting surface irrigation systems to the pressurized irrigation techniques such as drip or sprinkler irrigation method. Those techniques lead to greater water savings under well management. If farmers have to use surface irrigation methods, they should design the basin, border, and furrow irrigation systems in accordance with soil properties, availability of water resources, and field slope. For example basin and border size should be shortened under conditions of high field slope, light soils, and low stream size; 4- application of deficit irrigation strategy. The main goal of deficit irrigation is to get maximum yield or returns with unit-used irrigation water. It is more beneficial to apply water deficiency as crop growth cycles based than certain amount (standard) of water deficiency in each irrigation process. There are several studies showing the deficit irrigation effect on crop yield performance. For example; Acar et al. [15] reported no significant yield reductions between full and 75% of full irrigation in sugar beet, grain-maize, sunflower, dry bean, potato and so on under semi-arid Konya province of Türkiye; Seid et al. [16] obtained no notable yield difference between full and 85% of full irrigation at Melkassa province of Mauritania; and Gadedjisso-Tossou et al. [17] stated no remarkable yield reduction between full and 80% of full irrigation in maize crop under Northern Togo conditions; 5- crop pattern must be planned in accordance with current water resources. The farm-size cultivating less water consuming crops such as sunflower, chickpea, and cereals should be enlarged and landowners producing less water using crops should be subsidized more; 5- Farmers should be educated about agricultural water management at farm levels with sample practices under field conditions.

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Conflict of interests

The authors have not declared any conflict of interests.

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Optimization of Cooling Performance of Horizontal Commercial Refrigerator Prototype with Mathematical Modeling

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Abstract: In recent years, the problem of food safety and traceability has been one of the difficulties faced by restaurant and hotel businesses. Food spoilage can occur at any stage of the food chain; most of all food spoilage is due to improper storage conditions in refrigerators. Increasing attention to food quality and safety requires the development of sensitive and reliable analysis methods as well as innovative technologies for maintaining freshness and food quality. Freshness is a factor that has both health qualities and features that affect customer satisfaction and business profitability. Within the scope of this study, simulation studies were carried out to improve the cooling performance with mathematical modeling on a horizontal commercial refrigerator (operating temperature: $-2/+8^{\circ}\text{C}$) prototype. Along with the original design, evaluations were made with mathematical modeling according to many different variables (temperature, airflow distribution, etc.) that affect the cooling performance, and R&D gains were achieved in developing the prototype according to the most suitable working conditions.

Keywords: COMMERCIAL REFRIGERATOR, ENERGY EFFICIENCY, MATHEMATICAL MODELING

1. Introduction

Commercial cooler units are widely used in large enterprises (restaurants, hotels, shopping malls, dormitories, etc.) to keep food and beverages in suitable conditions for a long time. In particular, research and developments are continued on cooling products to ensure that the food chain is not interrupted and to provide adequate cooling properties according to many variable parameters (eg refrigerant [1-4], different condenser [5-7], evaporators [8-10] and compressors [11-13], etc.).

Our company's refrigerator products include commercial refrigerators and deep freezers (in different models and capacities). Within the scope of this study, activities aimed at improving the cooling performance were carried out with the original designs to be obtained by mathematical modeling and simulation studies on the horizontal-type refrigerator.

Since most of the studies in the literature are related to domestic refrigerator products, it has been determined that the studies on commercial products are limited. Various studies from literature research in this field in relation to refrigerator products are summarized below.

Mathematical applications for thermal analysis were carried out by Ledesma and Belman in order to keep the food in the fresh food compartment in the most suitable conditions by replacing the glass-sectioned shelves of the domestic refrigerator. The image of the displacement of the glass shelves for the thermal behavior examinations in the different compartments in the refrigerator is given in Fig. 1 [14].

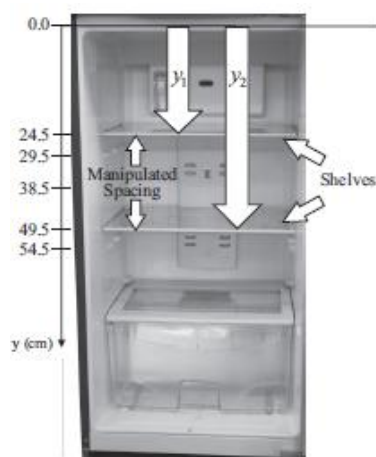


Fig. 1 Position of shelves (y_1 and y_2) and manipulated spacing in a domestic refrigerator [14].

Tests were carried out to evaluate the thermal difference of the domestic refrigerator according to the change of compartments. In the tests, the refrigerator was operated at a constant room temperature of $294\text{ K} \pm 1.5\text{ K}$ and a relative humidity of $60 \pm 5\%$. Evaluations were made to determine suitable shelf locations with 2D color temperature variance maps. A thermal map representing the temperature means in the fresh food compartment when the position of the glass shelves shown in Fig. 1 is changed (y_1 and y_2) is given in Fig. 2. High temperatures are shown in black. In contrast, the lowest temperatures (-3.3°C) are shown in dark blue [14].

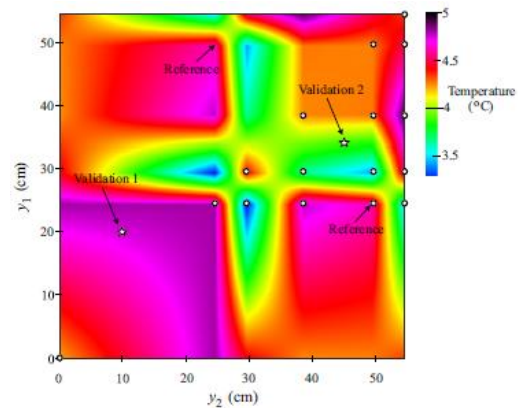


Fig. 2 Temperature mean in the fresh food compartment of the domestic refrigerator. [14].

Roccate and his work group evaluated probability density distributions for domestic refrigerator temperatures and storage times of refrigerated foods with long or short expiration dates for northern and southern European countries. They reported that storage time and temperature stand out as the two main factors affecting microbial growth and that there may be different approaches according to the purpose within the framework of food safety assessment. They emphasized the importance of shelf life studies and food safety risk assessments in domestic refrigerators [15].

Belman and Munoz analyzed a refrigerator compartment's flow and thermal behavior, where the cooling effect is based on diffusion-absorption technology using CDF. They compared the thermal behavior of a plate-evaporator design with a finned surface (reference refrigerator) and a plate-evaporator with a smooth surface (recommended design). The refrigerator's energy performance was analyzed according to the design of the plate evaporator [16].

A comparative analysis of the COP was made based on the design of the plate evaporator, and the cooling capacity of the refrigerator differed by about 0.5 W, while the COP difference was determined as 0.008. With this result, it was concluded that the plate evaporator design affected the thermal and energy performance of the refrigerator at a minimum level [16].

Belman and his work group evaluated various parameters that could increase the energy efficiency of a household refrigerator. It has been reported that the domestic refrigerator includes cabinet improvements (insulation, magnetic seals), refrigeration system improvements (adequate performance of components, use of harmless refrigerants), mechanism improvements (defrosting, temperature control), and minimal thermal stratification in compartments [17].

Chen and his work group were evaluated by comparing the modified ejector-enhanced dual temperature refrigeration cycle (MERC), and the basic ejector-enhanced dual temperature refrigeration cycle (BERC) with cycle performance by performing thermodynamic modeling with energetic and exergetic analysis methods. With the simulation data, they determined that the COP (coefficient of performance) and q_{ev} (volumetric refrigeration capacity (kJ.m^{-3})) values of MERC improved by 23.1% and 34.7% compared to BERC under general operating conditions of domestic refrigerator/freezer. The comparison of exergy destruction (E_{xd}) between the modified ejector-enhanced dual temperature refrigeration cycle (MERC) and basic ejector-enhanced dual temperature refrigeration cycle (BERC) is given in Fig. 3 [18].

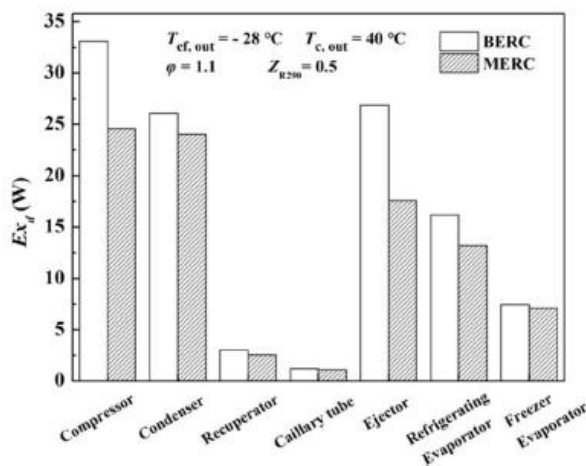


Fig. 3 Comparison of exergy destruction values between MERC and BERC (Z : mass fraction of R290 in the mixture, ϕ : cooling capacity ratio) [18].

Each component (see Fig. 3) of the modified ejector enhanced dual temperature refrigeration cycle (MERC) shows lower levels of exergy destruction compared to the basic ejector enhanced dual temperature refrigeration cycle (BERC). They conducted studies on energy modeling, exergy modeling, and ejector model validation within the scope of mathematical modeling [18].

2. Methodology

The temperature and airflow distribution in the horizontal commercial refrigerator compartments have been studied in the literature. Numerical methods have been used to predict the temperature distribution inside the refrigerator. By 3D simulations, temperature uniformity and airflow in a natural convection refrigerator can be improved. In this work, a numerical simulation is used to model the performance of a convection refrigerator. The compartments are investigated to improve operating conditions. Through simulations, we proposed a new design for the shelves in the air outlets achieving a better thermal profile. A thermal camera to observe the temperature distribution of the horizontal commercial

refrigerator as an alternative method to analyze the cooling system; we compared the thermal behavior of the new design with conventional thermocouples.

The temperature distribution in the compartments is investigated numerically and experimentally, and, in most cases, results with numerical simulations are compared. The method to reach temperature uniformity inside the horizontal commercial refrigerator and temperature distribution influenced by the geometric configuration, the accessory position in the compartment, and the refrigerating space are investigated.

3. Experimental procedure

The results of various simulation analyses (velocity distribution, distribution of streamlines, time-dependent thermal analysis) on the horizontal commercial refrigerator prototype are shown in Fig. 4-7.

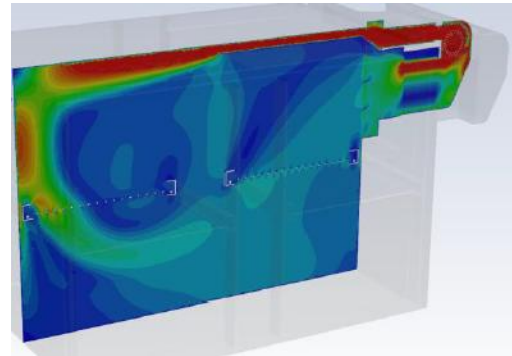


Fig. 4 Flow analysis (velocity distribution) of a horizontal commercial refrigerator prototype.

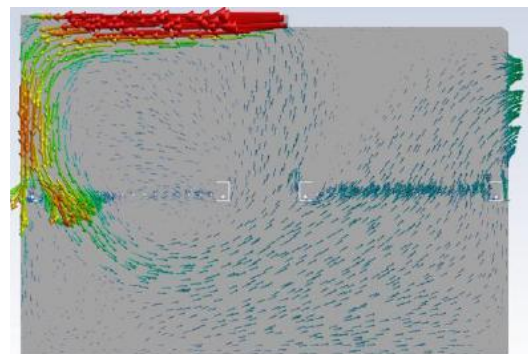


Fig. 5 Flow analysis (distribution of velocity vectors in the cross-section) of a horizontal commercial refrigerator prototype.

