

Mathematical Processing and Analysis of Sleep Signals Using a Portable and Cost-Effective Oculograph

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Abstract: *The study of sleep is crucial for understanding various physiological and neurological processes, yet research on different sleep phases often comes with high costs and requires specialized equipment. To address these challenges, we developed a portable and relatively inexpensive oculograph, which enables more accessible sleep studies. A critical technical aspect of using the device is the necessity for mathematical transformations to interpret the signals generated by eye movements, which are often complex and prone to noise. We implemented several mathematical procedures for noise reduction, signal filtering, and the extraction of key signal features. To assess the accuracy of the oculograph, we conducted 10 daytime experiments with predefined protocols involving specific eye movements. The results indicate that the oculograph successfully measures eye movements with high precision, which was further validated through comparison with graphical signal representations. Moreover, we performed tests for nighttime use of the device, and validation of REM sleep signals is planned using a camera to record the subject during sleep. These promising outcomes suggest significant potential for the oculograph to help sleep research by offering a more affordable and mobile solution, suitable for both laboratory and home environments. The mathematical procedures and signal processing techniques presented here are tailored to the needs of psychological and medical sleep studies. Additionally, practical applications of the oculograph for targeted sleep research, including tracking eye movements during various sleep stages, are proposed.*

Keywords: SLEEP MONITORING, OCULOGRAPH, EYE MOVEMENTS, REM VALIDATION

1. Introduction

Sleep research relies on a range of advanced methods to explore sleep phases and their roles in physiological and psychological health. The primary goal of sleep analysis is to classify different sleep stages based on physiological signals. Sleep is typically divided into several phases, including wakefulness, light sleep, deep sleep, and REM sleep. Of these, REM sleep is particularly interesting due to the intense eye movements that occur during this phase.

Electroencephalography (EEG) has historically been the foundational technique for monitoring sleep stages by detecting brainwave activity across NREM and REM stages [1, 2]. Yet, researchers have developed additional methodologies that vary in objectivity and use of contact or contactless devices, allowing for a multi-dimensional analysis of sleep, as reviewed in [3]. These methods, when used in conjunction, are part of what is known as polysomnography, an approach that combines diverse recording devices to enhance reliability and mitigate the limitations of single-method studies.

Polysomnography can incorporate various measures, such as EEG for brainwave tracking, electrooculography (EOG) for eye movements, electromyography (EMG) for muscle activity, and electrocardiography (EKG) for heart rate monitoring. Devices like pulse oximeters monitor oxygen saturation, respiratory monitors track breathing efforts, and capnography records CO₂ levels. Video recording and microphones serve as non-invasive tools to capture body movements and sounds like snoring, while other tools measure physiological responses, such as esophageal tests for GERD or blood pressure monitors. This range of tools enables a nuanced analysis of sleep parameters, including duration and quality of REM and NREM stages, as well as timing of different phases.

Alongside objective measures, subjective tools, such as questionnaires and sleep diaries, provide insights into experiences that instruments cannot capture, like dream content and emotional states. Studies on animal sleep often utilize more invasive techniques to complement human data, as described in [4].

Despite its advantages, the multi-method approach of polysomnography introduces challenges. The complexity of multiple datasets can lead to an excess of information, complicating the analysis and increasing the chance of false findings. Furthermore, high costs and the need for consumable supplies, particularly for EEG, can make this approach less accessible.

Electrooculography (EOG), which measures eye movements potentials using external electrodes, serves as a more cost-effective alternative to EEG, especially for detecting REM sleep. EOG is based on detecting horizontal eye movements through the electrical potential generated near the eyes. Its output, a stream of discretized data, shows positive or negative shifts based on eye direction. EOG excels in identifying REM sleep and has seen progress in automated REM detection through algorithmic advances ([5], [6], [7]). Although visual recognition of REM is straightforward in EOG data, developing reliable algorithms for automatic recognition remains a complex task, highlighting the evolving challenges and potential of sleep research techniques.

Analyzing a signal offline differs significantly from online analysis. Offline analysis occurs after sleep has concluded, allowing for more complex and detailed examination. In contrast, online analysis is conducted in real-time, during sleep, and requires rapid signal processing to keep up with ongoing activity.

2. Off-line Sleep Analyzer

We have already developed several tools for offline analysis, as reported in [8, 9].

