Control-relevant identification of the DC engine coupled with a reaction wheel

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Abstract: *This paper proposes to estimate the mathematical model of the DC motor coupled with a reaction wheel in two ways: open-loop identification and closed-loop identification. For open-loop identification, the experimental curve is approximated with the model of object with inertia first order, and inertia second order. In the case of closed-loop identification, it was obtained the model with inertia third order, that approximates the dynamic of the process, where the coefficients are calculated based on the simple analytical expressions according to the values that are extracted from the undamped step response of the closed-loop system with P controller. The results of the experimental model estimation were compared with the results obtained using the System Identification Toolbox from MATLAB. In addition, to the identified object model, it was proposed to tune a PID controller based on the maximum stability degree criterion.*

Keywords*: ESTIMATION OF THE MATHEMATICAL MODEL, EXPERIMENTAL IDENTICATION, TRANSFER FUNCTION, CLOSED-LOOP IDENTIFICATION, CONTROL SYSTEM.*

1. Introduction

A significant advancement in technological installations has been the development of electrical machines, which act as actuators in many systems. These machines, particularly electric motors, are crucial for driving industrial processes and typically operate based on electromagnetic forces acting on a current-carrying conductor within a magnetic field. Electric motors are generally categorized by the type of electric current they utilize: direct current (DC) motors and alternating current (AC) motors. Due to their linear characteristics and relatively straightforward speed control methods, DC motors are the most widely used actuators in control systems [1].

They offer precise control and adaptability, making them ideal for a wide range of industrial applications. In automatic control systems, the DC motor is commonly employed either as an actuator or as the control object itself. In both roles, controlling the motor's speed is essential, and this is typically achieved by implementing standard control algorithms designed for such purposes. To effectively control these systems, it is often necessary to derive the mathematical model of the industrial process. A detailed and accurate mathematical model that captures the dynamic behavior of an industrial process is fundamental for the successful design and optimization of control algorithms in industrial applications. This model serves as the foundation upon which control strategies are developed, enabling engineers to predict system responses under various operating conditions and disturbances. By providing a close approximation of the process dynamics, the model allows for the synthesis of control algorithms that can improve stability, enhance system performance, and minimize energy consumption or production costs. The mathematical modeling can be achieved through analytical or experimental approaches.

The analytical approach is based on well-established theoretical principles, such as physical laws and known relationships between variables involved in the process. On the other hand, the experimental approach involves collecting input-output data from the actual process, allowing for the development of models using statistical or data-driven techniques. These models describe the system's behavior based on empirical data, providing insight into its dynamics. The choice of estimation method depends largely on the objectives of system identification, whether for system analysis or control system design.

The analytical and experimental approaches to system identification are closely linked to the choice between open-loop and closed-loop identification methods. In the analytical approach, theoretical principles such as physical laws and established relationships between variables provide the foundation for deriving mathematical models. This method often aligns with open-loop identification, where the system's dynamics are analyzed without the influence of feedback. Open-loop identification techniques rely on linearization around a nominal operating point and are simpler to implement, especially when sufficient prior knowledge of the system exists. These methods are typically used when the system is assumed to behave predictably and when control objectives, such as real-time feedback, are not an immediate concern.

On the other hand, the experimental approach involves collecting input-output data from the actual process to develop empirical models using statistical or data-driven techniques. This approach is particularly useful when the system's dynamics are complex, nonlinear, or affected by external disturbances, making analytical modeling less feasible. Closed-loop identification is often preferred in such cases, where the system operates under feedback control during the identification process [2-3].

In recent years, there has been a growing interest in closed-loop identification, particularly in the field of control-relevant identification [5]. This specialized approach adapts the identification process to extract information directly from the system that is pertinent to real-time controller design and tuning. Control-relevant identification aims to ensure that the identified models effectively contribute to achieving the desired control objectives. Several methods have been developed to implement this approach, which has proven to be accurate for auto-tuning controllers [4]. These controllers improve system performance and perturbation signal rejection.

The goal of this paper is to estimate the mathematical model of a DC motor coupled with a reaction wheel using two distinct approaches: open-loop identification and closed-loop identification. For open-loop identification, the system is modeled with first- and second-order inertial dynamics, approximating the experimental response. In the closed-loop identification approach, a third-order inertial model is derived, with coefficients calculated analytically from the undamped step response of a closed-loop system controlled by a proportional (P) controller. The paper aims to compare the accuracy of the experimental models with those obtained using the System Identification Toolbox in MATLAB, and further proposes a PID controller tuning method based on the maximum stability degree criterion to enhance control performance.