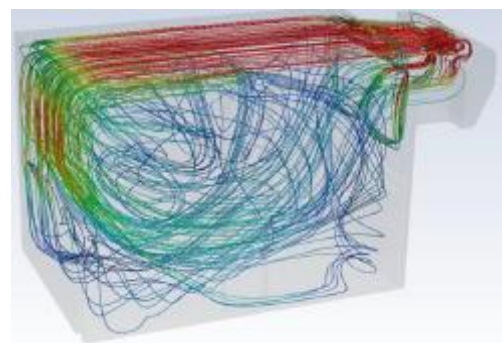


Fig. 6 Flow analysis (distribution of streamlines) of a horizontal commercial refrigerator prototype.

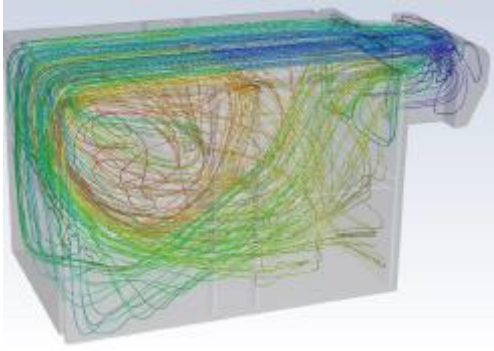


Fig. 7 Time-dependent thermal analysis (time: 9.6 sec.) of a horizontal commercial refrigerator prototype.

As seen in Figs. 4-6, flow analysis shows a good velocity distribution in the horizontal commercial refrigerator prototype. Fig. 7 shows that surfaces are colder than compartments near the trays. This is improved by adding sensors in several places.

4. Conclusions

Mathematical modeling and simulation studies were carried out to optimize the cooling performance of the horizontal commercial refrigerator prototype.

According to the positioning of the cooler group elements (compressor, evaporator, fan, etc.) on the existing product, evaluations were made regarding the distribution of the cooling air in the cabinet of the horizontal commercial refrigerator prototype (according to the blowing and suction conditions of the air).

Various flow analyses (velocity distribution, distribution of streamlines, time-dependent thermal analysis) on the horizontal commercial refrigerator prototype have provided data on improving the effective and efficient operating performance.

A mathematical method is considered to analyze the thermal behavior of the compartments in a horizontal commercial refrigerator. This work mainly focused on studying the thermal effect in the horizontal commercial refrigerator compartment when the shelves' location is changed. Finally, we proposed using thermal maps to find the most appropriate shelf location for a compartment as the specific application in this kind of fridge. The main conclusions of this paper are summarized as follows:

- The temperature analysis provides an easy way to evaluate the thermal change rate.
- To analyze the thermal distribution of the horizontal commercial refrigerator compartments, an algorithm is developed.
- The temperature variance map shows the temperature average in the compartments of the horizontal commercial refrigerator as a function of the shelves' location. This map is an excellent tool for finding suitable shelf locations to attain the proper temperature.

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Planning and analysis of materials handling processes in production company using modelling and simulations

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Abstract: In this paper is presented the method of planning and analysis of the materials handling line in one production company. The method consists of design, simulations, and analysis with the support of Arena simulation software. The design of the line is accomplished using modular modelling with interconnected modules and blocks, and simulations will be carried out to view the flow of boxes with materials and the functionality of main units and processes. The modelled line contains all the necessary units of the input processes, production and output processes, warehouses, materials handling vehicles, and the process flow of packages. Analysis will investigate key performance parameters and evaluate the functionality of the line. Production and logistics companies are implementing various software to plan and design their materials handling line, internal transports and production processes prior to the implementation and development of their plant. This enables them to plan the work, determine influential parameters and identify problems in the functionality. Results will be given in the graphical and tabular forms, which will represent the analysis of performance parameters and line efficiency.

Keywords: MATERIALS HANDLING, WAREHOUSE, FACTORY, BOXES, PALLETS, MODULAR MODELING, SIMULATIONS, PROCESS CONTROL, PERFORMANCE ANALYSIS

1. Introduction

The material handling systems and warehouse management are complex processes with many operations and devices that require adequate planning, proper design of infrastructure and deep analysis for good performance and functionality. Many logistics and production companies' deal with these issues, and search for the best solutions in order to increase production parameters. These processes can be planned before the development of materials handling and production lines and after them, if there are problems identified.

In order to support the companies for planning and analysis of materials handling lines, advancements are made towards the Industry 4.0 technologies using specialized software that help the modelling of these lines and apply simulations of working processes. In these software's, the model designed is a schematic representation of materials handling line.

The methodology in this paper is modeling and simulations of materials handling line for design and analysis. I took an example of materials handling company in Kosovo that produces canned food and has an input and output warehouses and production facilities. The line comprises of main structural units, vehicles like trucks, forklifts, conveyors, then warehouses, production units, stations and travel routes. The units are connected with transfer lines that define the entity flows, direction of movement and the interaction between main units. Main operations of the processes in the model must be exactly defined. Simulations should be carefully planned and tested to represent the correct behavior of the line and its units [1], [2]. Results will be acquired for Key Performance Parameters [3]. Analysis will identify the performance of the line and problems that require correction.

Design of the Materials handling line and simulations will be implemented using Arena Simulation software [4]. The software is developed from the Company Rockwell Automation, Inc. In the Fig. 1 is the view of the Software's User Interface and its main parts. Model design is carried by dragging and dropping modules in the Model window. These modules or blocks defines the process and are interconnected with connector lines, which represent the flow and direction of the entity [5], [6]. An entity is the product or package, or both, flowing in the process. Modules in this software can be *Flowchart Modules* and *Data Modules*.

Flowchart modules have various shapes depending on the functionality or operation. They are placed in the model window and interconnected between each other to form a flowchart, depending on the logic of the process [4], [5]. *Data modules* are defined in the spreadsheet interface of the Properties window and are not placed in the model window.

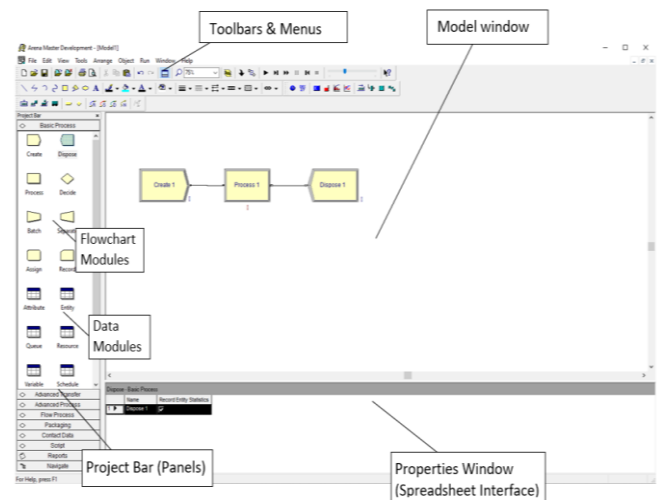


Fig.1. Arena Simulation software - User interface with main design and simulation parts [4]

Regarding the Literature review, several authors have published papers in the topic of modelling and simulation of materials handling and processing lines using Arena Simulation or other similar software.

S. Abedinzadeh et al. [1] in their paper present the simulation of a warehouse of an automotive company. They created a model and simulated it with ARENA software. The results presented bottlenecks in parts of the warehouse which lead to long waiting time for personnel. They presented strategies to reduce the average waiting time and improve the performance of the warehouse. Said M. et al. [3] identified Key performance indicators which were evaluated using Arena software. They developed a new improved production line layout a compared with real production line layout. Then analyzed the improvement of indicator values of new model compared to real model. Ch.Y. Liong et al. [10] in their paper presented ARENA simulation models for the loading and unloading systems in a warehouse. They analyzed for utilization of workers and waiting times at the various processes to identify the bottleneck in the system. They identified that the inter-arrival time of customers' trucks, number of forklifts, waiting time at the order picking, sealing, and loading process are the contributing factors towards the performance of the loading system. A.H. Abdul Rasib et al. [11] analyzed a real case study in food industry to construct simulation model in ARENA. Based on the issues found, they suggested improvements to the smoothness of the manufacturing system.

E. Pekel et al. [12] created a warehouse storage simulation model by using Arena Simulation Software of a hypothetical

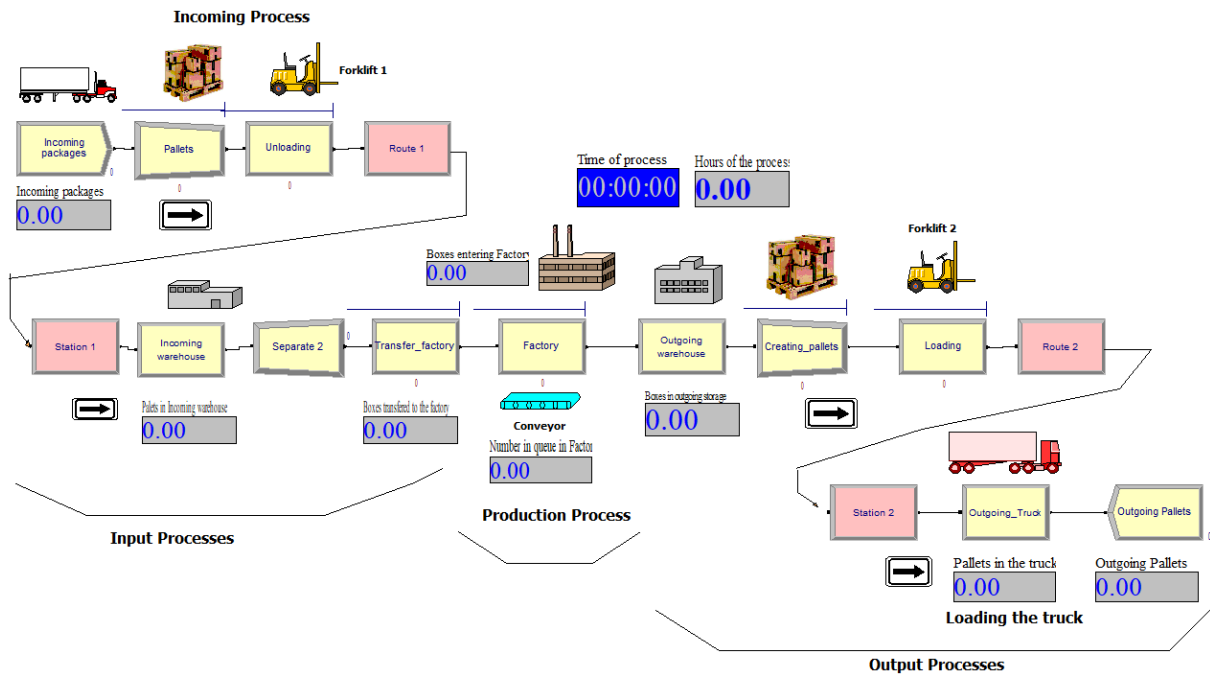


Fig.2. Model of the Materials handling Line

company. The aim was how to obtain the best possible result by fixing the departure time of the transporter and to determine how many pallets will be required monthly with the minimum and maximum bounds. Ch. M. L. Rahman et al. [13] created simulation model to a batch manufacturing system for process flow improvement. The model is an ARENA software of a UPS manufacturing line. They determined that the transformer making section was the main system bottleneck while other sections were running without delay. Changing the level of resources, adding extra workers and facilities re-layout have been suggested to improve the system performance. J. Saderova et al. [14] created simulation model for the receipt of goods into the warehouse. They observed and measured the forklift's work cycle, which unloaded pallets from the trucks. To create a simulation model they used EXTENDSIM8 software. They analyzed results of 3 experiments performed. Th. Wasusri et al. [15] analyzed case study in Arena Simulation in footwear warehouses. A. N. A. Ahmad et al. [16] present the method to increase the production output by improving conveyor layout, based on analysis using ARENA simulation.

