DEVELOPMENT OF METHODS FOR CALCULATING THE NONEQUILIBRIUM PHASE TRANSFORMATIONS DURING THE CONDENSATION OF SUPERCOOLED STEAM IN THE TURBINE FLOW PATH

РАЗРАБОТКА МЕТОДОВ РАСЧЕТА НЕРАВНОВЕСНЫХ ФАЗОВЫХ ПРЕВРАЩЕНИЙ ПРИ КОНДЕНСАЦИИ ПЕРЕОХЛАЖДЕННОГО ПАРА В ПРОТОЧНОЙ ЧАСТИ ТУРБИНЫ

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Abstract:
Methods for calculating the nonequilibrium phase transformations during the condensation of supercooled steam in the turbine flow path are developed. In the process of realization, the development of the classical Zel'dovich-Frenckel's theory for the case of nonstationary nucleation of a new phase with fast extensions of supercooled steam is considered. A numerical-analytical method for calculating condensation was designed and implemented in the form of a software package, which consistently takes into account the nonstationarity of the process. Numerical studies have shown high efficiency and accuracy of the method.
KEYWORDS: PHASE TRANSFORMATIONS, CONDENSATION, SUPERCOOLED STEAM, MOISTURE, TURBINE STAGE, MATHEMATICAL MODEL.

1. Introduction

Every year, the consumption of electricity increases, which in turn is compensated mainly by powerful thermal and nuclear power plants. One of the main problems arising in the operation of powerful power units is the work of the last stages of the low-pressure part in the wet steam area. Improvement of working processes in the two-phase flow, in which the formation, transformation and transfer of moisture has a significant negative effect on the characteristics of the stages, is one of the possibilities for further improving the economy and reliability of turbomachines [1, 2].

At the present time, at the modern period of development of turbine construction, the problems of the theory of the nucleation of moisture are analyzed and solved with taking into account the three-dimensional flow of steam in the turbine stages, the nonstationarity of the nucleation of a new phase and the energy exchange with the growth of condensed particles [3 – 5]. To solve the problems posed, it is necessary to obtain a system of equations that take into account nonstationarity, nonisothermal effects, and those that possess universality.

In this paper, we consider the problems of nonequilibrium homogeneous condensation in the expansion of supercooled steam in the turbine flow path. These studies are very relevant, since the nucleation of drops largely determines the nature of the further course of the working fluid. Its detailed investigation and description is necessary for the creation of appropriate gas-dynamic calculation schemes and for studying complex physical phenomena associated with the emergence of "condensation turbulence", the intensification of nonstationary processes, as well as the further transformation of water-droplets and their influence on working processes in the turbine flow path.

2. Fundamentals of the classical theory of the new phase nucleation

The new phase nucleation theory is closely related to the basic concepts of statistical physics and uses the fundamental results of Gibbs and Einstein. In the modern form it was developed by Zel'dovich [6], who received

(1) $$\frac{\partial f}{\partial t} + \frac{\partial j}{\partial g} = 0, \quad j = DN\frac{\partial}{\partial g}\frac{f}{N}.$$

Expression (1) has the form of the Fokker-Planck's equation and represents the basic equation of the new phase nucleation kinetics, where g is the number of molecules in the nucleating droplet ("size"); $j(g,t)$ is the flow of nuclei in the size space; $D(g)$ is the diffusion coefficient; $f(g,t)$ and $N(g,t)$ are the kinetic and equilibrium distribution functions, respectively.

With time-independent external conditions, the system establishes a stationary distribution of droplets in size

$$f_{\mathrm{ST}} = \frac{1}{2} N \operatorname{erfc}\left(\frac{g-g_*}{\Delta}\right),$$

where $\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^{\infty} \mathrm{e}^{-x^2} dx$ is additional probability integral; $\Delta = \left\{-\frac{1}{2K_{\mathrm{B}}T}\cdot\frac{\partial^2 W}{\partial g^2}\Big|_*\right\}^{-1/2}$ is critical region width; K_{B} is the Boltzmann's constant.

