Dynamic stability of a fluid-immersed, cracked pipe conveying fluid and resting on a Winkler elastic foundation

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Abstract: *The dynamic stability of a cracked pipeline resting on a Winkler elastic foundation and immersed in fluid that is moving with a particular velocity is investigated. The Galerkin method is employed to approach numerically the problem. Conclusions are drawn on the influence of the rigidity of the Winkler elastic foundation on the critical flow velocity of the pipe.*
Keywords*: STABILITY, FLUID, CRITICAL VELOCITY, IMMERSED PIPE, WINKLER ELASTIC FOUNDATION*

1. Introduction

Fluid-immersed pipes, conveying fluid are used in many areas of the industry. They are widely used in the petroleum industry for transportation of oil and gas.

Numerous articles nowadays analyze the linear and nonlinear dynamics of the fluid-immersed pipes conveying fluid, proving the actuality of the problem.

Deng and Yang [1] studied the dynamics of pipes with different types of flowing fluid. The tubes are immersed in fluid. The pipe is considered as a cylindrical shell.

In [2] is presented an investigation about the dynamic stability of a pipe with a flowing fluid immersed in a non-viscous fluid moving at a constant speed. It is also given an analytical solution for the same type of pipe with a rigid body attached at one of its ends.

Pipelines often rest with its entire length or with part of it on an elastic medium. The first suggested model of that medium is the Winkler elastic foundation. Although it has some shortcomings it is still being widely used in civil engineering since its introduction in 1867. According to the model, any point deflection at the surface of an elastic medium is proportional to the applied load in the point and is independent on the applied loads at other points of the surface. Thus, the mechanical model of the elastic medium consists of a series of closely spaced and mutually independent linear elastic springs with rigidity k .

Cracks are the most encountered damages in the structures. They reduce the stiffness of the structural element which causes decrease in its natural frequencies and change in the mode shapes. In pipes conveying fluid, cracks lead to decrease in the critical flow velocity. The cracks could be hazardous for the system. They might lead to loss of stability if the reduced, due to the crack, critical velocity of the transported fluid is exceeded. That's why crack detection is a topic of much interest in the scientific community. Some of the studies for crack detection deal with change in the natural frequencies and Eigen forms, other with dynamic response to harmonic loads.

The present paper investigates the dynamic stability of a cracked, fluid-immersed pipe resting on a Winkler elastic foundation. The results obtained reflect the dependence of the critical fluid velocity on the rigidity of the Winkler elastic foundation. The results also show the effect of an open crack on the critical velocity of the fluid.

2. Problem formulation

The present paper uses the Euler-Bernoulli beam theory to investigate the dynamic stability of a fluid-immersed pipe of length l , conveying fluid and resting on a Winkler elastic foundation. The pipe, shown in Fig.1, is hinged at both ends. The pipe is supposed to have an open edge crack, which dimensions (θ_c and b) are shown in (Fig.1). b is the length of the crack. θ_c is the half central angle corresponding to the chord b . The crack severity is usually measured by the ratio θ_c / π . The crack position along the length of the tube is fixed through the coordinate x_c . The crack is modeled as a rotational spring with a lumped stiffness k_r [3] (Fig.2).

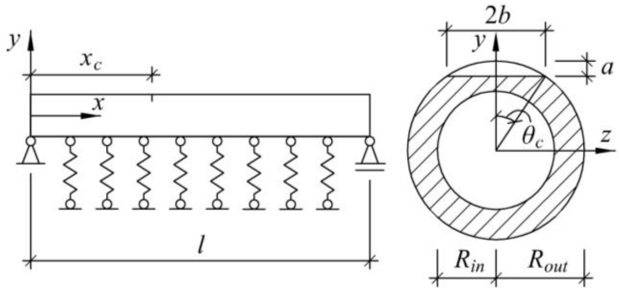

Fig. 1 *Static scheme of the investigated pipe*

The pipe is divided into two segments. The first segment is the left-hand side of the crack, and the second – is the right-hand side of the crack.

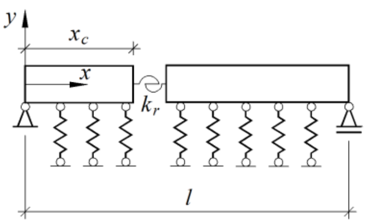

Fig. 2 *Mechanical model of the crack*

The transverse vibration of a fluid-immersed straight pipe conveying inviscid fluid and lying on a Winkler elastic foundation, with rigidity k , is governed by the following differential equation.

$$(1)\quad EI\frac{\partial^4 w}{\partial x^4}+\left(m_f V^2+m_e V_e^2\right)\frac{\partial^2 w}{\partial x^2}+2\left(m_f V+m_e V_e\right)\frac{\partial^2 w}{\partial x \partial t}+$$

$$+\left(m_f+m_p+m_e\right)\frac{\partial^2 w}{\partial t^2}+kw=0\ ,$$

where t is the time, $w(x,t)$ is the lateral displacement of the pipe axis, x is the coordinate along the axis, EI is the rigidity of the pipe. The mass of the pipe per unit length is denoted by m_p , the mass of the fluid per unit length of the pipe by m_f and m_e is the added mass of the external fluid. V is the flow velocity of the fluid in the pipe and V_e is the velocity of the external fluid.