2. The Materials Handling Line Model

Configuration of the Materials handling line is presented in Fig.2. The model of the Line consists of interconnected modules which represents an object, facility, resource or operation, and have their corresponding symbols. Modules are connected with wires. The model is designed in two rows, starts at the left and flows to the right, in one direction.

Other objects, like numerical results of key parameters are presented in gray color squares. There are textual boxes as a description of processes, and image description of facilities, loads, devices and vehicles. At upper part of the model are two important objects: *Time of the process* which shows the runtime of the simulation (Blue squared object), and *Hours of the process* which shows the hours of the line work (Grey color object).

The line is organized in three main parts: *Input processes*, *Production process* and *Output processes* (Fig.2) [2], [10], [12].

Units in the Input processes are:

1. *Incoming packages* – This is the name of the first module in the line, type *Create*. This module defines the startup processes

and initiation of the operations (Fig.3). In this module is defined the *Entity Type* that flows in the line. It is named *Box*.

Boxes are packages with raw material that are brought with the truck, and carry it for manufacturing in the factory. One box has dimensions 40 cm width, 50 cm length and 30 cm height (Fig.4). *Max Arrivals* = 780 is the number of boxes that can fit in a trailer truck and will be unloaded [7], [8]. The boxes arrive every 0.1 minutes each for discharge from the truck, in *Random* distribution. But boxes are not single handled. They are part of the standard pallet to be unloaded, defined in the next module.

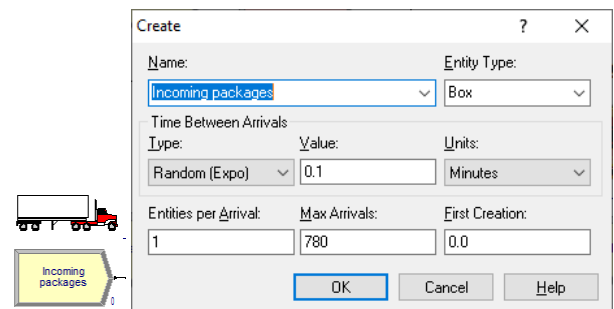


Fig.3. Create module symbol and properties

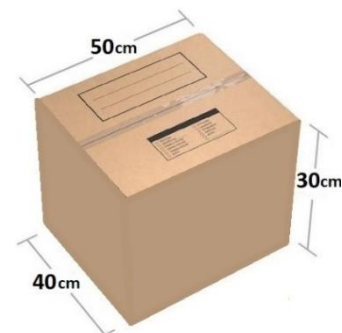


Fig.4. Example of the standard Box with its dimensions, as a sample in the model analysis

2. Second module is *Pallets*, type *Batch*. This is a grouping module of entities, in this case boxes. The boxes are grouped in one standard EURO2 pallet with dimensions 120x100 cm, and height

165 cm [9]. This is the most appropriate format of boxes which utilizes up to 99% the filling of the EURO2 pallet. One pallet consists of 30 boxes (Fig.5).

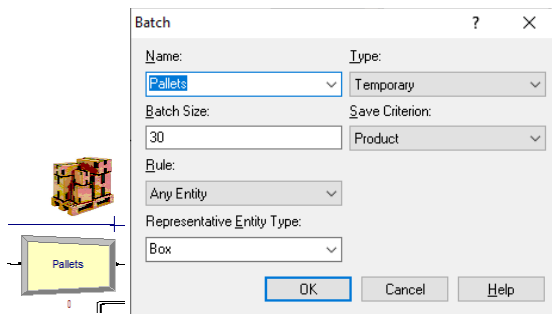


Fig.5. Batch module symbol and Properties

3. Third module in line is a process type, named *Unloading*. With this module is defined a process, in this case unloading of pallets from the truck. Unloading is done with one *Forklift*, which is defined as a *Reource*, named *Forklift1*. *Delay Type*, is the time required for the forklift to unload one pallet from the truck, and is defined by Uniform dispersion in minutes, between 2 and 3 minutes [4], [17] (Fig.6).

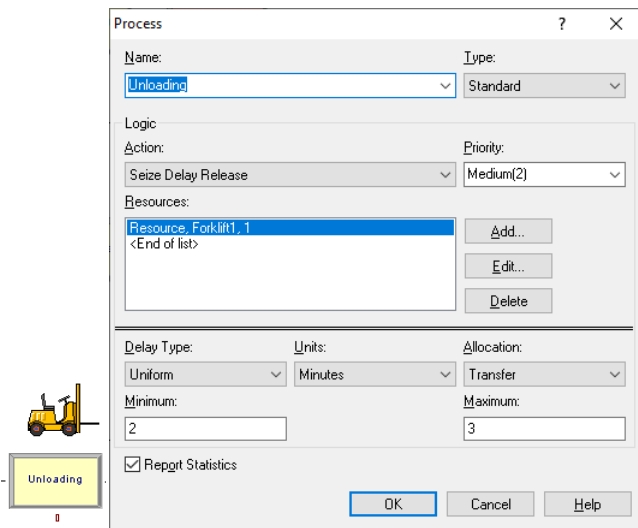


Fig.6. Symbol and Input parameters of the Unloading process

4. Next module is *Route 1* of type *Route*. The module represents the time the pallets are transferred by the forklift to the next station, in this case *Station1*. The route time of forklift travel is estimated 2 minutes (Fig.7).

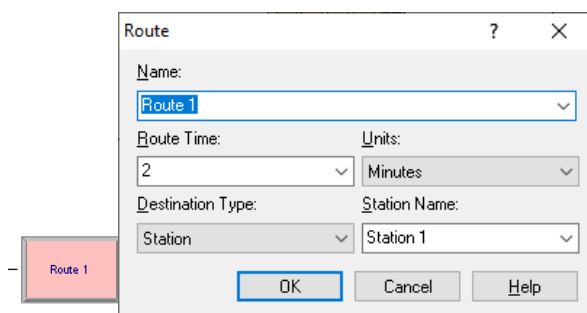


Fig.7. Symbol and Input parameters of the Route module

5. *Station* module, named *Station 1* defines a physical location, in this case the entry to the *Incoming warehouse* (Fig.7).

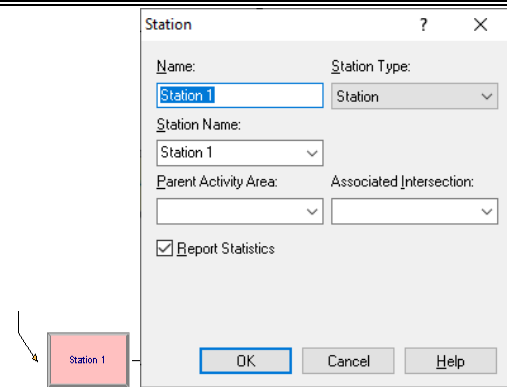


Fig.8. Symbol and Input parameters of the Station module

6. *Store* module, is used to add boxes to the storage, in this case *Incoming warehouse*. (Fig.8)

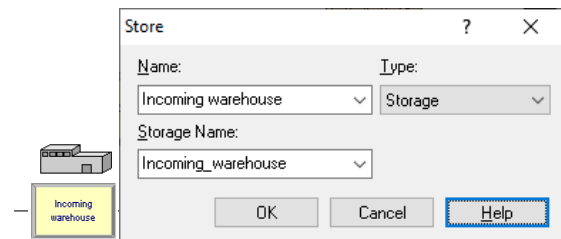


Fig.8. Symbol and Input parameters of the Store module

7. *Separate_boxes* is the name of the module type *Separate*, used to separate boxes from the previous batch created. In this case it means separating boxes from the pallets and sending them to the *Factory* for processing. (Fig.9)

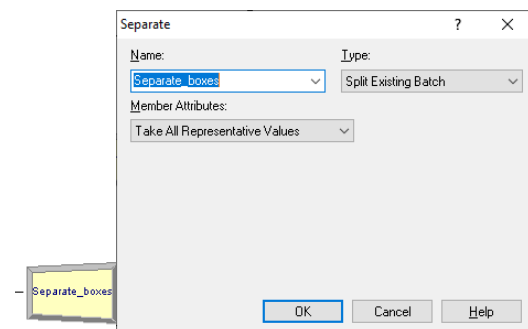


Fig.9. Symbol and Input parameters of the Separate module

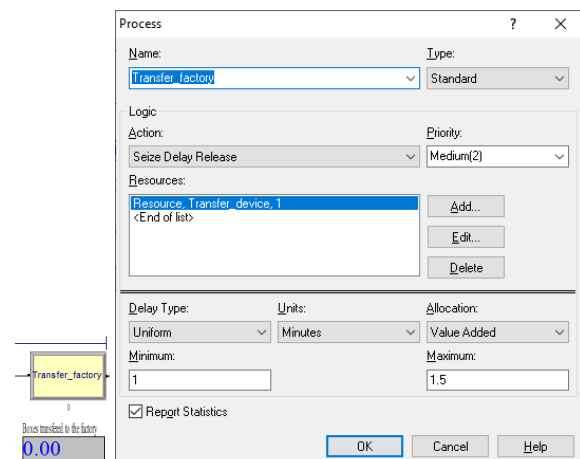


Fig.10. Symbol and Input parameters of the Transfer_factory module

8. *Transfer_factory* is the name of the process module that represents the time the input boxes will be transferred from the *Incoming warehouse* to the *Factory*, after being separated in the *Separate_boxes* module (Fig.10). Resource in this process is named *Transfer_device*. It is not defined exactly, while it can be a forklift,

cart or jack handled by workers, or even conveyor. *Transfer_device* is the resource that will transfer one box unit in Uniform dispersion in time 1 to 1.5 minutes (defined in *Delay Type*). This is an approximated time, while *Transfer_device* can carry more than one box. For example, if a cart carries 5 boxes, it will require 6 to 9 minutes to transfer them to the *Factory*.

9. Next module in the line is a process module named *Factory* (Fig.11). This is the main module of the *Production process*. Here, raw material is discharged from the input boxes, products are created in the factory, and output boxes are filled with final products for output. Conveyor is the main resource as an internal transportation device for transferring products from the *Factory* to the *Outgoing warehouse*. One conveyor is planned for the process. There can be more than one conveyors, depending on the manufacturing processes. Also, other resources as vehicles can be used, like forklifts, cranes and carriages.

Delay Type means the time spent for filling one box with ready made products, sealing the box and transferring it with conveyor to the *Outgoing warehouse*. This time is a function of triangular dispersion, and is estimated to be 10 to 14 minutes (Fig.11). This timeframe can vary, depending on the number and dimensions of the products (cans, jars, bottles, bags, etc.), format of the box they will fill in and velocity of the conveyor.

In this case, the manufacturing time starting from the raw materials processing to the final product is not included, while it can vary heavily on the type of products processed, number of machines, time between machines, packaging process, etc. If required, the manufacturing time can be added to the already estimated time.

