THE EFFECT OF PHASE TRANSFORM ON CREEP RESPONCES OF FGM ROTATING DISK

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Abstract: *In many applications, FGM rotating discs are subjected to severe operating temperature and high rotational speeds. In such conditions the incidence of creep is inevitable. To study some aspects of this phenomenon, using the Sherby's law the creep relaxation of a group of FGM rotating discs is modeled. Results show that even thought the temperature is uniform entirely, the stress variation leads to a phase transformation and so the creep mechanism is changed. In this paper the effect of this change of creep mechanism on the steady state creep behavior of FGM rotating discs made of Al-SiC is studied.*
KEYWORDS: *CREEP, FGM, ROTATING DISK, PHASE TRANSFORM*

1. Introduction:

In many types of machineries such as the turbines, pumps, compressors and jet engines, rotating discs are the basic part. Generally it happens that such instruments are subjected to severe simultaneous thermo-mechanical loads. In these cases the study of creep as a kind of mechanical behavior makes sense. While considering the creep is one aspect of this research, the other standpoint is the study of non-homogeneity effects upon the load carrying capacities of the rotating discs.

Now a day in various industries, productivity implies that many efforts must be made to improve the quality of the manufactured parts and the performance of different machineries while keeping the costs as low as possible. This in turn leads to an essential need for the optimization of different material characteristics such as the weight, thermal and mechanical durability and strength. A new technique to overcome this requirement is the administrated use of FGMs (Functionally Graded Materials). In a structural view FGMs are inhomogeneous materials usually composed of the mixture of seemingly homogenous parts or particles. These compositions can bring about different physical characteristics and functional capabilities such as the light weight, high strength, high conductivity and thermal strength as well as the resistance against the wear, corrosion or burning. So in severe thermo-mechanical loadings their FGM compositions can behave ideally.

At high temperatures the creep deformations are inevitable and consequently the creep must be considered in a careful modeling attempt. The first studies of the creep phenomenon as a mechanical property dates back to the early 1920s. The importance of the material high temperature time dependent behavior or creep relaxation has been recognized during the World War II and following the development of the jet engines [1]. Schweiker [2] has investigated the creep relaxation of high temperature pressurized thick-walled tubes which is necessary to ensure a safe operational condition of the vessels. Owing to the extensive utilization of the discs in rotating machineries, the study of creep in rotating discs covers a wide range of the researches and investigations. Among the first studies we can refer to the work of Wahl et al. [3] in 1954, which has theoretically analyzed the steady state creep deformations of a rotating turbine disc using the von-Mises and Tresca's yield criteria and evaluating the results by comparing with experimental data. In 1959, Ma [4] analyzed the creep deformations and stresses in a rotating gas turbine disc with variable thickness exposed to isothermal conditions and high temperatures, using the maximum shear stress theory. In 1960 and 1964, Ma [5, 6] studied the creep relaxation in the variable thickness discs operating under the conditions of radial temperature gradients. In this analysis Ma used both exponential and power law creep models to find the steady state creep relaxations. Using a time hardening creep law in 1979 Arya and Bhatnagar [7] investigated the creep of orthotropic rotating discs. They have indicated that by increasing the anisotropy of the disc material, the tangential stresses at any radius and the tangential strain rate at the inner radius are decreased. In 1986, Biakiewicz [8] offered a theoretical method to find the finite strains of a rotating disc using the damage and creep rupture front motion as described by Kachanov's theory [9]. In 1986, Bhatnagar et al. [10] studied the steady state creep behavior of the orthotropic rotating discs comprising the linear and hyperbolic types of thickness variations. By selecting a certain type of anisotropy they derived the best profile of the disc such that stresses are decreased and life expectancy is increased. In 1999, Durodola and Attia [11] obtained the creep results for several forms of the reinforcement gradation with unique overall volume fraction.