The nucleation rate I_{ST}, that is, the number of stable nuclei formed in the system per unit time, is defined as the size-independent stationary flow

(2) $$j_{\mathrm{ST}} = I_{\mathrm{ST}} = \frac{D_* N_*}{\Delta\sqrt{\pi}} = \frac{1}{2\sqrt{\pi}}\cdot\frac{\partial \dot{g}}{\partial g}\Big|_* N_* \left\{-\frac{1}{2K_{\mathrm{B}}T}\cdot\frac{\partial^2 W}{\partial g^2}\Big|_*\right\}^{-1/2},$$

where $\dot{g}$ is the macroscopic rate of change in the size of the nucleus, which can be determined through the diffusion coefficient D in accordance with the well-known Einstein's relation

$$-\frac{D}{K_{\mathrm{B}}T}\frac{\partial W}{\partial g} = \dot{g}.$$

Concretizing the input quantities in the equation (2) as applied to the condensation of supercooled steam, Ya. I. Frenckel obtained an expression for the stationary nucleation rate (the Zel'dovich-Frenckel's formula) [7]

(3) $$I_{\mathrm{ST}} = \alpha_c W \left(\frac{2}{\pi}\right)^{1/2} \frac{p}{K_{\mathrm{B}}T}\cdot\frac{1}{\rho'}(\sigma m_{\mathrm{M}})^{1/2} N_1 \exp\left(-\frac{4\pi\sigma}{3K_{\mathrm{B}}T} r_*^2\right),$$

where p is the steam pressure; ρ' is the liquid density; σ is the surface tension coefficient; m_{M} is the molecule mass; r_* is the radius of the critical nucleus; α_c is the condensation coefficient.

An analysis of the above theory shows that the assumption of stationarity of the nucleation process is violated in many practical cases, in particular, with the expansion of supercooled steam in the turbine flow path.

In this connection, it is advisable to consider the refinement of the kinetic aspects of nucleation associated with the situation noted above, when the steady-state approximation for the solution of equation (1) is not applicable. It should also be noted the need for detailed consideration of heat- and mass-transfer between a separate nucleus and steam.

3. Development of the classical theory as applied to working processes in the stages of wet-steam turbines

Let us further consider the development of the classical theory for the case of a nonstationary nucleation of a new phase with fast extensions of supercooled steam. The process of the nonstationary nucleation of a new phase is described by the Zel'dovich's equation (1). The nonstationarity of the problem is related to the time dependence of the external conditions and, consequently, the coefficients of equation (1).

To characterize the nonstationarity level, we introduce a certain quantity connecting the relaxation time to the stationary distribution τ_{rel} and the change rate of the nucleation barrier $\frac{\partial}{\partial t}(\frac{W_*}{K_\text{B}T})$, it is the nonstationarity coefficient *n*

(4) $$n = -\tau_{\text{rel}} \frac{\partial}{\partial t}(\frac{W_*}{K_\text{B}T}).$$

To describe the initial period of condensation during the expansion of steam in the turbine flow path, it is necessary to solve the problem of determining the nucleation rate I under nonstationary external conditions, taking into account the release of heat in the growth of nuclei. The correctness of the obtained result is confirmed by the transition of the solution to the stationary solution if $n \to 0$. The solution of this problem for both isothermal (one-parameter) and nonisothermal nucleation was considered in [8, 9].

For us, the so-called "effect of the presence" of a new phase, which occurs only after a significant increase in the size of the nuclei formed, is of special interest. For such nuclei, the macroscopic growth equations and the change in the total number of molecules that have entered the new phase N' are valid, which are described as follows:

(5) $$\frac{dN'}{dt} = \int_0^\infty I(t-t')g(t')\,dt',$$

where $I(t-t')$ is the nucleation rate, which in the general case should be determined from the Zel'dovich's equation (1); $g(t')$ is the number of molecules in the nucleating droplet that was formed at the time $(t-t')$ and is observed at time *t*.

The methods available in the literature for an approximate analytical study of equations of the type (5), together with the drop growth equation, in spite of their main advantage, which consists in obtaining closed analytic expressions for the total number of nuclei formed, have not found universal application. This is due to the use of a number of serious assumptions in their implementation, as mentioned above.