Two primary adjustable parameters govern the algorithm's performance and allow for customization depending on specific participant needs. The first parameter, smoothness, manages the level of smoothing applied to the raw signal to reduce random fluctuations and prevent misinterpretations of sporadic eye movements. By increasing this parameter, the algorithm removes a greater number of anomalies, enhancing the specificity of the signal but potentially sacrificing some sensitivity. The second parameter, the width of the time window, affects the accuracy of stage detection; wider windows improve the accuracy of the detected sleep stages but introduce a delay in the responsiveness of the signal interpretation. Together, these parameters enable the program to balance precision with noise reduction, adapting to diverse use cases.

The algorithm itself is composed of four main procedures: smoothing the input data, calculating the first derivative of the data, isolating individual eye movements (EMs) and measuring their characteristics, and categorizing these movements into distinct sleep stages. In the first procedure, the program reads incoming data points in sequence, performing a smoothing operation based on a set number of previous points, determined by the window width. For example, with a window width of five, the algorithm calculates the median of the five most recent points at each data intake, reducing the influence of outliers and high-amplitude noise. This

produces a smoothed data output that retains the original wave structure while omitting extraneous spikes and frequent oscillations. The resulting signal closely resembles the primary pattern of eye movements in the original data but in a more stable and refined format.

Following smoothing, the second procedure calculates the approximate first derivative of the smoothed signal to determine the slope of each eye movement waveform. For each data point in a given window, the algorithm computes the difference between that point and the mean of all values within the window, dividing this by the time interval separating the point from the central element in the window. This calculation, based on the analogy with a first derivative, results in an averaged rate of change, which indicates the direction and slope of each wave in the smoothed signal. This derivative information is integral to the next procedure, which identifies and characterizes individual EMs.

In the third step, the algorithm isolates individual eye movements, measuring each one's amplitude, duration, and slope. This process transforms the smoothed data into distinct, segmented linear sections that correspond to separate eye movements. By structuring the data into these intervals, the algorithm enables a clearer analysis of each movement, filtering out extraneous fluctuations and focusing on main eye movement patterns. This segmentation process effectively converts the data into discrete episodes, which facilitates subsequent classification into sleep stages.

The final procedure involves classifying each detected eye movement into one of four sleep stages: wakefulness, deep sleep, REM, or an undefined phase. This classification is determined by the calculated slope, duration, and amplitude of each EM segment. A parameterized tolerance is built into this step to prevent rapid switching between stages, ensuring that the program recognizes a phase only after detecting consecutive EMs indicative of that stage. For instance, if REM phase characteristics are detected repeatedly across several EMs, the algorithm concludes that REM sleep is occurring rather than oscillating between stages based on single, isolated readings. This tolerance helps maintain the reliability of stage categorization by reducing the likelihood of brief disturbances or fluctuations from interrupting an established sleep stage.

This algorithm demonstrates how data smoothing, derivative calculation, EM segmentation, and sleep stage classification can be combined into a cohesive system for automated REM detection. By continuously monitoring incoming data, this approach can detect sleep stages with minimal delay, though some latency is introduced as a trade-off for accuracy. The program's reliance on configurable parameters makes it adaptable to different sleep patterns and requirements, offering a versatile tool for real-time sleep analysis.

3. On-line Sleep Analyzer

The development of an algorithm for the automatic, real-time detection of the REM phase of sleep offers a streamlined approach to monitoring sleep stages using an incoming data stream. This Python-based algorithm interprets a continuous sequence of numerical data, transforming it into categorized time intervals corresponding to different sleep stages. Through this process, the algorithm categorizes sleep into REM, deep sleep, wakefulness, and undefined intervals based on key parameters and calculated eye movement characteristics.

The main steps of the procedure start with reading each number from a designated data file. If the absolute value of a data point exceeds defined maximum amplitude, it is ignored to eliminate extreme outliers. For each valid data point, the algorithm compares it with the previous point to detect whether the trend is increasing or decreasing. This directional information helps determine the beginning and end of peaks. Peaks are only logged if they meet certain criteria, such as a threshold for change from the previous point and a minimum length, helping isolate significant eye movement events.

As each peak is detected, it is stored with details such as its start point, length, and amplitude, all of which are useful for distinguishing between sleep phases. During ongoing data collection, the algorithm also performs statistical checks within predefined time intervals. These include mean peak length, mean peak amplitude, and a ratio of high-to-low amplitude peaks (based on a set threshold). These statistics provide insights into sleep phase characteristics, enabling better phase detection accuracy. Additionally, a custom function checks if certain criteria are met during each statistical cycle. If they are, the algorithm logs the corresponding time, possibly marking the start of a significant sleep phase.