2. Description of the System

The FK130SH DC motor was chosen to test multiple reaction wheels, with the system implemented on the NUCLEO-F303K8 platform from ST Microelectronics. The reaction wheel is directly coupled to the motor, and its speed is regulated by the STM32F303K8 microcontroller. The block diagram of the system is presented in Fig. 1. To begin data recording, the computer sends a start signal to the microcontroller, which then controls the electronic switch connected to the motor. The microcontroller measures the motor's instantaneous rotational speed and transmits the data to the computer in real time. The EE-SX4235A-P2 transmissive photomicrosensor is used as the speed sensor. This sensor outputs a logical high signal when an opaque object, such as a marker on the reaction wheel, passes between the photoemitter and photoreceptor, generating a pulse with each marker crossing through the sensor's aperture [9].

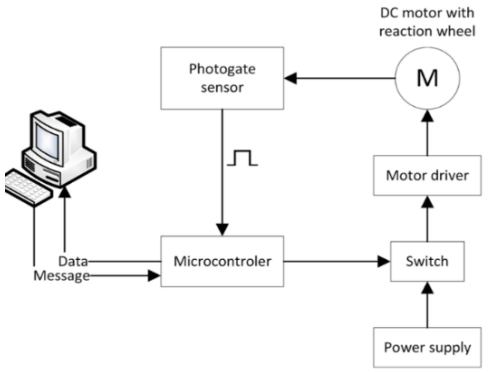

Fig. 1 *Structure diagram of the designed system.*

The experimental identification in the open-loop involves the acquisition of data so that the experimental variation of the DC motor speeds at the reference speed of 12000 rpm was obtained as presented in Fig. 2.

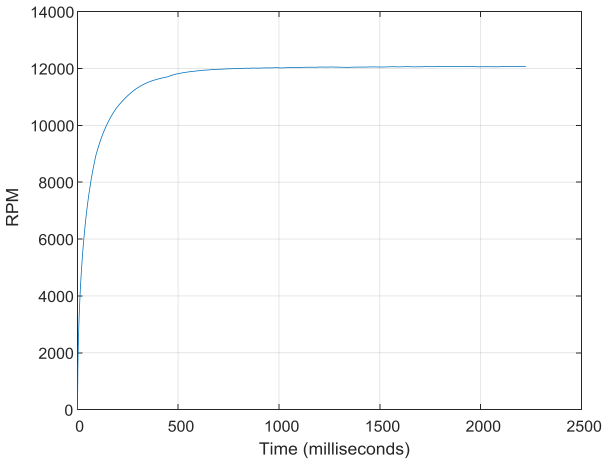

Fig. 2 *Experimental curve.*

3. Experimental Identification

3.1. Open-loop Identification

Based on the experimental curve presented in Fig. 2, it was performed the experimental identification using the System Identification Toolbox from MATLAB and there were obtained the following transfer functions:

$$H(s) = \frac{k}{Ts+1} = \frac{1}{937.58s+1}, \quad (1)$$

where k - transfer coefficient, T – time constant.

$$H(s) = \frac{k}{(T_1s+1)(T_2s+1)} = \frac{1}{(1466.794s+1)(240.31s+1)}, \quad (2)$$

where k - transfer coefficient, T_1, T_2 – time constants.

Figure 3 presents the comparison between the original experimental curve – curve 1; the step response of the model of the object (1) – curve 2; the step response of the model of the object (2) – curve 3.

Next, based on the step response of the open loop system presented in Fig. 2, it is calculated the value of the $y(t)$ at level 0,632k, according to this value it is calculated the value of the time constant T. According to this it was obtained the following transfer function:

$$H(s) = \frac{k}{Ts+1} = \frac{1}{812.5s+1}, \quad (3)$$

where k - transfer coefficient, T – time constant.

The transient response presented in Fig. 2 can be approximated with the transfer function with inertia second order:

$$H(s) = \frac{k}{(T_1s+1)(T_2s+1)} = \frac{k}{a_0s^2+a_1s+a_2}, \quad (4)$$

where T_1, T_2 - time constants, k - transfer coefficient and $a_0 = T_1T_2, a_1 = T_1 + T_2, a_2 = 1$.