The added mass of the external fluid per unit length of the pipe m_e in the case when the pipe is close to a horizontal plane (Fig.3) is calculated by the following formula, given in [4]:

$$(2)\quad m_e=\pi\rho_e r^2\left(1+\frac{r^2}{2h^2}\right),$$

where ρ_e is the density of the external fluid.

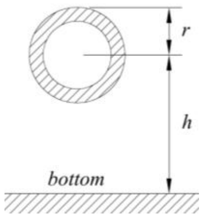

Fig. 3 *A scheme for obtaining the added mass of the external fluid*

The spectral Galerkin method is applied to approximate the solution of the boundary value problem (1). According to this method, an approximate solution is sought in the form [5]:

$$(3)\quad w(x,t)=\sum_{i=1}^{n} y_i(x) z_i(t),$$

where:

$z_i(t)$ are unknown functions;

$y_i(x)$ are basic functions that satisfy the boundary conditions of the pipe. Such functions are the functions describing the *i*-th mode of vibration of a beam with the same static scheme as the immersed pipe.

On the basis of the differential equation, describing the lateral vibrations of an immersed tubular beam, filled with stationary fluid ($V=0$) is obtained [5]:

$$(4)\quad y_i^{IV}(x)=\gamma_i^4 y_i(x),$$

where:

$$(5)\quad \gamma_i=\sqrt[4]{\frac{(m_f+m_p+m_e)\omega_i^2}{EI}},$$

where ω_i is the circular frequency of the beam.

Substituting equation (3) into equation (1), one obtains the residual function:

$$(6)\quad R(x,t)=\sum_{i=1}^{n}\Big\{(m_f+m_p+m_e)y_i\ddot{z}_i+2(m_fV+m_eV_e)y_i'\dot{z}_i+$$
$$+\left[(EI\gamma^4+k)y_i+(m_fV^2+m_eV_e^2)y_i''\right]z_i\Big\}$$

In (6) and in the sequel, primes denote derivatives with respect to x and dots with respect to the time t.

The Galerkin method requires the residual function $R(x,t)$ to be orthogonal to the basic functions in the interval $x\in[0;l]$:

$$(7)\quad \int_0^l R(x,t)y_k(x)dx=0,\ \text{for}\ k=1,\dots,n$$

Equation (6) is rewritten in the following form:

$$(8)\quad \sum_{i=1}^{n}\int_0^l\Big\{(m_f+m_p+m_e)y_i\ddot{z}_i+2(m_fV+m_eV_e)y_i'\dot{z}_i+$$
$$\left[(EI\gamma^4+k_w)y_i+(m_fV^2+m_eV_e^2)y_i''\right]z_i\Big\}y_k\,dx=0\ \text{for}\ k=1,\dots,n.$$

Equation (8) represents a system of n differential equations with n unknown functions $z_i(t)$. In order to solve the system, the described in [5] method is applied. According to it the pipe is divided to sections with length Δx. The following relationships are taken into account:

$$(9)\quad \int_0^l y_i\,y_k\,dx=\{y_i\}^T\{y_k\}\Delta x$$

$$(10)\quad \int_0^l y_i'\,y_k\,dx=\{y_i'\}^T\{y_k\}\Delta x$$

$$(11)\quad \int_0^l y_i''\,y_k\,dx=\frac{1}{EI}\{M_i\}^T\{y_k\}\Delta x$$

where in (9),(10) and (11):

$\{y_i\}$ - is a column vector consisting of the lateral displacements of the stations on the axis of the pipe, corresponding to the i-th Eigen form in the case of stationary fluid ($V=0$);

$\{y_i'\}$ - is a column vector consisting of the rotations of the cross-sections in the stations on the axis of the pipe, corresponding to the i-th Eigen form in the case of stationary fluid ($V=0$);

$\{M_i\}$ - is a column vector consisting of the bending moments in the stations on the axis of the pipe, corresponding to the i-th eigen form in the case of stationary fluid ($V=0$).

Substituting (9),(10) and (11) in (8) the following system of n differential equations with n unknown functions $z_i(t)$ is obtained:

$$(12)\quad \sum_{i=1}^{n}\Big\{(m_f+m_p+m_e)\{y_i\}^T\{y_k\}\ddot{z}_i+\left[2(m_fV+m_eV_e)\{y_i'\}^T\{y_k\}_i\right]\dot{z}_i+$$
$$+\left[(EI\gamma^4+k_w)\{y_i\}^T\{y_k\}+(m_fV^2+m_eV_e^2)\frac{1}{EI}\{M_i\}^T\{y_k\}\right]z_i\Big\}\Delta x=0$$