The program provides visualization functions to illustrate detected peaks, statistics of peak lengths and amplitudes, and high-to-low amplitude ratios over time. These graphs assist in analyzing how eye movements and their characteristics fluctuate during sleep, contributing to a deeper understanding of the REM phase and other sleep stages. Overall, this online analyzer identifies peaks and calculates meaningful statistics, supporting a real-time approach to sleep analysis.

4. Examples

This section illustrates how the sleep analysis algorithm processes data and identifies different sleep stages.

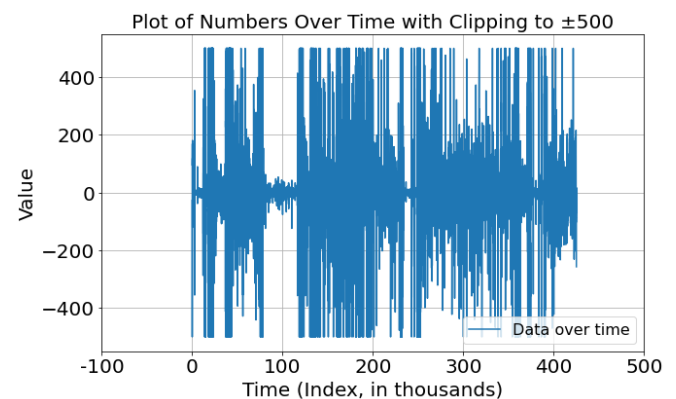


Figure 1: The original signal, limited to a range of -500 to 500. The horizontal axis represents time, measured at a rate of 50 units per second (the signal frequency). The vertical axis shows the output from the oculograph, capturing the difference in electrical potentials that cause muscle contractions responsible for horizontal eye movements in the left and right eyes.

Figure 1 shows the raw signal representing a continuous stream of data collected over time. This signal exhibits fluctuations that reflect eye movements but also includes noise and high-amplitude outliers, which can obscure meaningful patterns. To enhance the analysis, the signal is limited to a range of -500 to 500, focusing on relevant data while excluding extreme values that could interfere with the detection of sleep stages.

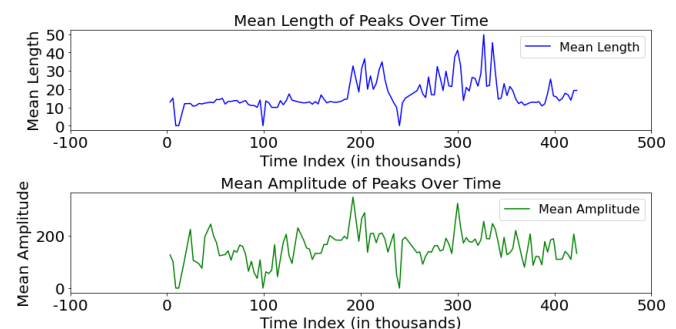


Figure 2: The processed signal from Figure 1. The top panel shows the mean length of detected peaks over a time window of 3000 units (equivalent to 1 minute). The bottom panel shows the mean amplitude of these peaks over the same time window.

In Figure 2, the signal is processed to highlight the statistical properties of the detected peaks. The mean length of the peaks provides insight into their duration, while the mean amplitude reflects their strength. These metrics are crucial for differentiating between various sleep stages, as the duration and intensity of eye movement change across wakefulness, deep sleep, and REM sleep.

The preprocessing steps are explained in detail as follows: First, the data is read from a file, where each line represents the difference in electrical potentials responsible for horizontal eye movements. Each number is converted into a float and processed sequentially. The algorithm compares each new number with the previous one to detect changes in the signal's direction. A peak is identified when the current value increases beyond a threshold or decreases below it, marking the end of a peak.

Once a peak is detected, its length is measured by the number of data points within the peak, and its amplitude is calculated as the difference between the maximum and minimum values within the peak. The peaks are then stored with their starting index, starting value, length, and amplitude for further analysis. To ensure that extreme values do not interfere with peak detection, any values exceeding set maximum amplitude are ignored.

After the peaks are identified, the algorithm calculates statistics, such as the mean peak length and mean peak amplitude, over a specified time window (about one minute). These statistics are updated each time the set time window completes, resulting in a processed signal, as shown in Figure 2, where the top panel represents the mean peak length, and the bottom panel shows the mean peak amplitude.

Once peaks are categorized, the algorithm computes statistics such as the mean length and amplitude for detected peaks within a given time window. Additionally, it classifies peaks as high-amplitude or low-amplitude based on a predefined threshold and records the counts for each category. A key statistic is the high-to-low amplitude ratio, which reflects the distribution of different peak types over time. This ratio is updated and stored for further analysis.