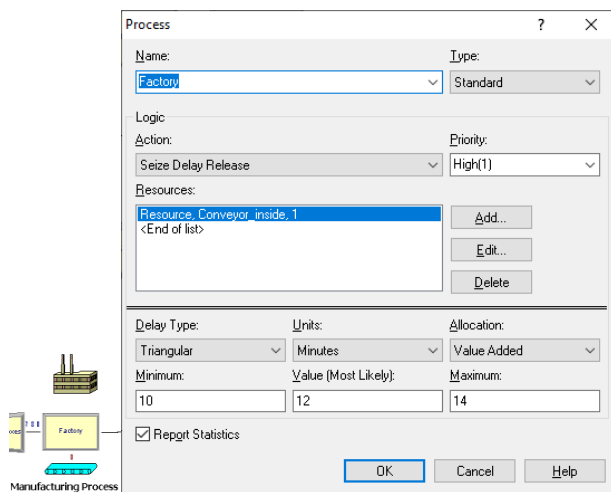


Fig.11. Symbol and Input parameters of the Factory module

Next modules in the line are units of the Output processes. The modules are:

10. *Outgoing warehouse* – The module is of type *Store*, which represents the warehouse where the output boxes come from the *Factory*.

11. *Creating pallets* – The module is the type *Batch*, where the outgoing boxes are grouped in 30 in one standard pallet EURO2. The number of outgoing boxes has the same box format as the incoming boxes for the purpose of pallet utilization (Fig.4).

12. *Loading* - The module is of type *Process*, which represents the loading of pallets from the *Outgoing warehouse*. Loading is done with one *Forklift*, which is defined as a *Resource*, named *Forklift2*. *Delay Type*, which is the work of Forklift is defined by Uniform dispersion in minutes, between 2 and 4 minutes. During this time it is estimated the forklift will take one pallet from *Outgoing warehouse* and load it on the truck.

13. Next module is *Route 2* of type *Route*. The module represents the time the pallets are transferred by the forklift to the next station, in this case *Station2*. The route time of forklift travel is 2 minutes.

14. *Station* module - named *Station 2* defines a physical location, in this case the entry of pallets to the *Outgoing truck*.

15. *Outgoing truck* – The module is of type *Store*, which represents the truck loaded with pallets.

16. *Outgoing Pallets* – is the last module in the line, type *Dispose*. This is the ending point of the model, in this case for the materials handling line (Fig.12).

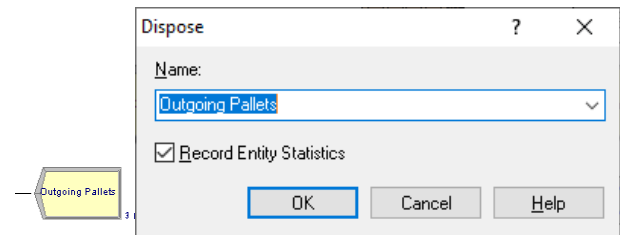


Fig.12. Symbol and Input parameters of the Dispose module

3. Simulation planning of the process flow in the line

After the model is designed and modules are connected, next step is planning the simulation scenario for the process flow.

The simulation is planned to run until all the boxes and pallets of raw materials as the input process are sent to the *Factory*, and all the boxes and pallets of outgoing process are loaded in the *Truck*. The main measuring parameter is the time of the main processes. The two timing boxes, first one in blue named *Time of process* shows the actual time of the process flow, and second one in gray named *Hours of process* shows the total hours of the process (Fig.13).

Results of boxes and pallets flow will be shown in the gray boxes around the corresponding modules during the simulation run. After the simulation, the software will generate report of results.

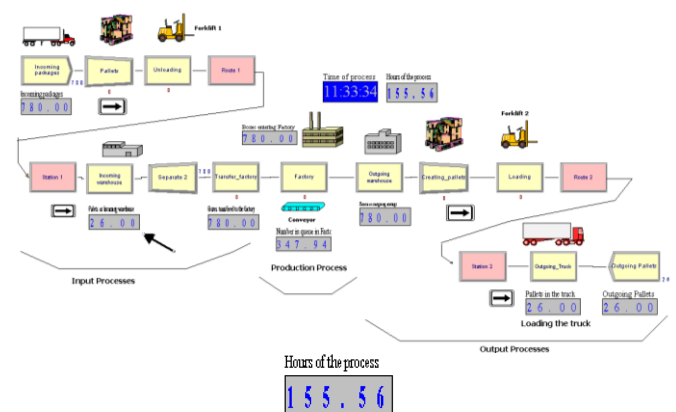


Fig.13. Model after the simulations and results in boxes close to the modules. Below are shown hours of the process

4. Results, analysis, and discussions

After the simulation, results are generated in the form of numerical results and can be viewed in the gray boxes of the model (Fig.13).

In the first unit of the Input processes, *Incoming packages*, is the truck carrying 780 boxes of raw materials, grouped in pallets that can carry 30 boxes each. There are 26 pallets with boxes in the truck to be unloaded.

The main parameter to analyze is the hours of the entire process duration or simulation in this case. The number of hours of the entire process is 155.56 hours. If we consider that one working day has two working shifts, or 16 working hours, the process will last 9.72 days.

The time the first pallet is unloaded from the truck with *Forklift1* lasts 3 minutes and 32 seconds (0.06 hours) (Fig.14).

All 780 boxes (26 pallets) carried by the *Forklift1* arrived in the *Incoming_warehouse* for approximately 1 hour and 24 minutes (1.40 hours, or 84 min) (Fig.15).

While this is a line with uninterrupted and continuous flow, the first box to enter the *Factory* is after 9 minutes and 36 seconds.

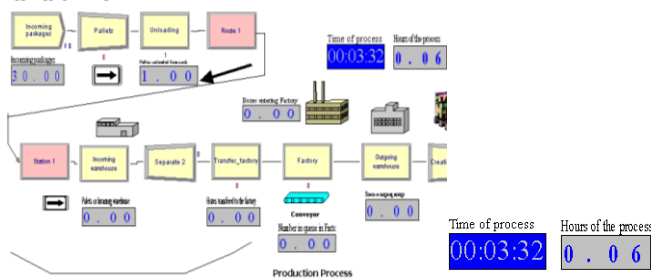


Fig.14. The time of the first pallet is unloaded from the truck

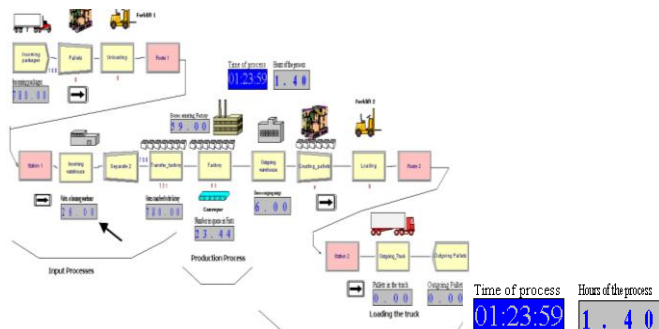


Fig.15. The time all the pallets arrive in the Incoming warehouse

The total time of the *Input processes* lasts around 16 hours 26 minutes or 16.44 hours (Fig.16). During this time, all the 780 boxes will exit the *Incoming warehouse*, are transferred and enter the *Factory*.

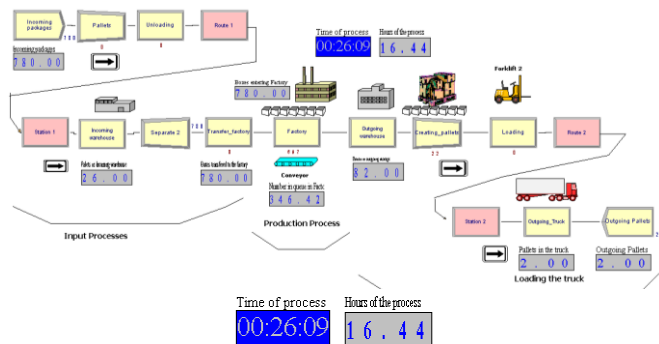


Fig.16. The total work time of the Input processes

Comparing the time all the pallets arrive in the *Incoming warehouse* (1.40 hours) and total work time of the *Input processes* (16.44 hours), difference is $16.44 - 1.40 = 15.04$ hours. This is the time boxes spent in transfer between the *Incoming warehouse* and *Factory*. This is considered a bottleneck in the process.

The remaining total time of the process, $155.56 - 16.44 = 139.12$ hours goes to the *Production process* and *Outgoing processes*.

By the end of *Input processes time*, 16.44 hours, in the *Factory* there will be in average 346 boxes in the queue for outgoing. There are only 82 boxes that exited the *Factory* and arrived in the *Outgoing warehouse* (Fig.16).

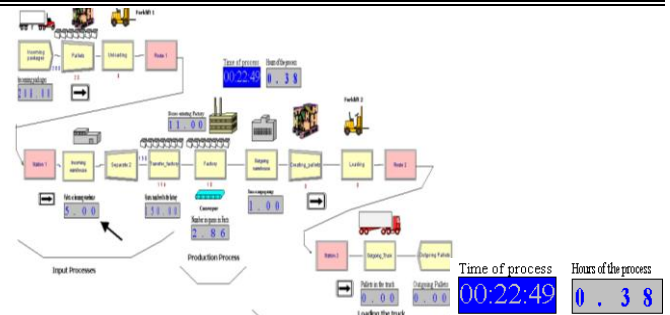


Fig. 17. The time the first box exits the Factory and enters Outgoing warehouse

The initial work in the *Outgoing processes* starts after 22 minutes and 49 seconds (0.38 hours) when the first box exits the *Factory* and enters the *Outgoing warehouse* (Fig.17). In this time 11 boxes entered the *Factory*, and an average of 2.86 are in the queue.

As shown in Fig. 18, pickup of prepared pallets in *Creating_pallets* module starts after 5 hours and 59 minutes (5.99 hours), when the first pallet with 30 boxes is assembled and ready to be loaded with the *Forklift 2*.

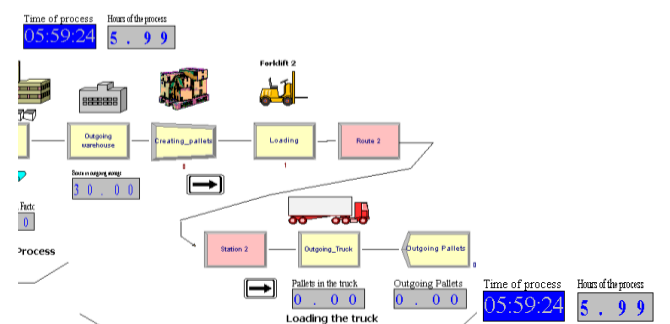


Fig.18. The start of work in the Outgoing processes – after the first pallet is assembled with 30 boxes

Based on last two figures, we can draw a conclusion that the time difference between the first box exiting the factory and the first outgoing pallet assembled with 30 boxes is approximately $5.99 - 0.38 = 5.61$ hours (5 hours and 37 minutes). This is a long delay and the main drawback of the line, which identifies that transfer from the *Factory* to the *Outgoing warehouse* is the longest process and a bottleneck in the line.

Following the process flow after the *Factory*, the pickup of one assembled pallet with *Forklift 2* from the *Outgoing warehouse*, travel to the *Outgoing_Truck* and loading it in the truck will last approx. 4.5 min.

4.1. Presentation and Discussion of reported results

After the simulations, the Report with results was generated by the software [4]. Results will be shown in tabular form.

Table.1. Waiting times and Queues of boxes in the processes

Process name and type	Average waiting time of boxes (hours)	Maximum waiting time of boxes (hours)	Average number of boxes in queue-	Max. number of boxes in queue-
Loading (Process)	0	0	0	0
Pallets (Batch)	0.0248	0.064	0.124	30
Transfer_factory (Process)	7.465	14.936	37.434	720
Factory (Process)	69.39	138.92	347.94	697
Creating_pallets (Batch)	2.888	5.882	14.84	30
Unloading (Process)	0.002	0.0174	0.0003	1

In Table 1, in the first two columns is given the waiting time of boxes in the main processes of the line. It can be concluded that the longest waiting times are in the *Factory*, then in the *Transfer_factory*, which proves previous conclusion that bottlenecks are present there. Other units show better performance. Best performance is in the *Loading* process.