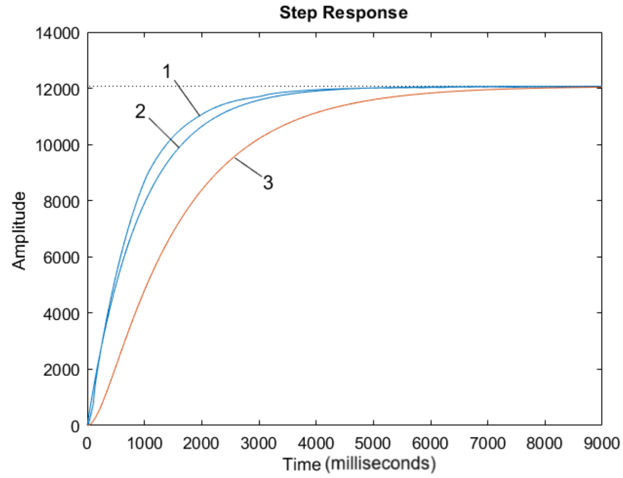

Fig. 3 *Transient processes.*

And it is used the following procedure for the determination the parameters k, T_1, T_2 [8]:

1. It is calculated the value of the $y(t)$ at the level 0.632k, according to this value it is calculated the value of T (Fig. 2).

2. It is calculated the values of the time constants T_1 and T_2 for the model of the object (4) [7-8]:

$$T_1/T_2=0.5,$$

$$T_2=0.64T=520 \text{ ms.}, \quad (5)$$

$$T_1=0.5T_2=260 \text{ ms.} \quad (6)$$

According to this calculation, there is obtained the following transfer function:

$$H(s) = \frac{k}{(T_1s+1)(T_2s+1)} = \frac{1}{(260s+1)(520s+1)}. \quad (7)$$

In Fig. 4, it is presented the comparison between the original experimental curve – curve 1; the step response of the model of the object (4) – curve 2; the step response of the model of the object (7) – curve 3.

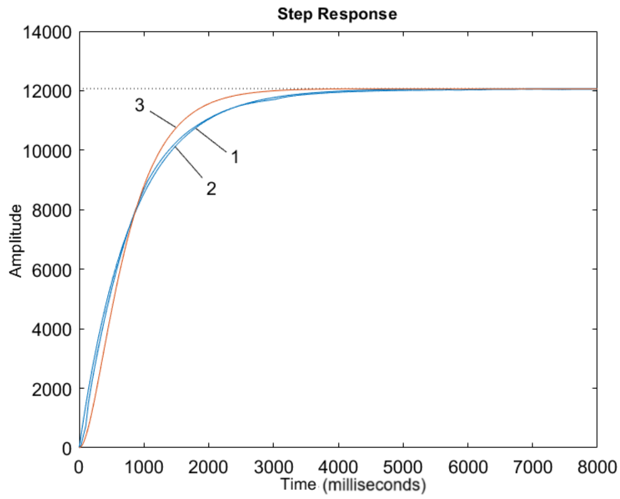

Fig. 4 *Transient processes.*

3.2 Closed-loop Identification

In author's paper [6], it was proposed an algorithm for closed-loop identification, where the transfer function that approximates the control object is:

$$H(s) = \frac{k}{(T_1s+1)(T_2s+1)(T_3s+1)} = \frac{k}{a_0s^3+a_1s^2+a_2s+a_3} = \frac{B(s)}{A(s)}, \quad (8)$$

where T_1, T_2, T_3 are time constants; k is transfer coefficient; $a_0 = T_1T_2T_3, a_1 = T_1T_2 + T_1T_3 + T_2T_3, a_2 = T_1 + T_2 + T_3, a_3 = 1$.

According to the closed-loop identification algorithm, there are the following steps:

1. Implementation of the feedback control system with P controller.

2. Variation of the proportional tuning parameter $k_p > 0$, until the system achieves the limit of stability and further determines the value of critical transfer coefficient k_{cr}, period of oscillations – T_{cr}, the amplitude of oscillations - A_{cr}. According to this step, it was obtained the experimental step response presented in Fig. 5.