The computed statistics, including mean peak lengths, mean amplitudes, and the high-to-low amplitude ratio, are visualized over time. Figure 3 shows the high-to-low amplitude ratio, providing insights into how the signal's characteristics evolve throughout the analyzed period.

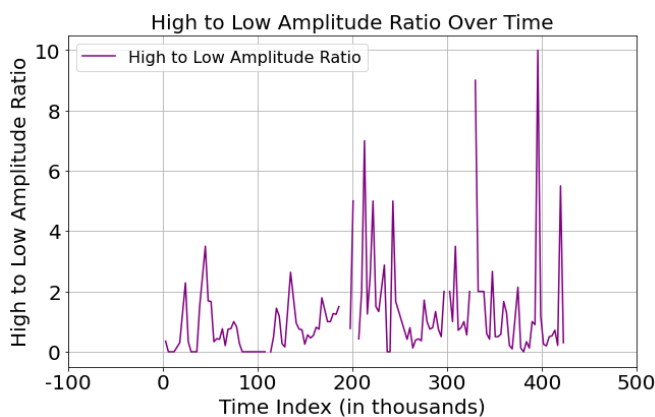


Figure 3: High-to-Low amplitude ratio over time. Higher values indicate periods dominated by peaks with high amplitudes, while lower values correspond to periods dominated by low-amplitude peaks.

Figure 4 and Figure 5 illustrate two distinct periods of sleep. Figure 4 shows a 15-minute interval with only a few detected peaks, corresponding to a period of calm sleep, likely without rapid eye movements. In contrast, Figure 5 displays a 15-minute interval with a significantly higher number of detected peaks, indicative of a highly probable REM sleep phase, characterized by frequent rapid eye movements.

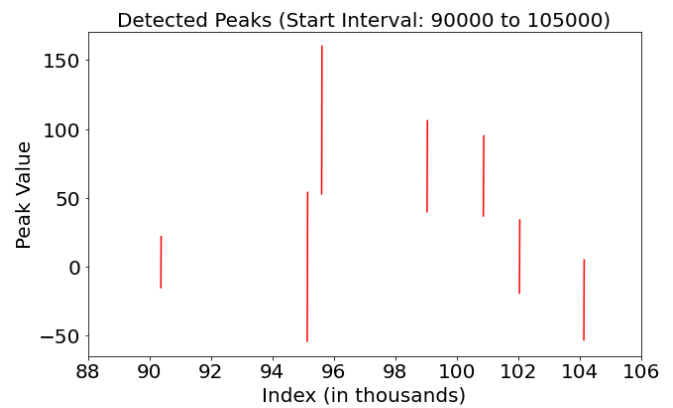


Figure 4: A period of calm sleep, with few peaks detected, characterized by relatively low amplitudes.

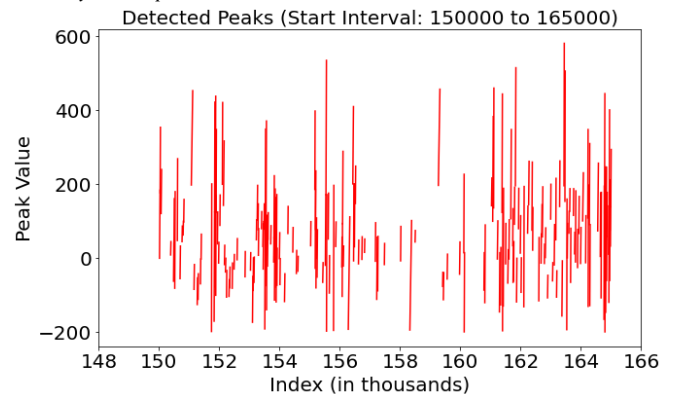


Figure 5: A period of probable REM sleep, marked by intense rapid eye movements and frequent peak detections.

In conclusion, the online sleep analyzer efficiently processes an incoming stream of data in real-time. It detects peaks in the signal, measuring their length and amplitude, and calculates the high-to-low amplitude ratio. By analyzing these parameters, the system can distinguish between periods of calm sleep, where peak activity is infrequent and of lower amplitude, and periods likely corresponding to rapid eye movements (REM) sleep, which are characterized by frequent and intense peaks. This automated approach allows for continuous, real-time monitoring and classification of sleep stages.

5. Conclusions

This paper presents a comprehensive method for analyzing sleep stages using a mathematical tool that specifically detects Rapid Eye Movement (REM) sleep from oculographic data. By examining the length and amplitude of eye movement peaks detected in the oculograph signal, we introduce a systematic approach to differentiate between calm sleep and probable REM sleep phases. This tool processes data efficiently, offering a robust and accurate method for sleep stage classification.