In the columns 3 and 4 are given the number of boxes waiting in queues. The conclusion is similar; the highest number of queues are in the *Transfer_factory* and in the *Factory*. Maximum number of boxes in the *Pallets* and *Outgoing_pallets* is 30, but this is a scheduled number, while one pallet has 30 boxes.

Table.2. Time in hours the boxes spent in the line

Entity of the line	Average Total time	Average transfer time*	Average VA time**
Box	80.178	2.297	6.65

*Transfer Time: It is the time when the box incurs a delay at a process whose allocation has been designated as transfer [4].

**VA Time: Value added time is the time when a box incurs a delay at a value added process [4].

Based on the results in Table.2, one box spent in average 80.178 hours in the line. The time of transfer between units is 2.297 hours. This draws a conclusion that boxes spent much more time inside the units rather than in transfer between them.

Table.3. Usage of Resources

Name of Resource	Usage in %
Forklift 1	1.47
Transfer_device	10.4
Conveyor	99.80
Forklift 2	1.34

In Table 3 is given the usage of resources during the total time of the process (155.56 hours). It can be concluded that *Conveyor* was busy almost all the time of the process, 99.8% or 155.33 hours transferring boxes, *Transfer_device* was busy 10.4% or 16.17 hours, *Forklift 1* was busy 1.47% or 2.28 hours unloading 26 pallets, *Forklift 2* was busy 1.34% or 2.08 hours loading 26 pallets. Forklifts are low utilized compared to conveyor and transfer device.

For this line it is useful to shorten the delays in the transfer to the *Factory*, plan more than one conveyor, or shorten the delays in the *Factory*.

5. Conclusions

After the results and discussions, it can be concluded that using Arena Simulation software is useful to design materials handling line and analyze it after performing simulations. It is important to design the line, processes it adequately and plan the process flow accurately.

Most important parameter measured and analyzed is the number of boxes and pallets in the *Incoming processes*, *Production Process* and *Outgoing processes*. This division of processes in the line is better for study and analysis. The results gave us a view about the performance of the line and possible issues. The bottlenecks in the line were identified in the transfer from the *Incoming warehouse* to the *Factory*, and in the production process, in the *Factory*. The first one happens due to the slow transfer of boxes, and the second one mainly due to the long packaging of products, placing them in the box and transferring with conveyor to the *Outgoing warehouse*.

The *Incoming processes* and *Outgoing processes* of the line gave good results, while the time of the process flow is considerably short and there are no identified blockades.

With this model, it was possible to analyze also the performance of the *Resources* or vehicles like forklifts for loading and unloading, conveyor for transporting products inside the *Factory* and particular transfer devices. Conclusion is that the usage of conveyor was high, and Forklifts were used less compared to the conveyor in entire process.

The model design of the line can be further developed with addition of various production processes and testing various packages. That would increase the complexity of the analysis but also contribute to more research and investigation on this the topic.

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Schedule process modelling of a “wood processing SME” based on IEC/EN 62264 standard

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Abstract: The purpose of the research is to present a solution to production planning optimization problems, in cases of short-term customer orders and variable production load. The main reasons for the emergence of the task are the introduction of SMEs to unregulated electricity market and the increasing price of electricity due to the imposition of additional components in the formation of the price. The lack of traditional market and the small number of regular customers of SMEs make it difficult to forecast electricity consumption, and therefore impractical to request electricity for long-term periods of time. A possible solution related to the implementation of operational management systems, which, through the introduction of standard models, will enable the implementation of optimization algorithms for production planning.

Keywords: MODEL DRIVEN DEVELOPMENT, IEC/EN 62264, MODELLING, SCHEDULE PROCESS

1. Introduction

The need to improve the performance of production systems, related to increasing the level of coordination and integration of all resources and production functions in order to achieve flexibility and higher competitiveness, causes a growing interest in planning methods. Short-term and detailed production planning can be defined as a decision-making process that answers the questions: how, where and when to produce. How it relates to the required resources, where - to the allocation of each operation to a separate apparatus or machine, and when to predicting the start and end time of each operation. The most important resources are equipment, labour (personnel), raw materials, electricity, production or warehouse space, required capital, etc. The optimal distribution of resources leads to a shortening of production time, and hence to a reduction of production costs, which increases the competitiveness of production. In short-term, detailed planning, the focus is on satisfying the customer's requirements or completing the necessary tasks in the shortest possible time.

Optimal planning of production operations is part of the task of short-term, detailed planning and is a process of allocating available production resources to perform certain production tasks and determining the sequence and time parameters for performing this work. The manufacturing operations themselves are defined in the product design stage. Regardless of the many efforts to tackle the task of optimal planning of operations, the variety of tasks in the field and their high dimensionality cannot be solved by universal means. In recent years, there has been an effort to use the results obtained in the field of formal verification and more precisely from solving the problem of model checking.

The main goal of the research, presented in this paper, is to propose a solution for the process planning task in a “Wood Processing SME”, based on IEC/EN 62264 standard.

The paper is structured in 4 parts. After the introduction, in Part 2, a short analysis of the optimization methods for optimal process scheduling is done, in Part 3 a short overview of proposed techniques is done and in Part 4 the case study is presented. At the end some conclusions are made.

2. Short analysis of the optimization methods for optimal process scheduling

Production operations planning is an optimization task that requires the definition of one or more objective functions. Typical objective functions used in the field of manufacturing systems are:

- Number of orders fulfilled per unit of time (throughput).
- Number of unfinished orders (work-in-process inventory).
- Duration of order execution (lead time, mean order flow time).

- Difference between the completion time of the order execution and the execution time (mean order tardiness). If this difference is negative, it is assumed equal to 0.
- Minimization of the execution time of all orders (makespan), etc.

The main difficulty in the field of optimal planning of production operations is that there is a wide variety of tasks that prevent the development of a universal method for its solution. Different perspectives on the task of optimal planning lead to different approaches to its solution, systematised and presented in Fig.1. The traditional perspective in the academic world is static and deterministic, which has led to the development of numerous algorithms for off-line optimal planning. However, these methods, as well as those of deterministic dynamic programming and static stochastic planning, are inappropriate for the case under consideration, as they inadequately reflect the real aspects of the task. Reactive optimal planning algorithms accounting for disturbances in a dynamic environment are methods that embrace both the dynamic and stochastic nature of the optimal planning task. However, most reactive optimal planning methods do not predict the stochastic effects, but only react to them. Only proactive planning takes into account the risk of disruption. All methods listed so far are algorithmic. An alternative approach for many of the tasks can be structural solutions, which are called optimal planning architectures. They focus more on the practical aspects of the optimal planning task, such as software reusability, and are called optimal planning environments.

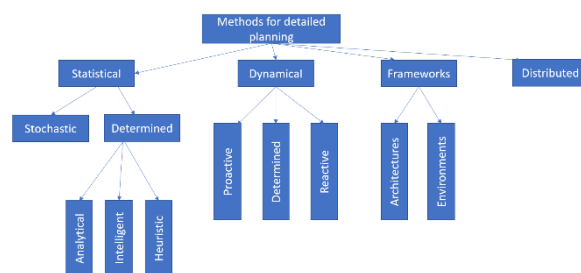


Fig. 1: Methods classification for detailed planning

3. Short overview of proposed techniques

Manufacturing Operations Management systems (MOM) are mainly applied for real-time visual representation and control of production processes they interconnect the business information systems with the manufacturing and are middleware of the pyramid for automation [1].

3.1. IEC/EN 62264 standard

Based on ANSI/ISA 95, the international standard IEC/EN 62264 is enterprise-control systems integrational standard. IEC/EN 62264 involves UML models and objects to solve the data inconsistency for transferring and aggregating data between different data levels [2]. It is separated on five main parts: models and terminology [3], objects and attributes for integration of enterprise-control system [4], activity models of manufacturing operations management [5], object models attributes for integration of manufacturing operations [6], and business to manufacturing transactions [7]. Through providing standard models and terminology the standard supports the development and usage of systems for manufacturing operational management.

According to IEC/EN 62264 the operations in manufacturing are separated on several levels (Fig. 2). The actual production is on Level 0; Level 1 is for detecting and control of the production processes; Level 2 is for supervising the production process and its time frame is measured in hours and minutes; Level 3 is for control of manufacturing operations through workflow control of production and optimizations of the processes, the time frame is measured in shifts; Level 4 is for developing the operational scheduling including actual production, shipping, etc., the time frame is measured in weeks/days [8].

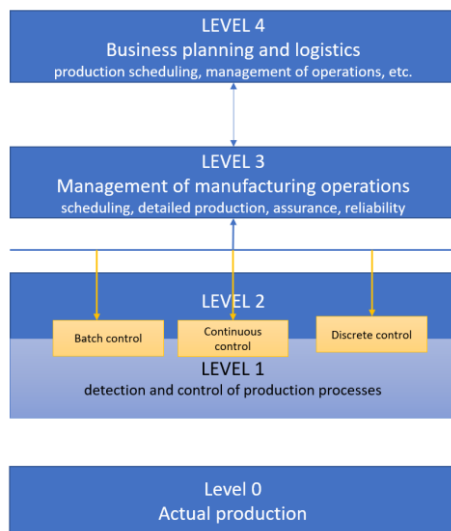


Fig. 2: IEC/EN 62264 production process hierarchy [8]

According to the standard the activities in management of manufacturing operations can be separated into several areas for management of inventory, quality operations, maintenance operations and production operations. IEC/EN 62264 standard facilitates the separation of business process from production ones and segregates the exchanged information from business systems and specific manufacturing implementations. [1]

3.2. Model Driven Development and Model Driven Architecture

Model-driven development (MDD) is a technique that moves model development on the front line instead of code first [9,10]. In [11] values symbolise models and variations map models to models and MDD is visualised as meta-programming architecture. One of the most promising realisations of MDD is Model Driven Architecture (MDA) of Object Managements Group (OMG). It ensures a conceptual framework for model development and transformation, which includes several types of models like Computation Independent Model (CIM), Platform Independent Model (PIM) and Platform Specific Model (PSM).

- **Conceptual Independent Model (CIM)** – is the domain model which previews the system expectations through a vocabulary. It excludes the specifications for implementing the system.
- **Platform Independent Model (PIM)** – helps with logical data modelling, specifies workflows and processes, defines dependencies. Creates a view of the system excluding the specifications required from the implementation.
- **Platform Specific Model (PSM)** – this model easE the code generation for specific platform by combining the details for the execution platform and PIM model.

3.3. Unified Modelling Language

Unified Modelling Language serves to meet the requirements of Model Driven Architecture (MDA) and MDD. It is a modelling language used to create the solution specifications, documentation, design needed for the process of software development [12]. “Visual Paradigm” is a tool which supports UML, DevOps tools, SysML, etc. Models and their components are presented with UML [13]. UML 2.5 includes 15 diagrams separated in two main groups: Structural and Behaviour diagrams. Structural diagrams represent the static structure of the system and its elements on different levels of abstractions. Part of the structural diagrams are the class, object, deployment and component diagrams. Behaviour diagrams describe the dynamic behaviour of the system. These diagrams are used to describe the functionality of a software system, part of them are the use case and activity diagrams. Use case diagram describes the system operations from users’ perspective through actors and cases (Fig. 3). Activity diagram represents the operational flows in a system.

