Finite Elements Modelling for Calculation of Magnetic Fields in Magnetoencephalography

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Abstract: *The present study concerns the simulation, in two dimensions, by the finite element method, of the behaviour of the magnetic field created by sources of equivalent neuronal currents by considering a simplified model of a diagnostic device used in MEG (Magnetoencephalography). The simplified geometric model is composed of the head, the magnetic field sensors and the sources of equivalent currents representing the neurons. The geometric model of the head is represented by 3 layers which are brain, bone and skin. The sensors placed above the part of the skin make it possible to calculate the electric voltage induced by the magnetic field resulting from the sources of equivalent currents produced by the neurons. Neurons are represented by equidistant point current sourcesThe Maxwell-multi physics software has been used.*

***Keywords**: FINITE ELEMENT METHOD, MAGNETOENCPHALOGRAPHY, INDUCED VOLTAGE, SQUID SENSOR, HEAD AND SOURCE MODEL.*

1. Introduction

The human brain is the most complex organized structure known to exist, and, for us, it is also the most important. There are at least 10' neurons in the outermost layer of the brain, the cerebral cortex. These cells are the active units in a vast signal handling network, which includes 10' interconnections or synapses. %'hen information is being processed, small currents fIkow in the neural system and produce a weak magnetic fie1d which can be measured noninvasively by a SQUID magnetometer, placed outside the skull, provided that thousands of nearby neurons act in concert. This method of recording, called magnetoencephalography (MEG), has already produced several pieces of new information about the functioning of the human brain. In many MEG measurements, an area in the brain that has been activated by a stimulus, such as a sound or a picture, can be located by analyzing the detected magnetic fields [1].

MEG also aids in diagnosing and delineating multiple novel findings in other neuropsychiatric disorders, including Alzheimer's disease, Parkinsonism, Traumatic brain injury, autistic disorders. If the primary source and the surrounding conductivity distribution are known, the resulting magnetic field can be calculated from Maxwell's equations. [2-3]

In the current study, we focus on the simulation 2D of the behavior of the magnetic field generated by equivalent neuronal current sources using the finite element method. It considers a simplified model of a diagnostic device used in MEG (Magnetoencephalography). The simplified geometric model comprises the "head," magnetic field sensors, and equivalent current sources representing neurons. The "head" model is represented by three layers: the brain, bone, and skin. Sensors placed above the skin layer allow for the calculation of the electrical voltage induced by the magnetic field resulting from the equivalent current sources produced by the neurons. The neurons are represented by equidistant point current sources. The study investigates the effect of the sensor position relative to the outer layer (skin) on the intensity of the calculated signal. The variation of the electrical voltage at the sensors as a function of the distance between the outer layer and the sensors is thus provided. The electrical voltage and magnetic field distribution were calculated for different situations related to the activation of a subset of neurons. The results obtained are considered consistent with those already existing in the scientific literature.

2. Geometric model of the head

Most models used today consider the head to be composed of a series of interdependent layers, each with its own physical properties, and then the model is characterized by the geometry of the interface between two continuous media [4].

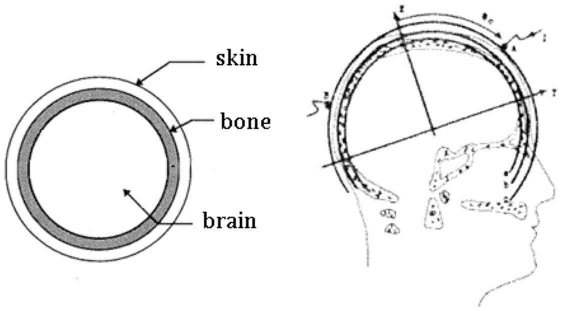

Fig. 1 *Model of concentric spheres.*

This model is only a very rough approximation of the head's geometry. The value of the normalized radii that we considered for our spherical model are given in Table 1.

Table 1: *Normalized radii values for a three-layer model.*

Medium	Scalp	Bone	Cortex
Normalized radii	1	0,92	0,87

3. Magnetostatic field computation

The modeling of the electromagnetic problems is based on the Maxwell's equations. In case of magnetostatic hypothesis we have Ampere's law, and the law of conservation of magnetic flux [5]:

$$\vec{\nabla} \wedge \vec{H} = \vec{J} \qquad (1)$$

$$\vec{\nabla} . \vec{B} = 0 \qquad (2)$$

$\vec{H}$, $\vec{B}$ and $\vec{J}$ are the magnetic field [A/m], magnetic flux density [T] and current density [A/m^2] vectors respectively.