Sleep analysis has long been an area of significant interest for researchers and clinicians alike, as understanding sleep stages—particularly REM and non-REM sleep—provides valuable insights into human health and well-being. Accurate identification of these stages is crucial for understanding sleep quality and health. Offline techniques for analyzing oculograph data have been widely used to study eye movement patterns and classify sleep stages. These methods typically involve post-hoc analysis of recorded data, allowing researchers to classify sleep based on signal characteristics such as peak amplitude, length, and frequency. However, these systems are inherently limited by their reliance on pre-recorded data and lack of real-time processing, which is problematic for continuous monitoring and immediate intervention, especially in clinical settings.

To address these limitations, we introduced a novel online sleep analyzer that processes incoming data in real time, offering substantial advantages over offline systems. By continuously analyzing oculograph data, the system detects and characterizes peaks corresponding to eye movements, measuring their length, amplitude, and the high-to-low amplitude ratio. These parameters provide crucial information about sleep stage transitions, distinguishing between quiet sleep and periods of REM sleep, where the frequency and intensity of eye movements increase. This tool provides continuous feedback on the sleep state, updating classifications dynamically as the system receives data.

A key contribution of this work is the development of an algorithm that processes oculographic signals in real-time, detecting peaks and computing statistical properties such as peak length, amplitude, and the high-to-low amplitude ratio. The ability to calculate and monitor the high-to-low amplitude ratio offers a new dimension in detecting dynamic transitions between sleep stages, providing deeper insights into sleep architecture. REM sleep detection from oculograph data has historically been challenging due to the noisy and dynamic nature of the signals, which often include outliers and artifacts. The mathematical tool presented here addresses these challenges by filtering out extreme values, enabling a more accurate detection of meaningful peaks that correspond to eye movements during REM sleep. By focusing on peak characteristics such as length and amplitude, the tool can track changes in the frequency and intensity of eye movements, which are critical indicators of REM sleep.

The high-to-low amplitude ratio, computed over a defined time window, is a particularly useful statistic in distinguishing between periods of calm sleep and REM sleep. Periods with higher ratios indicate dominance by high-amplitude eye movements, typical of REM sleep, while lower ratios are indicative of calm, non-REM sleep, which is characterized by fewer and lower amplitude peaks. This approach allows for a quantitative and dynamic classification of sleep stages, offering a significant advantage over qualitative or visually assessed methods. The real-time capability of the system provides immediate feedback, making it suitable for both clinical diagnostics and research purposes.

In clinical settings, the tool offers the potential for continuous monitoring of sleep stages, enabling healthcare providers to detect REM sleep and other phases in real-time. This ability to track the high-to-low amplitude ratio and peak characteristics offers immediate insights into the patient's sleep state, allowing for timely intervention when needed. In research settings, the ability to analyze REM sleep in real-time opens up new avenues for studying how eye movements correlate with various physiological, psychological, and environmental factors. The real-time data stream provides researchers with dynamic insights into how sleep evolves over the course of the night, potentially revealing novel links between sleep stages and health outcomes.

Figures presented in this paper demonstrated how the system processes the raw signal data, calculating mean peak lengths, amplitudes, and high-to-low amplitude ratios over time windows. These metrics are crucial for identifying sleep stages, with changes in peak characteristics correlating with transitions between different sleep phases. For example, periods of calm sleep, with relatively few detected peaks, contrast with periods of REM sleep, where rapid eye movements lead to frequent and intense peaks. This ability to differentiate between these two stages in real-time offers significant potential for clinical diagnostics and sleep research.

Moreover, the real-time nature of the online analyzer opens up new opportunities for continuous sleep monitoring. In clinical settings, this could provide real-time data on sleep disorders, enabling more immediate interventions. In research settings, it facilitates the collection of continuous sleep data, allowing researchers to study sleep patterns in a more dynamic and immediate context.

In conclusion, this paper presents a powerful mathematical tool for detecting REM sleep from oculograph data. By focusing on the statistical properties of eye movement peaks, the tool provides an efficient and accurate means of analyzing sleep stages, distinguishing calm sleep from REM sleep with high precision. This tool can be applied in both clinical and research settings, enabling real-time sleep monitoring and offering valuable insights into sleep architecture and its relationship to health. As the tool evolves and integrates with other technologies, it has the potential to further enhance our understanding of sleep and improve the diagnosis and treatment of sleep disorders.

6. References

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