An alternative to the analytic description is the rejection of all sentences, except for the equations (1), (5) used in the derivation of the equations, and the subsequent numerical solution of these equations by a computer. This procedure was implemented by us to obtain an "exact" solution, which is then used to control the results obtained in the work and compare it with the results of other authors. However, in connection with the cumbersome numerical solution of the partial differential equation (1) in conjunction with the integral relation (5), the application of the method in problems characteristic of turbomachines, where condensation is only a part of very complex gas-dynamic processes (including multidimensional flow, its nonstationarity, turbulence and etc.) or in problems of synthesis of the optimal flow path is very difficult. In other words, in real calculation schemes, an increase in the "dimensionality" of the problem by adding an additional coordinate r_d is highly undesirable. The way out of this situation is to look at the evolution of several initial moments instead of the evolution of the distribution function

$$\Omega_\nu = \int_{r_*}^\infty r_d^\nu \cdot f(r_d)dr_d,\ \nu \geq 1.$$

Such an "intermediate", between analytical and numerical, the method has become quite widespread. In this method, while preserving the first two assumptions of the analytical approaches on the possibility of using the stationary expression for the nucleation rate and on the independence of the drop growth rate on their size, the integral equation (5) reduces to a system of differential equations for the moments of the distribution function

(6) $$\dot{\Omega}_\nu = \nu \dot{r}_d \Omega_{\nu-1},\ \nu = 1,2,3; \qquad \dot{\Omega} = I_{\text{ST}}.$$

The system (6) is actively used to study condensation in a variety of situations [8].

At the same time, the rejection of the assumption of stationarity of the nucleation process requires the refusal of the direct use of the system (6).

Thus, the problem of creating a sufficiently accurate and simple method for calculating the condensation process, taking into account the peculiarities of the nucleation of a new phase in the case of fast steam expansion in the turbine flow path and convenient for inclusion in gas-dynamic calculation schemes and in the synthesis of optimal constructions for wet-steam turbines, remains very relevant.

We note here that in carrying out numerical studies it is convenient to construct a general system of equations that makes it possible to perform calculations uniformly for arbitrary values of the coefficient λ, which characterizes the heat exchange between the steam and the nucleus. For the stage of moisture nucleation, the system of equations in the final form is represented as follows:

(7) $$\dot{\Omega}_3 = 3\dot{r}_{d^\infty}\Omega_2,\ \dot{\Omega}_2 = 2\dot{r}_{d^\infty}\Omega_1 + J\delta^2,$$
$$\dot{\Omega}_1 = \dot{r}_{d^\infty}\Omega_0 - J\delta, \qquad \dot{\Omega}_0 = J,$$

where

$$J = I_{\text{ST}}\frac{\mu}{\ln S}\Gamma(n+1)\cdot(6\frac{W_*}{K_\text{B}T}\cdot e^{-\frac{25}{12}})^{-n};$$
$$\delta = \left\{\frac{\mu}{\exp(\mu-1)}\cdot\ln\left[\frac{2}{n'}\cdot\frac{1-\exp(-\mu)}{\mu}\right]-1\right\}r_*;$$
$$\dot{r}_{d^\infty} = \alpha_c\beta V'_\text{M}[1-\exp(-\mu)],\ \beta = \frac{p}{\sqrt{2\pi m_\text{M}K_\text{B}T}},$$
$$n = \left(\frac{4\pi}{3}\right)r_*^4\left(\frac{q}{K_\text{B}T}\right)\cdot\left(\frac{1}{V'_\text{M}}\right)^2(\alpha_c\beta\mu)^{-1}\chi;$$
$$\mu = \frac{\lambda\ln S}{\lambda + \frac{K_\text{B}}{C_\text{M}}(\frac{q}{K_\text{B}T})^2};$$

$\dot{r}_{d^\infty}$ is the growth rate of a "large" droplet ($r_d \gg r_*$); β is the collision frequency with a unit surface; Γ is the gamma function; S is the degree of supersaturation; V'_M is the molecular volume; χ is the cooling rate; q and C'_M are the heat of a phase transformation and the heat capacity of a liquid per molecule; n' is the nonstationarity parameter, determined by the adiabatic expansion.