The relation between magnetic flux density with the magnetic field whish represent the medium law is given follow:

$$\vec{B} = \mu \vec{H} \qquad (3)$$

μ is the magnetic permeability [H/m].

The solution of field problems that include discrete current carrying regions demands the use of the magnetic vector potential $\vec{A}$ concept.

From (2); we deduce: $\vec{B} = \vec{\nabla} \wedge \vec{A}$ (4)

4. Finite elements formulation

The behavior of the magnetic field produced by the sources of equivalent currents relative to the neurons is translated by partial differential equations, which are obtained from Maxwell's equations (1) and (2) associated with the relations of the medium given by equation (3). In this case, the unknown variable to be considered is the magnetic vector potential $\vec{A}$.

The magnetostatic equation developed in cartesian coordinates is given by the following expression:

$$-\left(\frac{\partial}{\partial x}\left(\upsilon\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\upsilon\frac{\partial A_z}{\partial y}\right)\right)=J_{sz} \qquad (5)$$

Where $\upsilon=\frac{1}{\mu}$ is the reluctivity.

The integral formula of equation (5) is given bellow:

$$\iint_\Omega \alpha_i\left(\left(\frac{\partial}{\partial x}\left(\upsilon\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\upsilon\frac{\partial A_z}{\partial y}\right)\right)+J_z\right)dx\,dy=0 \qquad (6)$$

Equation (6) can be rewritten in the following form:

$$\iint_\Omega \alpha_i\left(-\frac{\partial}{\partial x}\left(\upsilon\frac{\partial A_z}{\partial x}\right)-\frac{\partial}{\partial y}\left(\upsilon\frac{\partial A_z}{\partial y}\right)\right)dx\,dy=\iint_\Omega \alpha_i J_z\,dx\,dy \quad (7)$$

The application of Green theorem in the first term of equation (7) allows us to written:

$$\iint_\Omega \upsilon\left(\frac{\partial\alpha_i}{\partial x}\frac{\partial A_z}{\partial x}+\frac{\partial\alpha_i}{\partial y}\frac{\partial A_z}{\partial y}\right)dxdy - \int_\Gamma \upsilon\frac{\partial A_z}{\partial n}\alpha_i d\Gamma=\iint_\Omega \alpha_i J_z\,dxdy (8)$$

By considering homogeneous boundary conditions, the previous relation is reduced to:

$$\iint_\Omega \upsilon\left(\frac{\partial\alpha_i}{\partial x}\frac{\partial A_z}{\partial x}+\frac{\partial\alpha_i}{\partial y}\frac{\partial A_z}{\partial y}\right)dxdy=\iint_\Omega \alpha_i J_z\,dxdy \qquad (9)$$

$$A_z=\sum_{j=1}^{n}\alpha_j A_j \qquad (10)$$

$A_Z(x,y)$ is the unknown.
α_i is the projection function at node i.
α_j is the shape function at node j.
A_j is magnetic vector potential unknown at node j.

after introducing the magnetic vector potential discretized formula in equation (1) and with some arrangements we obtain the complex magnetostatic algebraic system to solve:

$$[M][A]=[K] \qquad (11)$$

Where [M] is stiffness matrix, [K] is source vector and $[A]$ is vector of magnetic vector potential unknowns

$$M_{ij}=\iint_\Omega \upsilon\left(\frac{\partial\alpha_i}{\partial x}\frac{\partial A_z}{\partial x}+\frac{\partial\alpha_i}{\partial y}\frac{\partial A_z}{\partial y}\right)dxdy \qquad (12)$$

$$K_i=\iint_\Omega \alpha_i J_z\,dxdy \qquad (13)$$

5. Induced voltage computation

The induced voltage is evaluated using Lenz's and Faraday's law, given by:

$$e=-\frac{d\vec{\Phi}}{dt} \qquad (14)$$

$$\phi=B\,ds \qquad (15)$$

By combining equation (14), (15) and (4), we obtain the relation which allows us to calculate the induced voltage given by:

$$V=\frac{1}{\Delta t}\int\vec{A}\,\vec{dl} \qquad (16)$$

6. Application and results

The configuration shown in figure (2) consists of three layers representing the skin or scalp, the skull bone, and a medium for all brain tissue. The sensors are distributed over the external surface of the head.

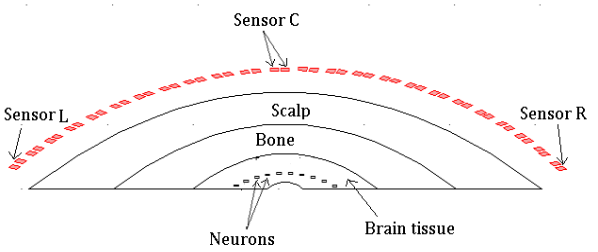

Fig. 2 *Geometry of the studied problem.*

The physical and geometrical proprieties for each layer are listed in Table 1.