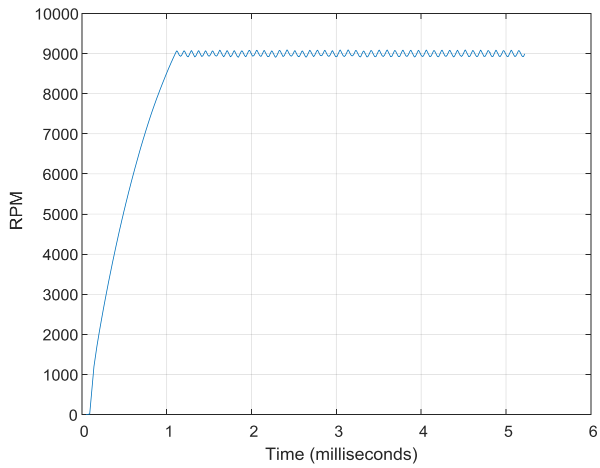

Fig. 5 *Oscillatory step response.*

According to the step response presented in Fig. 5, there were obtained: k_{cr} =500, A_{cr} – 0.0093,T_{cr} = 0.0935 ms.

3. Calculation of the value of natural frequency according to the expression:

$$\omega_n = \frac{2\pi}{T_{cr}} = 67.2.$$

4. Calculation the parameters' values of the control object (8):

$$\begin{cases} a_0 = \frac{(k_{cr}+1)\sqrt{1-A_{cr}^2}}{A_{cr}\,\omega_n^3} = 0.1775; \\ a_1 = \frac{k_{cr}+1}{{\omega_n}^2} = 0.1109; \\ a_2 = \frac{(k_{cr}+1)\sqrt{1-A_{cr}^2}}{A_{cr}\,\omega_n} = 801.5616; \\ a_3 = 1. \end{cases} \quad (9)$$

Based on the expressions (9), there are calculated the transfer function of the control object:

$$H(s) = \frac{1}{0..1775s^3+0.1109s^2+801.561s+1}. \quad (10)$$

In Fig. 6, it is presented the comparison between the original experimental curve (Fig. 2) – curve 1; the step response of the model of object (10) – curve 2.

From Fig. 6, it can be observed that closed-loop identification offered so high precision in the process of identification of the mathematical model of the DC motor.

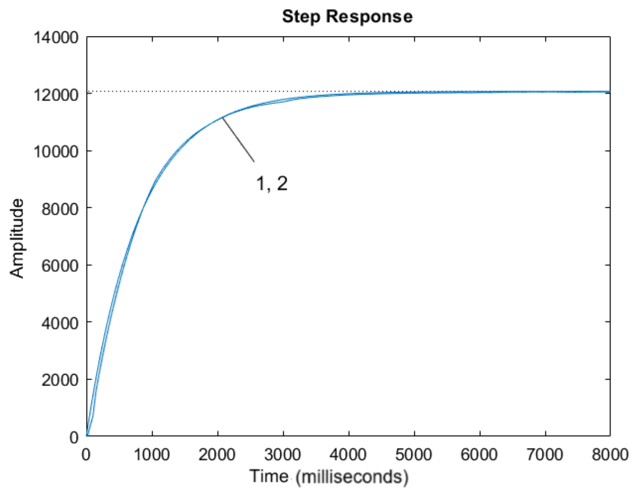

Fig. 6 *Transient processes.*

4. Tuning the PID controller

It is considered that the control system is presented in Fig. 7 and is formed from the controller with transfer function $H_R(s)$ and control object (7).

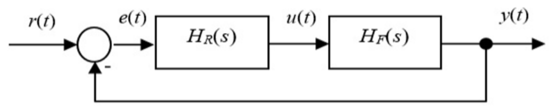

Fig. 7. *Structural scheme of the automatic control system.*

The control algorithm PID is described by the following transfer function [1]:

$$H_R(s) = k_p + \frac{k_i}{s} + k_d s, \quad (11)$$

where the tuning parameters of PID controller - k_p, k_i, k_d.