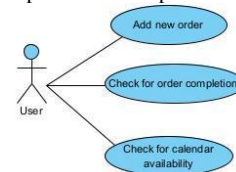


Fig. 3: Example of a Use Case diagram

4. Case Study

All resource models are already developed and described in [14].

4.1. Brief description of the “Wood Processing SME”

Two main types of wood materials are processed - dry softwood and broad-rounded dry wood (oak) of different lengths and widths. One of the main processes that takes place is the process of grinding wooden boards, which is carried out on a wide belt grinder, consisting of a conveyor belt along which the sanding material is moved at a feed rate from 1 to 20 m/min (depending on the processed surface or the type of processed material) and a grinding speed in the range of 23 – 30 m/s. The maximum working width of the belt is 1200 mm. The grinding tool is an abrasive belt placed between two horizontal shafts, one of which is working and the other is winding. The purpose of sanding is to calibrate the exact size of the wooden board and the preparation for varnishing. Drive power is from 45 to 55 kW. The technological time for processing (sanding) 1 m² of wood parts is as follows: 5 m/min - for coniferous, and 2.5 m/min - for broad-leaved. The waste raw material from this process is wood shavings, which are stored in a hopper, which has a capacity of 5 m³, which, according to the relative weight of the processed wood, means 550 kg/m³ softwood and 900 kg/m³ - hardwood (oak). When processing 1m² of timber, about 2.2 kg of coniferous sawdust or 1.8 kg of broad-leaved sawdust are obtained.

Wood bran is used to produce pellets, which are highly efficient, clean, renewable energy leading to energy savings and reduced carbon emissions. Pelletizing is carried out with a press, which is a round, fixed matrix made of thick steel (about 30 mm.) and perforated with numerous holes, with a diameter corresponding to the diameter of the pellets. These holes fulfil the role of matrix(mold) in which the pellets are formed. The sawdust entering the press is pressed against the holes of the die by means of pressing devices (usually two to three rollers) which are self-propelled. Before entering the pellet press, the wood bran is moistened with water, after which they are fed into the press chamber of the pellet press. During the rotation of the pressing devices and under the pressure exerted by the toothed rollers, the material is pressed into the aversion of the matrix, during which the pellets are formed. In doing so, the friction heats up the die and the pellets leave the press very hot at around 95 °C. After they are cut, they go through a cooler and then they are packed. The productivity of the pellet press is 120 kg/h. The total power of the entire pelletizing plant is 15 Kw.

4.2. Defining the Operations Planning Task.

The goal of the task is optimal planning of the operations in a “Wood Processing SME” described in point 4.1 and is expressed in determining an optimal time schedule of the operations performed by a woodworking wide belt grinder and the pelletizing press, utilising the waste material generated from the grinding machine and filling the hopper as an intermediate storage with limited capacity. The aim is to optimise the production process based on certain conditions and technological limitations. The technological limitations are as follows:

- Power supply limitation – due to energy limitation and insufficient electrical power supplying the furniture production, the wide belt grinding machine and the pelletizing plant cannot work at the same time.
- Hopper capacity limitation - when the hopper for collecting the waste material from the grinding machine is filled in the range of 80-95%, the pelletizing plant must be started.

There are P of wood grinding orders and Q of pellet orders. Each grinding order l is characterised by an arrival time (al) and a lead time (d_l) and includes one operation that is performed on one machine. Each pellet order k is characterised only by delivery time (d_k), involving 3 operations that are performed on three separate machines. The duration of all operations (l, j_k) is set. The first operation can start only after the order arrives. The following conditions should also be considered during planning:

- An operation can be executed only if all previous operations have been completed.
- Machines cannot execute more than one order at the same time.
- Once started, orders cannot be interrupted and replaced by others.

The goal of optimal planning of operations is to determine the time sequence of execution of orders, which ensures a minimum value of the time for execution of all orders, the characteristics of which are presented in Table 1 and Table 2.

Monthly requests for grinding							
N	Date	Client	Type of wood material	Width [m]	Area [m ²]	Time [min]	Delivery Time
1	08:00 15.05	Bulter	Coniferous	0.28 0.48	68 192	128	13:00, 16.05
2	10:00 15.05.	Euromebel	Coniferous	0.28 0.48	162 284	234	17:00 15.05
3	08:00 16.05.	Cotzevi	Coniferous	0.28 0.48	29 98	62	17:00 16.05
4	09:00 16.05.	Sankevi	Deciduous	0.28 0.48	18 48	65	15:00, 16.05

Table 1: List of orders for grinding of wood material

Monthly requests for pellets					
N	Date	Client	Quantity [kg]	Time [min]	Delivery time
1	10:00 15.05	A.N.N. 2	2000	1000	17:00, 16.05.
2	14:00 15.05	ARNI	3000	1500	15:00, 17.05.
3	12:00 16.05	Kos Hol	1000	500	11:00, 17.05.
4	14:00 16.05	Hol Les	6000	3000	13:00, 18.05.

Table 2: List of orders for pellets

4.3. Models based on IEC/EN 62264

For the aim of the task an activity diagram is developed based on IEC/EN 62264 (Fig. 4). It includes two group elements: “Infinite Planning” and “Fine Capacity Scheduling”. “Infinite Planning” includes the processes “Create Work Order”, which includes “Create Material Requirements”, “Create Personnel Requirements”, “Create Equipment Requirements”, and “Calculate Lead Time”. “Finite Capacity Scheduling” includes “Calculate Dependencies”, “Check Calendar Availability”, which leads to the need of “Reserve Material”, “Reserve Personnel”, “Reserve Equipment”, “Reserve Physical Assets”, after all of the above are finalised only then the process can continue with “Reserve Capacity” and finalise the order.

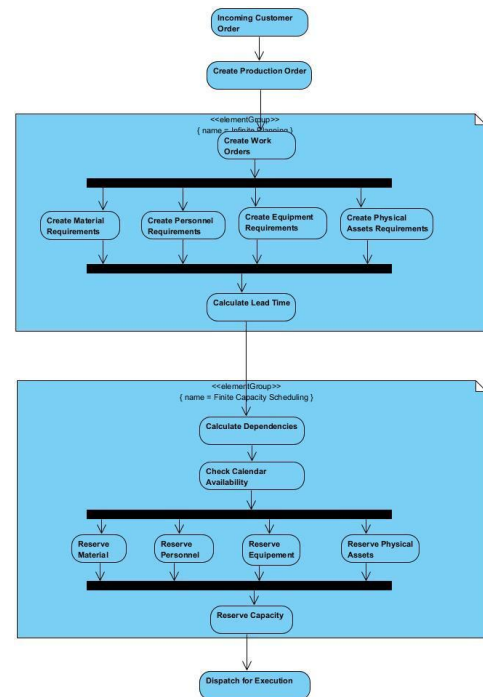


Fig. 4: Proposed activity diagram

Conclusions

The proposed approach for optimal planning of production operations using IEC/EN 62264 standard has the unique advantage of solving the task in real time and guaranteeing a quick response of the system in the conditions of changes in the technological structure and changes to the production program. Successfully solving the task of optimal planning of operations will allow more effective energy cost planning and construction of an efficient operational production management system that monitors pellet orders and starts the pelletizing machine when a certain amount of waste biomass is stored in the hopper for storage generated by the wide belt grinder. Pellet orders should be quantitatively tracked, and information should be given on the lead time for the ordered quantity of pellets.

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Assessing the impact of intellectual capital on economic growth

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Abstract: The article explains the elements that have the largest share in intellectual capital and explains the possibility of quantitative assessment of the specific weight of these elements in intellectual capital. Here on the example of the Republic of Azerbaijan on the basis of statistical data the value of works and services on intellectual capital in Azerbaijan is analyzed, as well as the dynamics of added value created at the expense of intellectual capital. In the article, the impact of added value created by intellectual capital on economic growth in Azerbaijan has been evaluated in the software package Eviews-12.

KEYWORDS: INTELLECTUAL CAPITAL, VALUE ADDED, GLOBALIZATION, FIRM, CORRELATION, REGRESSION, MODEL, ADEQUACY, CRITERION.

1. Introduction

In the context of economic globalization, exporting knowledge-intensive products is an enhancement of competitiveness among the countries of the world for every industrialized country and one of the most important means of further improvement is direct investment and the sale of licenses. High technology, based on intellectual capital, plays a decisive role in maintaining and strengthening the competitive position of these countries among the world states in modern conditions. The main result of financial and economic activity for each enterprise is to provide customers with information and successful solution of organizational problems. At present, for this purpose, international and local consulting companies are operating in most countries of the world, covering various spheres. When evaluating the activities of consulting firms, it is important to consider the high importance of personnel qualifications aimed at creating profit as the final result of the financial and economic activity of the enterprise. All this shows that the modern approach to assessing the real value of the company, in addition to determining how those or other decisions affect the value of the enterprise, also includes the type of assets (tangible and intangible) that represent the value of the company [1].

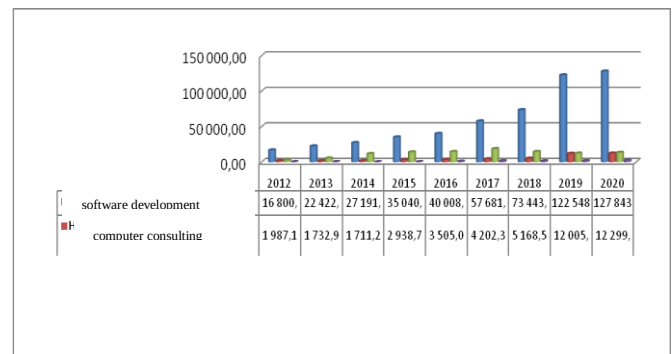
2. Main part

Since intellectual capital is an intangible asset, as an economic category it also includes other elements of national wealth in the form of technology, know-how, which cannot be separated from fixed assets, and cultural value. The elements with the largest share of intellectual capital in the structure of the national wealth of each country are advanced technology, fundamental science and university science. These elements can be quantified by their weight in intellectual capital.

It should be noted that the quality of intangible assets plays a key role in shaping the value of companies. Since intangible assets play a decisive role in the formation of enterprise value in the long term, enterprises without intangible assets will have no value in the long term [2, 3]. Intangible assets are primarily social human capital, which affects the economy as a whole, and individual human capital, which is directly related to individual corporations. In a globalized world economy, modern firms rich in intangible assets are producers of knowledge rather than goods. The efficiency gains achieved in many organizations are the result of specialized knowledge, long-term learning, and interaction with partners and counterparties. Instead of physical assets or financial capital, intellectual capital becomes a sustainable competitive advantage. The structure of intellectual capital includes human, organizational and consumer capital [4, 5].

As seen from the structure of intellectual capital, the use of technical software in organizational capital, patents, trademarks and technical software on elements of the organizational structure is important as one of the factors affecting the organization of optimal regulation of the financial and economic activity of enterprises.

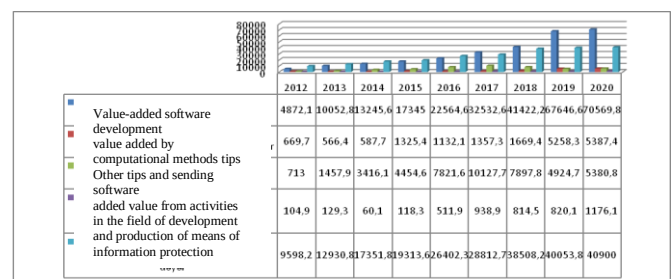
The diagram below shows the dynamics of added value due to the provision of technical software in the Republic of Azerbaijan.