The system (12) could be rewritten in matrix form:

$$(13)\quad M\,\ddot{z}+C\,\dot{z}+K\,z=0$$

The general solution of the system (12) is expressed through the roots ($\lambda_1,\dots,\lambda_{2n}$) of the equation:

$$(14)\quad \det X=0$$

The elements of the matrix X are given by:

$$(15)\quad X_{ik}=\lambda^2M_{ik}+\lambda C_{ik}+K_{ik}$$

$$(16)\quad M_{ik}=(m_f+m_p+m_e)\{y_i\}^T\{y_k\}\Delta x,\quad M_{ik}=0\ (\text{when}\ i\neq k)$$

$$(17)\quad C_{ik}=2(m_fV+m_eV_e)\{y_i'\}^T\{y_k\}_i\Delta x$$

$$(18)\quad K_{ik}=\left[k_w\{y_i\}^T\{y_k\}+(m_fV^2+m_eV_e^2)\frac{1}{EI}\{M_i\}^T\{y_k\}\right]\Delta x+E_{ik}$$

$$(19)\quad E_{ik}=EI\gamma^4\Delta x,\quad E_{ik}=0\ (\text{when}\ i\neq k).$$

On the basis of obtained roots ($\lambda_1,\dots,\lambda_{2n}$) could be drawn conclusions about the stability of the system. The system is stable if the real part of all the roots of the characteristic equation (14) is negative.

The roots ($\lambda_1,\dots,\lambda_{2n}$) depend on all the parameters of the system. If all of them are fixed except the velocity of the conveyed fluid V or the velocity of the external fluid V_e, one could obtain the corresponding critical velocities.

3. Crack modeling

It is considered that the bending vibrations of the Euler-Bernoulli beam is in the plane $x-y$ (Fig.1), which is also a plane of symmetry for the cross-section. The crack is assumed to be open. Castigliano's theorem is used to obtain the local flexibility in the presence of the crack [6]

(20) $c = \frac{\partial^2 U}{\partial M^2} = \frac{1-\nu^2}{E} \int_{-b}^{b} \int_{0}^{a} \frac{\partial^2 \left(K_I^2\right)}{\partial M^2} dx\, dy$,

where E and ν are respectively Young's module and Poisson's ratio. K_I is the stress intensity factor of bending. a and b are the crack dimensions as shown in (Fig.1). M is the bending moment.

(21) $K_I = \frac{M}{\pi R^2 t_p} \sqrt{\pi R \theta_c}\, F(\theta_c)$,

where $R = 0{,}5(R_{in} + R_{out})$, t_p and θ_c are respectively thickness of the pipe and the half central angle of the crack (Fig.1). $F(\theta_c)$ is calculated from the following formula [7]

(22) $F(\theta_c) = 1 + A_t \left[4{,}5967 \left(\frac{\theta_c}{\pi} \right)^{1,5} + 2{,}6422 \left(\frac{\theta_c}{\pi} \right)^{4,24} \right]$

(23) $A_t = \sqrt[4]{\frac{1}{8} \frac{R}{t_p} - \frac{1}{4}}$ for $5 \le \frac{R}{t_p} \le 10$

(24) $A_t = \sqrt[4]{\frac{2}{5} \frac{R}{t_p} - 3}$ for $10 \le \frac{R}{t_p} \le 20$.

The equivalent rotational spring stiffness

(25) $k_r = \frac{1}{c}$.

4. Numerical results

Numerical studies have been carried out for the system in Fig. 1.

The geometric and the material characteristics of the pipe are: the inner and the outer radii of the cross-section of the pipe are $R_{in} = 0.012m$ and $R_{out} = 0.014m$, Young's modulus $E = 210GPa$, the density of the material of the pipe $\rho = 7800 kg/m^3$. The density of the flowing fluid in the pipe is $\rho_f = 900 kg/m^3$. The density of the external fluid is $\rho_e = 1000 kg/m^3$.

The dimensions of the crack are $a = 0.001m$, $b = 0.005m$. The position of the crack is fixed with the coordinate $x_c = 1{,}1m$.

In the present paper 16 eigenfunctions $y_i(x)$ are used in the approximate solution (3).

For the pipe in Fig.1 is obtained the critical value of the flowing fluid V_{cr} for different rigidity of the Winkler elastic foundation. The velocity of the external fluid is assumed to be $V_e = 2m/s$ The results are shown in Fig.4.

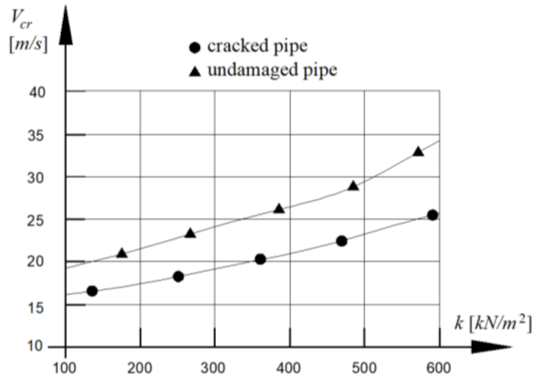

Fig. 4 *Critical value of the fluid velocity versus the rigidity of the Winkler elastic foundation*

4. Conclusions

The obtained results show that the Winkler elastic foundation has a stabilizing effect on the pipe - with increasing the rigidity of the foundation the critical velocity of the conveyed fluid increases. The crack has a destabilizing effect on the system, leading to decreasing of the critical velocity of the pipe flow.

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