The analytical expressions for the calculation the tuning parameters of the PID controller were obtained in concordance with the following analytical expressions [7]:

$$k_p = \frac{(T_1+T_2)^2}{2T_1T_2} = 2.25, \quad (12)$$

$$k_i = \frac{(T_1+T_2)}{2kT_1T_2} = 0.0029, \quad (13)$$

$$k_d = \frac{T_1+T_2}{2k} = 390. \quad (14)$$

After tuning the PID controller according to the expressions (12)-(14), it was obtained transient process presented in Fig. 8

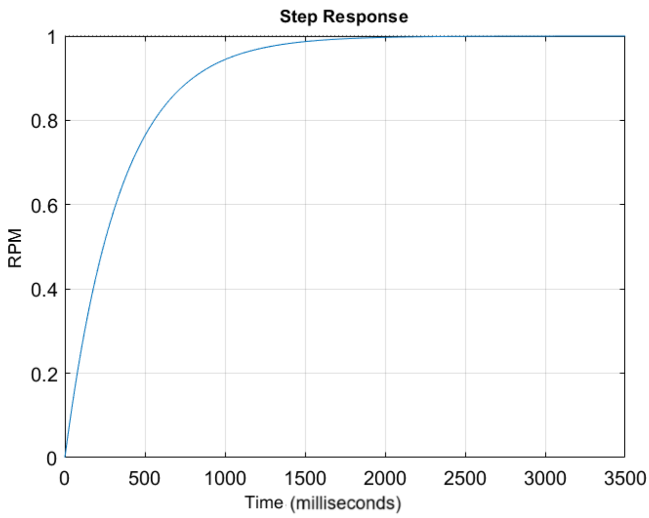

Fig. 8. *Structural scheme of the automatic control system.*

From Fig. 8, it can be observed that it was obtained the transient response with the following performances: settling time t_s – 1500 ms., rise time t_r – 1500 ms., overshoot σ – 0.

5. Conclusions

In this paper, it is presented the analysis of the two types for estimating the mathematical model of a DC motor coupled with a reaction wheel: open-loop and closed-loop identification. For open-loop identification, first and second-order inertia models were obtained to approximate the experimental system's dynamics, for analysis it was proposed to use the System Identification Toolbox from Matlab, which demonstrates good results in model estimation.

In the closed-loop approach, a third-order inertia model was obtained using simple analytical expressions based on the undamped step response of the system with a P controller. In this case, the identified model estimated with a high degree of precision the control object. Additionally, a PID controller was proposed, tuned according to the maximum stability degree criterion. The findings demonstrate the effectiveness of the proposed models in accurately approximating the system's dynamics and provide a reliable method for tuning PID controller. The proposed control-relevant approach can be implemented as an auto-tuning procedure of the PID controller.

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7. References

1. I. Dumitrache, *Control engineering (in romanian).* Bucharest: Politehnica P, V. **1**, (2016)
2. J. Ertveldt, J. Sanchez-Medina, Z. Jardon, M. Hinderdael, P. Guillaume, Experimental identification of process dynamics for real-time control of Directed Energy Deposition, Procedia CIRP, Vol. **111**, pp. 321-325, (2022)
3. W. Jianhong, R. A. Ramirez-Mendoza, The practical analysis for closed-loop system identification, Cogent Engineering, **7:1**, pp. 1-16, (2020)
4. A. O'Dwyer, *Handbook of PI and PID controller tuning rules*. 3rd Edition, Imperial College Press, (2009)
5. J. D. Álvarez, J. D. Guzmán, D. E. Rivera, D. E. Berenguel, S. Dormido, Perspectives on control-relevant identification through the use of interactive tools, Control Engineering Practice, **21(2),** pp. 171–183, (2013)
6. I. Cojuhari, Data-Driven Model Identification and Control of the Inertial Systems, In: Intelligent Control and Automation, **14**, pp. 1-18, (2023)
7. I. Cojuhari Algorithm for Self-Tuning the PID Controller. În Journal of Engineering Scienc. Vol. **XXVIII (4),** pp. 63-73, (2021)
8. B. Izvoreanu, M. Potlog, I. Cojuhari, I. Fiodorov, D. Moraru Metodă de Identificare a Modelelor Obiectului de Reglare de Ordinul Unu şi Doi după Răspunsul Experimental al Procesului [The method of identification the model of object with first and second order inertia by the experimental response of the process]. In: *Proceedings of the IX International Conference on Microelectronics and Computer Science (ICMCS-2017)*, October 19-21, Chişinău, pp. 206-209, (2017)
9. V. Melnic, I. Cojuhari Estimation of the Mathematical Model of the DC engine coupled with a reaction wheel. În ROMAI Journal, **14**, no.**2** pp, 103–112, (2018)