Graph 1: Dynamics of the cost of works and services on intellectual capital in the Republic of Azerbaijan, in thousands manats

Source: prepared by the author on the basis of data [3].

As can be seen from the diagram, the works and services performed on software development within the organizational capital of knowledge-intensive intellectual capital developed with increasing dynamics during 2012-2020. There was a 7.9-fold increase in 2020 compared to 2012. Despite the effects of the COVID-19 pandemic, the value of software development work and services increased by 4.3% in 2020 to AZN 127.8 million compared to 2019. Computer consulting, other consulting and activities in the field of design and production of information protection tools and services were also observed with a similar increasing trend during the study period. The increase in work and services performed in the design and production of information protection tools in software processing, computer consulting, other consulting and information protection tools in the organizational capital of knowledge-intensive intellectual capital from 2012-2020 contributed to the formation of value added in these areas with increasing dynamics. This can be seen more clearly in the graph below.



Graph 2: Dynamics of added value created at the expense of intellectual capital in the Republic of Azerbaijan, in thousands manats

Source: prepared by the author on the basis of data [3]

As can be seen from the diagram, the added value created by works and services performed in the field of software development, computer consultancy, other consultancy and activities in the field of design and production of information protection means within

the organizational capital for 2012-2020 in the Republic of Azerbaijan has developed with increasing dynamics.

3. Econometric evaluation of the impact of knowledge-intensive intellectual capital on economic growth

It should be noted that an important role in the formation of economic growth in the country plays an added value created at the expense of work and services performed in the field of design and

manufacture of information protection, software development, consulting on computer technology, other consulting and information protection. According to statistical data on GDP and science-intensive intellectual capital the dependence between them can be estimated by conducting a regression analysis. For this purpose ready packages of mathematical programs EViews, MatLab, MS Excel, MathCad and others can be used. In this regard, using the software package Eviews-12 for the above analysis, we obtain the following result.

Table. The result of the regression relationship between GDP and knowledge-intensive intellectual capital in Azerbaijan

Dependent Variable: Y				
Method: Least Squares				
Date: 02/11/22 Time: 21:20				
Sample: 2008 2020				
Included observations: 13				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
X	0.061289	0.012207	5.020846	0.0002
C	-3059.061	954.2056	-3.205872	0.0004
R-squared	0.696207	Mean dependent var		1716.985
Adjusted R-squared	0.668590	S.D. dependent var		470.5494
S.E. of regression	270.8870	Akaike info criterion		14.18192
Sum squared resid	807177.6	Schwarz criterion		14.26883
Log likelihood	-90.18247	Hannan-Quinn criter.		14.16405
F-statistic	25.20889	Durbin-Watson stat		1.082202
Prob(F-statistic)	0.000030			

Source: developed by the author on the basis of the Eviews software application

Based on the result obtained with the Eviews application software package, the regression equation will have the following form:

$$Y = 0.0312885943991 * X - 2036.06110137 \quad (1)$$

According to Eviews application software package, there is a high correlation relationship, expressed by the model $Y = 0.0313 * X - 2036.061$, between the dependent variable Y, which represents GDP in Azerbaijan, and the explanatory variable X, which represents knowledge-intensive intellectual capital ($R^2=0.696$).

Based on this established relationship equation, it can be concluded that a one unit increase in the factor (X), which represents the amount of knowledge-intensive intellectual capital, led to a 0.031 unit increase in the volume of GDP in Azerbaijan. As we can see, the standard errors of the coefficients of the dependent variable X and the free limit C in the data of table (1), obtained by the software complex Eviews-12, are less than the average value of the coefficients. This means that model (1) is statistically significant [6, p. 315].

It is important to check the adequacy of the established model, which is determined by the F-Fisher test. To check the statistical significance of the model (1), which expresses the regression equation $F_{table}(a; m; n - m - 1)$ as a whole, we should compare it with the value of Fisher's F-criterion [6, p. 315]. According to table (1), reflecting the result of the software package Eviews,

$$F\text{-statistic (Fisher's criterion)} = 25,21$$

If you determine the tabular value of F in EXCEL using the formula $F_{table}(a; m; n - m - 1) = F_{inv}$, for a 99% confidence interval $F_{table}(a; m; n - m - 1) = F_{inv}(0,01; 1; 11) = 9,65$ Fisher's F-criterion, $F_{table}(a; m; n - m - 1)$ if you compare its value, you can see that Fisher's F-criterion ($25,21 > 9,65$). This means that the regression equation as a whole is statistically significant [6]. It means the adequacy of established model (1).

The conclusion about the presence or absence of autocorrelation in the model can be made on the basis of Darbon-Watson statistics in Table 1, obtained on the basis of the EViews

application software package. As can be seen from the table, it is equal to $DW=1,082$. In this case the Darbon-Watson critical points for $1 \alpha = 0,05$ explanatory variable ($m=1$) and $n=13$ observations would be as follows [6, 7].

$$d_l = 1,010, \quad d_u = 1,340$$

$$d_l = 1,010 \leq DW = 1,082 < d_u = 1,340$$

as the conclusion about the presence of autocorrelation is not established [6, p. 311]. It means, that regression equation as a whole is statistically significant and established model $Y = 0,0313 * X - 2036,061$ is adequate.

As a result of the study for the linear regression equation (1) we can determine how much the result factor will change because of the cause factor by calculating the elasticity coefficient, which expresses the percentage change in the dependent variable as a result of a 1% change in the independent variable. This coefficient is calculated according to the following formula [5, 6].

$$E = \frac{\alpha_i \times \bar{x}_i}{\bar{y}} \quad (3.2)$$

Here α_i coefficients of the above relational equation, $\bar{x}$ - average score of the causal factor for the periods under study, $\bar{y}$ - average score of the resulting factor for the periods under study. The elasticity coefficients calculated on the basis of these scores, in accordance with the established model, will be as follows.

$$E_{inv} = \frac{\alpha \times \bar{x}}{\bar{y}} = \frac{0,0313 \times 1716,985}{65742,046} = 0,000817$$

4. Conclusions

The calculations show that a 1% increase in knowledge-intensive intellectual capital leads to a 0.00082% increase in GDP in Azerbaijan.

For the linear regression equation (1) above, the study can determine how much the outcome factor will change due to the cause factor by calculating the elasticity coefficient, which expresses the percentage change in the dependent variable as a result of a 1% change in the independent variable.

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Hemorheological models applied to data analysis in groups of healthy individuals and in patients with type 2 diabetes mellitus

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Abstract: The study aims to analyze the experimental haemorheological data with mathematical models and to analyze the interrelationship between the rheological parameters of the blood and blood rheology determinants. The study aims to develop this experimental basis of prognostic mathematical and hemorheological models for the interrelationship between the set of parameters. Two known models are used to describe the non-Newtonian rheological properties of blood – the shear stresses - shear rate dependences. The parameters of the simplest models - the power law and the Herschel-Bulkley law were used to analyse the differences between the hemorheological characteristics of blood from healthy individuals and a group of patients with diabetes mellitus type 2 (T2DM). The experimental data are statistically processed using the Mann-Whitney U-test for intergroup comparison of independent variables, as well as t-test. The Statistica program was used for statistical processing.

Keywords: HEMORHEOLOGICAL MODELS, BLOOD VISCOSITY, DIABETES MELLITUS TYPE 2

1. Introduction

Blood is a non-Newtonian fluid and under flow exhibits shear-thinning, viscoelastic and thixotropic behaviour. The complex rheology of blood and hemorheological parameters as shear stresses and blood viscosity are influenced by numerous factors including plasma viscosity, rate of shear, hematocrit, level of erythrocytes aggregation and deformability [1-7].

The rheological properties of blood play an important role in blood circulation under normal conditions, and the hemorheological changes can cause circulatory disorders, especially in the microcirculatory system [1-4]. The assessment of the nature and degree of rheological disorders of the blood in various diseases is essential for clarifying the pathogenesis of the morphological and biochemical changes occurring in them and for the application of rational therapy [6-10].

Newtonian fluids are, from a microscopic point of view, continuous, homogeneous and isotropic media. Their viscosity η is intrinsically linked to the fluid constitution but not to the type of flow, it is possible to determine its value under accurate experimental conditions using different types of viscometric flows as a Poiseuille flow in the capillary viscometer and a Couette flow in the rotational viscometer with coaxial cylinders. In the case of a Newtonian fluid, whose viscosity is not a function of the shear rate, the relationship between the flow rate and the pressure gradient, $\Delta P/L$, is given by the Hagen–Poiseuille equation (1):

$$\eta = (\pi a^4 / 8Q) / (\Delta P / L) \quad (1)$$

where Q is the corresponding flow rate, ΔP is the pressure drop, a is the capillary radius and L is the capillary length.

The following rheological equation (2) is the example of equation describing the behaviour of blood in shear flow :

$$\tau = \eta(\dot{\gamma}) \cdot \dot{\gamma} \quad (2)$$

Once the rheological data of a fluid are available, one is then able to develop predictive relationships for the laminar flow of fluids in many different physical situations. These predictions can be developed analytically in many cases, but almost any flow situation can be handled with computer simulation programs.

The microrheological properties of blood cells and their micromechanics are realized in the microvascular flow, where the fundamental metabolic processes between flowing blood and tissue cells occur, i.e., where the transport function of the cardiovascular system is realized and the transcapillary metabolism is provided, creating the necessary for the normal functioning of the body tissue homeostasis [1-6].

The relevance of the research is also determined by the fact that the mechanisms of development of pathological changes in tissue perfusion in diseases such as type 2 diabetes mellitus (T2DM), cerebrovascular and peripheral vascular diseases are related to the fundamental processes of blood flow, cell microrheology and intravascular hemocoagulation. Accumulated in the latter years' facts and knowledge on these issues allow for the creation of conceptual models that have predictive potential.

2. Materials and Methods

2.1. Subjects

The study included 13 patients (9 women and 4 males, mean age 66.92 ± 11.10 years) with type 2 diabetes mellitus T2DM, with mean duration of diabetes the disease 12.33 ± 5.3 years and 9 healthy controls (8 women and 1 male, mean age 52.3 ± 9.4 years). The diagnosis of Diabetes mellitus type 2 T2DM was based on clinical examination and estimation of blood glucose, insulin and HbA1C. The clinical examinations of the patients were performed in the University Hospital of Neurology and Psychiatry "St. Naum", Medical University in Sofia, Bulgaria. Blood samples were collected in Li heparin tubes and rheological measurements were completed within 3 hours after the blood sample withdrawal. Both groups gave voluntary informed consent to participate in the study.

The study was carried out in compliance with the requirements of the Helsinki Declaration; preliminary informed consent of the experiment participants was obtained following the recommendations.

2.2. Methods

Statistical analyses were performed using the scientific graphing and data analysis software packages SigmaPlot 11.0 (Systat Software Inc., 2008) и Origin 6.1 (OriginLab Corporation, 2009). The data were presented as mean $\pm$ standard deviation (SD). According to normality of the data distribution (Shapiro-Wilk's W test) the significance of inter-group differences was evaluated by using the Student's t-test and Mann-Whitney U test. Pearson correlation coefficient was used to evaluate the relationship between the whole blood viscosity and the hemorheological and the microrheological parameters). A p value less than 0.05 was considered as statistically significant.

2.2.1. Rheological measurements

A rotational viscometer Contraves LS30 (Switzerland) with MS 1/1 standard measurement system was used to investigate the rheological properties of blood. Whole blood viscosity (η) was

examined under conditions of a steady blood flow at 12 shear rates from $0,0237 \text{ s}^{-1}$ to $94,5 \text{ s}^{-1}$ at 37°C .