For the phase of recondensation, the analogous system of equations has the form

(8) $$\dot{\Omega}_\nu = \nu\,\alpha_c\beta V'_\text{M}\Omega_{\nu-1}\left\{1-\exp[(\mu\cdot\frac{r_*\Omega_{\nu-2}}{\Omega_{\nu-1}}-1)]\right\},\ \nu = 2,3,$$
$$\dot{\Omega}_0 = -\frac{3}{4}\,\alpha_c\beta V'_\text{M}\mu\Omega_0 r_* R^{-2},$$
$$\dot{\Omega}_1 = (\frac{8}{9}\Omega_0\Omega_2)^{1/2},\ R = (\frac{9}{8}\cdot\frac{\Omega_2}{\Omega_0})^{1/2}.$$

4. Numerical studies

In the comparative analysis, the simplest case of isothermal nucleation under expansion with constant velocity Q is considered

$$V = V_0(1 + Qt),$$

so that the cooling rate equation has the form

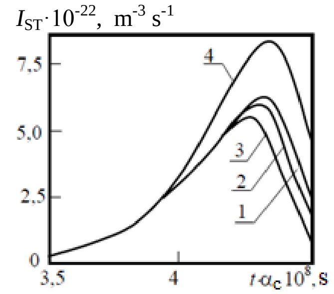
$$\chi = (k-1)\cdot Q\frac{V_0}{V} - \frac{q}{t}\cdot\frac{1}{N_1 C_v + N' C_M'}\cdot\frac{dN'}{dt}.$$

Based on the two models presented in [9], a stationary description was made. In the first case, the system of equations for the moments of the distribution function is considered, and in the second case the initial size of the emerging nuclei is assumed to be equal to the critical one, and the growth is given by the following equation

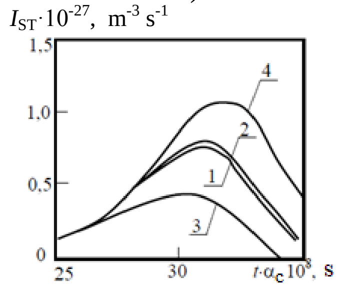
$$\dot{r_d} = \alpha_c \beta V_M' \left\{1 - \exp[\ln S(\frac{r_*}{r_d} - 1)]\right\}.$$

In the integral equation, the nucleation rate I is replaced by the stationary value I_{ST}, this method was called the "stationary integral", in contrast to the "nonstationary integral" corresponding to the numerical experiment [9].

In Fig. 1 shows the results of the description of the initial period of the condensation process in the form of the time dependence of the quantity I_{ST} calculated in accordance with the process parameters obtained in calculations by various methods (the use of I_{ST} is related only to the clarity of comparison of various methods for describing the condensation process).



a)



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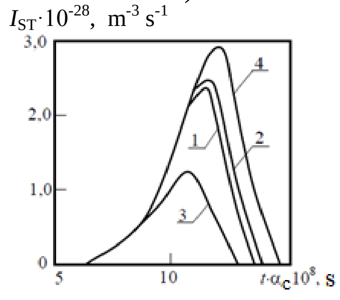

c)

Fig. 1. Dependence of I_{ST} on time, determined on the basis of various methods:
a) $Q/\alpha_c = 1\cdot 10^5,\ s^{-1}$; b) $Q/\alpha_c = 2{,}5\cdot 10^6,\ s^{-1}$;
c) $Q/\alpha_c = 1\cdot 10^7,\ s^{-1}$.

In Fig. 1, curve 1 corresponds to a direct numerical solution of the system of equations, curve 2 corresponds to the proposed method, curve 3 corresponds to a stationary calculation using the method of moment equations and curve 4 corresponds to a "stationary integral" method.

Apparently, the presented method has the best agreement with the results of a numerical experiment.

In Fig. 2 shows the results of a description of the condensation process under expansion with a time-periodic velocity. The expansion rate is given by expression

$$Q = Q'(1 + A\sin\{\omega(t - t_0)\}),$$

where ω is the angular velocity of rotation.

In Fig. 2, "solid lines" correspond to the numerical experiment; "dots" correspond to the calculation by the proposed method.