Table 2: Geometrical and *physical parameters.*

Medium	Scalp	Bone	Cortex	Sensor	Neurons
Electrical conductivity [S/m]	0,33	0,00492	0,33	$5,6*10^8$	0
Width [mm]	-	-	-	0,01	0,005
Thickness [mm]	0,1	0,092	0,087	0,01	0,005
Current density [A/m^2]	-	-	-	0	0,001

For this application, the equivalent current sources are considered constant (J=0,001 A/m^2), and the magnetic flux density distribution is shown in Fig. 3.

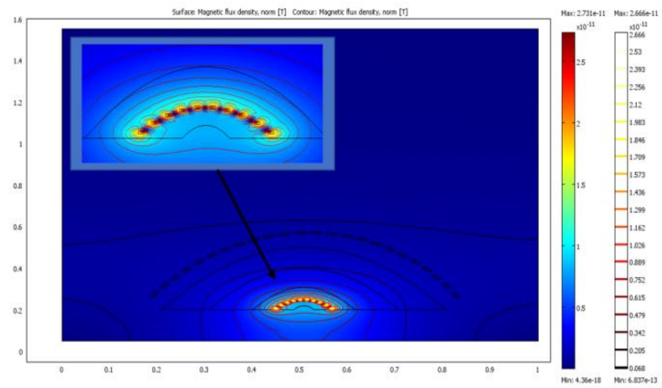

Fig. 3 *Magnetic flux density distribution.*

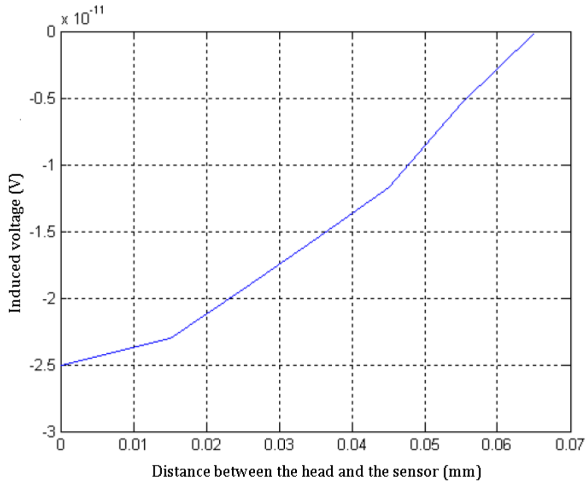

Fig. 4 *Induced voltage variation as a function of distance.*

The study of the effect of the position of the sensors relative to the external layer which is the skin on the intensity of the calculated signal has been highlighted, we observe a decrease in the intensity of the magnetic field (see Fig 4). The result obtained is considered to be consistent with those already existing in the scientific literature.

The magnetic flux density distribution for several values of the distance separating the sensors from the head is shown in Fig. 5.

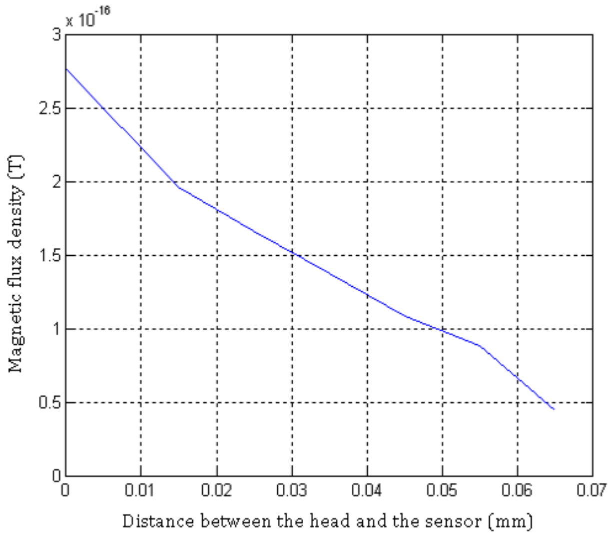

Fig. 5 *Magnetic flux density variation as a function of distance.*

7. Conclusion

In this work, a simplified model representing a diagnostic device used in magnetoencyphalography (MEG) was analyzed using a numerical method (finite element method), and the magnetic behavior of the system was highlighted. The effect of the distance between the sensors and the outer layer of the "head" was presented, as well as the distribution of the magnetic quantity in the [x,y] plane of study. The results obtained are consistent with those reported in the scientific literature.

8. References

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