2.2.2. Equations

The analysis of the relationship shear stress–shear rates under steady flow was done by means of non-linear regression applying the model of Ostwald–de Waele or the power law:

$$\tau = k \cdot \dot{\gamma}^n \quad (3)$$

where τ is shear stress, $\dot{\gamma}$ is shear rate, k and n are its parameters; as well as by the Herschel–Bulkley law:

$$\tau = \tau_0 + b \cdot \dot{\gamma}^m \quad (4)$$

where τ is shear stress, $\dot{\gamma}$ is shear rate, τ_0 is the yield shear stress, b and m are parameters. The parameters of Eq. 3 and Eq. 4 were determined from the experimental data for T2DM patients and for the controls (Table 1). The use of low shear rate viscometry and a Herschel–Bulkley equation provides a yield shear stress that has a defined unites and is known to reflect RBC aggregation.

3. Results

Effect of shear rate on blood viscosity

A group of 13 patients with diabetes mellitus type 2 (T2DM) with a mean age of $66,92 \pm 11,10$ years was studied by means of a rotational Contraves Low Shear 30 viscometer and the results were compared with a control group of 9 healthy subjects with a mean age $52,3 \pm 9,4$ years. It was found that the mean whole blood viscosity values of the investigated T2DM patients' group were elevated compared to that of healthy persons

over the whole shear rate range and fell by more than ten orders of magnitude (Fig.1). The coefficients of the models of Ostwald–de–Waele (power law) and Herschel–Bulkley law, which describe whole blood flow curves ($\tau - \dot{\gamma}$) within the range of shear rates from 10^{-2} s^{-1} to 10^2 s^{-1} and itself incorporate whole blood viscosity data in the entire shear rate range. A significant difference in the mean yield shear stress τ_0 values of the T2DM patients' group compared to the controls was found.

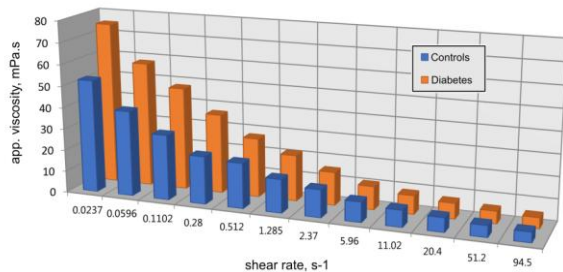


Fig.1. Whole blood viscosity–shear rate dependence in a group of patients with type 2 diabetes mellitus ($n = 13$) and the control group ($n=9$), LS 30 Contraves, $T = 37^\circ\text{C}$. (***) $p < 0.001$ at $\dot{\gamma}=0,0237 \text{ s}^{-1}$ and ** $p < 0.01$ at $\dot{\gamma}=0,0596 \text{ s}^{-1}$, $\dot{\gamma}=0,1102 \text{ s}^{-1}$ and $\dot{\gamma}=0,28 \text{ s}^{-1}$

Power law model

Table 1: Mean values and standard deviations of the coefficients of the power law and of the Herschel–Bulkley law in T2DM patients' group and in controls, compared with a non-parametric Mann–Witney test

Ostwald–de–Waele or power law (Eq. 5)		
	T2DM	Controls
$k [\text{mPa} \cdot \text{s}^n]$	16.83 ± 7.04	13.55 ± 4.53
n	$0.74 \pm 0.05^*$	0.77 ± 0.03
Herschel–Bulkley law (Eq. 6)		
$\tau_0 [\text{mPa}]$	$4.84 \pm 1.51^{***}$	2.51 ± 0.97
$b [\text{mPa} \cdot \text{s}^n]$	14.79 ± 6.59	12.88 ± 5.02
m	0.77 ± 0.05	0.78 ± 0.06

k is the flow consistency index [$\text{mPa} \cdot \text{s}$], n is the flow behaviour index (dimensionless) between 0 and 1 in Eq.3; the parameters of the Herschel–Bulkley law (Eq. 4) are τ_0 – the yield shear stress [mPa], b is the flow consistency index [$\text{mPa} \cdot \text{s}$], m is the flow behaviour index (dimensionless) between 0 and 1 in Eq. 4. * $p < 0.05$ (ns), *** $p < 0.001$

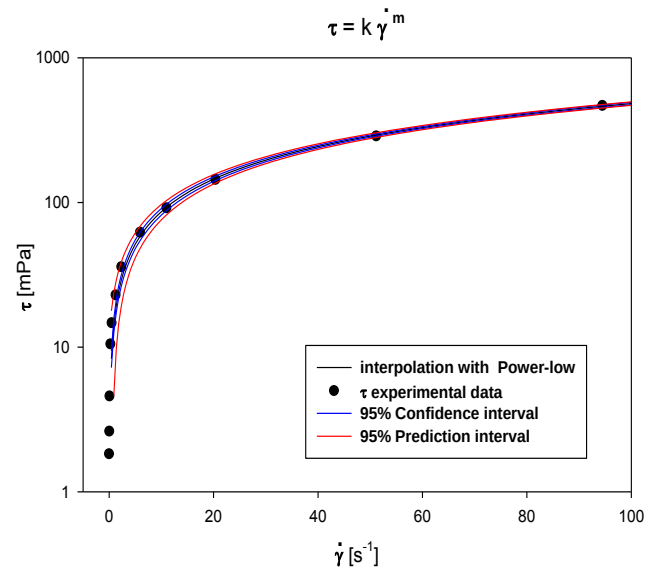


Fig.2. Interpolation of the experimental data obtained for the shear stress–shear rate ($\tau - \dot{\gamma}$) dependence by the Power law in a patient with T2DM, Low Shear 30 Contraves, $T=37^\circ\text{C}$.

Increased mean values for consistency coefficient (k) were observed in the T2DM group compared to the control group, although no statistically significant difference was found. The mean values of the flow index (m) were statistically significantly increased ($p < 0.05$) in the control group of healthy individuals compared to the diabetic group. This increase is represented graphically in Fig. 3, by means of a "box-plot" type diagram (Box-plot diagram, Vertical Box-plot).

The diagram in Fig. 3, presents the distribution of the obtained values in both studied groups. Shown is the sample median (straight horizontal black line), confidence interval (25%–75%, "boxed" values), and minimum and maximum value obtained (errors).

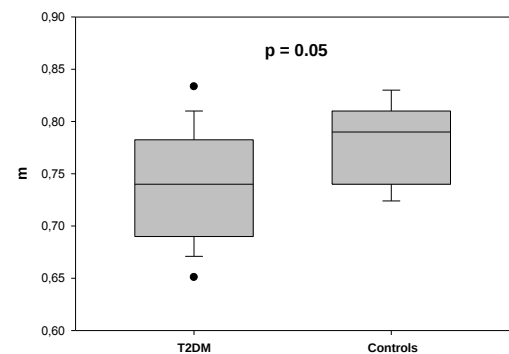


Fig.3. Box-plot diagram for the distribution of flow consistency index values (m) obtained from the Power-law model in the two studied groups

From the graph presented in Fig. 3, an asymmetry in the distribution of values is observed. A clustering of frequencies (repeating values) is seen in the higher parts of the median. It is known that the Power-law model does not describe the flow limit (τ_0). One of the disadvantages of the Power-law model is that it does not describe with sufficient accuracy the viscosity of non-Newtonian fluids in regions with very low and very high strain rates. Since m is usually less than unity, the viscosity (η) tends to infinity at very low shear rates and is practically constant at high rates, which is also confirmed experimentally (see Fig.1).

Herschel–Bulkley model

Since whole blood exhibits non-Newtonian pseudoplastic fluid behavior, i.e., it does not flow at very low tangential (shear)

stress is applied. The tangential stress must exceed a critical value known as the flow threshold for blood to flow [7-11].

The Herschel-Bulkley model extends the Power-law model, describing the dependence of the tangential stress (τ) on the strain rate ($\dot{\gamma}$) and the yield shear stress (τ_0), again introducing the parameters k and m (presented in Table 1).

The data were obtained by a high-accuracy approximation ($R>0.999$) of the experimental data obtained for the dependence of the tangential stress on the strain rate by applying the Herschel-Bulkley constitutive equation (Fig. 4). Increased mean values for consistency coefficient (k) were observed in the T2DM patients' group compared to the control group, although no statistically significant difference was found. For the flow index, lower values were found for the patient group compared to the control group. A statistically significant increase in mean flow index values (m) was not observed in either group. The mean values for the yield shear stress (τ_0) were increased significantly ($p<0.001$) in the T2DM group compared to the control group.

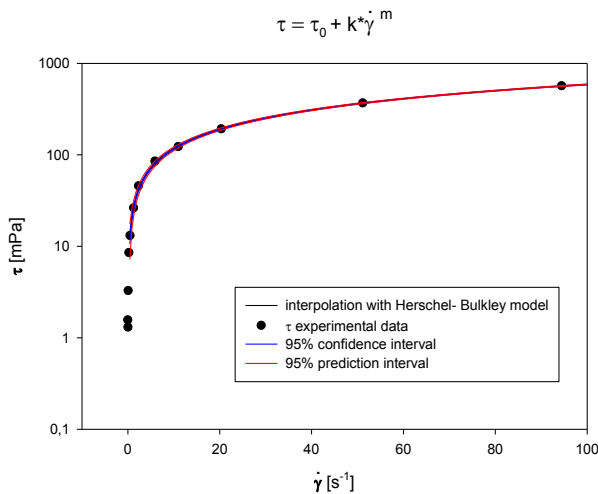


Fig.4. Interpolation of the experimental data obtained for the shear stress-shear rate (τ - $\dot{\gamma}$) dependence by the Herschel-Bulkley law in a patient with T2DM, Low Shear 30 Contraves, $T=37^{\circ}\text{C}$.

The data were obtained by a high-accuracy approximation ($R>0.999$) of the experimental data obtained for the dependence of the shear stress on the shear rate by applying the constitutive equation of the yield shear stress (τ_0) and for both groups is presented graphically in Fig.5, by means of a "box-plot" diagram, visualizing the distribution of the obtained values. The sample median (straight black line), confidence interval (25%–75%, boxed values), as well as the minimum and maximum value (errors) are shown.

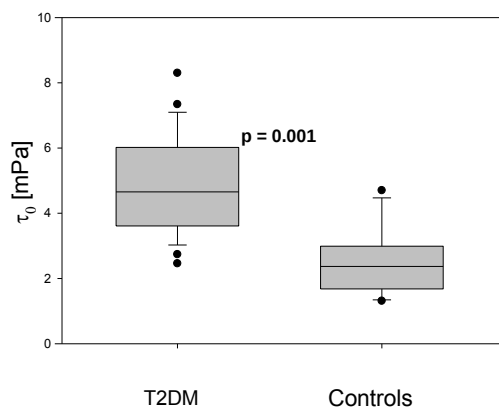


Fig.5. Box-plot diagram for the distribution of values of the yield shear stress (τ_0) obtained from the Herschel-Bulkley model in the two studied groups.

From the graph in Fig. 5 an asymmetry in the distribution of values, as an accumulation of frequencies (repeating values) is found in the lower parts of the median.

Since the Herschel-Bulkley model is suitable for describing non-Newtonian viscosity under shear-thinning non-Newtonian rheological behavior of the liquid and also reflects the yield shear stress, it is preferable for describing the rheological behavior of blood.

4. Conclusion

Two known models - the power law and the Herschel-Bulkley law were used to analyse the differences between the hemorheological characteristics of blood from healthy individuals and a group of patients with diabetes mellitus type 2 (T2DM). Their parameters, obtained based on the experimental data were analysed for the ability and accuracy for describing the non-Newtonian viscosity shear-thinning rheological behavior of the investigated groups.

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