Apparently, the proposed method well "follows" the changes in the flow parameters, which determines its prospects when using the calculations for the condensation process in conditions of unsteady flows.

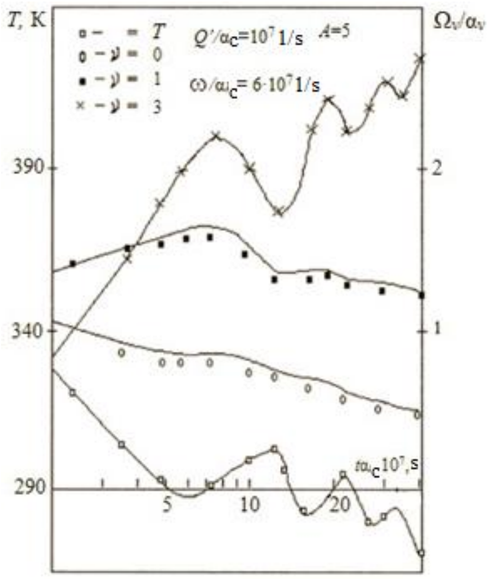

Fig. 2. Dependence of temperature and nondimensionless moments on time for the phase of recondensation at a periodic rate of expansion
$Q = Q'(1 + A\sin\{\omega(t - t_0)\})$

5. Conclusion

1. An analytical solution is obtained for the nucleation rate of the nucleating droplets, which is valid practically at an arbitrary level of nonstationarity; the well-known Zel'dovich-Frenckel's expression follows from it for weak nonstationarity.

2. The developed numerical-analytical method for describing the condensation process, based on a consistent consideration of the nonstationarity of nucleation is practically accurate (within the framework of the basic assumptions of the classical theory of nucleation) both in the period of nucleation and in the period of recondensation.

3. The simplicity of the method makes it possible to significantly accelerate calculations in comparison with numerical experiment (the time of the account is greatly reduced). This circumstance determines broad possibilities for using it in problems of synthesis of the optimal flow path of a wet-steam turbine or in direct research problems, where

condensation is only a part of complex gas-dynamic processes.

6. Literature

1. Danmei, Xie. Numerical Simulation of Water Droplets Deposition on the Last-Stage Stationary Blade of Steam Turbine. – Energy and Power Eng, № 2, 2010, P. 248–253. (Danmei Xie, Yu Xinggang, Li Wangfan and etc.).

2. Ismailov, B. Mathematical Modeling, Dynamic and Mass-Transfer Calculation of Gas-Drop Mixture in the Mass-Transfer Apparatus Multistage Channels. – Appl. Math. Sci. Vol. 18, № 92, 2014, P. 4561–4570. (Ismailov B., K. Ismailov, A. Urmatova, T. Koyshieva).

3. Garmathy, G. Grundlagen einer Theorie der Naßdampfturbine. Zürich, 1962, 196 s. (Garmathy. G).

4. Кириллов, И. И. Основы теории влажнопаровых турбин. Л.: Машиностроение, 1968, 264 с. (Кириллов И. И., Р. М. Яблоник).

5. Дейч, М. Е. Газодинамика двухфазных сред. М.: Энергия, 1968, 424 с. (Дейч М. Е., Г. А. Филиппов).

6. Зельдович, Я. Б. К теории образования новой фазы. Кавитация. Журн. эксперимент. и теорет. физики, Т. 12, № 11–12, 1942, С. 525–534. (Зельдович Я. Б.).

7. Френкель, Я. И. Кинетическая теория жидкостей. Л.: Наука, 1975, 598 с. (Френкель Я. И.).

8. Шубенко, А. Л. Нестационарное неизотермическое зарождение жидкой среды при конденсации. Докл. АН УССР. Физ.-мат. и техн. науки, № 1, 1988, С. 69–72. (Шубенко А. Л., В. А. Шнейдман).

9. Шубенко, А. Л. Моделирование процесса зарождения новой фазы при расширении переохлажденного пара в проточной части турбомашин. Изв. АН СССР. Энергетика и транспорт, Т. 36, № 6, 1990, С. 90–97. (Шубенко А. Л.).