In 2002, Singh and Ray [12] studied an FGM disc with linear distribution of silicon carbide particles in the radial direction, reinforced in aluminum matrix. They concluded that the FGM disc having linearly reducing particle contents along the radial direction exhibits significantly lower strain rates when compared to a similar disc but with uniform distribution of particles and the same total amount of particle contents. In 2003, Jahed and Bidabadi [13] presented a general axisymmetric method by using the variable material properties (VMP) method to analyze the primary and secondary creep in the axisymmetric rotating discs and pressure vessels. In 2004, Singh and Ray [14] studied the steady state creep in an anisotropic composite disc considering the Bauschinger's effect, Norton's creep power law and a new yield criterion. They have also considered the disc with residual stress to study the secondary creep. In 2004, Gupta et al. [15], used the Sherby's law to examine the steady state creep behavior of a constant thickness disc, made of isotropic functionally graded material. The disc is composed of linear distributions of the silicon carbide particles in a matrix of pure aluminum. The disc is assumed to operate under a radial thermal gradient and the results are compared with the creep relaxation of a disc with uniform particle distribution. The steady state creep behavior in an isotropic rotating disc made of Al-SiC has been analyzed by Gupta et al. [16] in 2004. They have used Sherby's creep law to look into the trends of the stress and strain rate variations in a particulate composite disc. In recent years Hafeez et al. [17] investigated the creep compliance of straight run (neat) and polymer-modified asphalt binders at their high performance grade temperature using a multi-stress creep recovery test.

In all references exist in the literature the effect of creep mechanism change is ignored. In this paper we intend to study the effects of the change of creep mechanism on the creep behaviors that can occur at the disc made of Al-SiC composite. For this purpose, the steady state creep is modeled and analyzed in FGM rotating disc.

2. Creep mathematical modeling

Similar to the procedures outlined in earlier works [15 , 16], a mathematical model is developed to study the second order creep relaxation in a rotating FGM disc. Further, it is assumed that the material of the disc is isotropic and incompressible. With these assumptions we will have,

$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0 \qquad (1)$$

In which the subscripts r, θ and z indicate the radial, tangential and axial directions and dot symbol designates the time derivative. Compared to other dimensions, the disc thickness is assumed to be small. So, we can suppose that the problem is plane stress and the assumption of $\sigma_z = 0$ is prevailing.

Moreover it is assumed that, in comparison with creep deformations, the elastic or plastic deformations of the disc are negligible. The study of creep is limited to the analysis of the secondary phase or steady state creep relaxations. In keeping with these assumptions, the Sherby's law [18] has been used for the description of material behavior. That is,

$$\dot{\bar{\varepsilon}} = A_s[\frac{\bar{\sigma}-\sigma_0}{E}]^n \qquad (2)$$

Where $\dot{\bar{\varepsilon}}$, $\bar{\sigma}$, n , E and σ_0 are effective strain rate, effective stress, stress exponent, Young's modulus and threshold stress respectively. A_s is a constant defined as,

$$A_s = \frac{AD_L\lambda^3}{|\mathbf{b}|^5} \qquad (3)$$

λ stands for the sub-grain size, $|\mathbf{b}|$ is the magnitude of the Burgers vector, A is a definite constant and D_L is the lattice diffusivity. The strain rate relation in Sherby's law can be written as,

$$\dot{\bar{\varepsilon}} = [M(\bar{\sigma}-\sigma_0)]^n \qquad (4)$$

In which, M equals $\frac{A_s^{\frac{1}{n}}}{E}$ and according to the von-Mises isotropic yield criterion the effective stress is given as [15],

$$\bar{\sigma} = \frac{1}{\sqrt{2}}[\sigma_r^2 + \sigma_\theta^2 + (\sigma_r - \sigma_\theta)^2]^{1/2} \qquad (5)$$

In a particulate composite, the creep parameters M and σ_0 are related to the particle size P, particle distribution V(r) and local temperature T. Following the experimental data reported by Pandey et al. in 1992 [19] and regression technique developed by Gupta et. al. [16], the parameters of Sherby's law in Eqs. (6) and (7) are found to be,

$$\ln M = -35.38 + .2077\ln P + 4.98\ln T - 0.622\ln[V(\mathrm{r})] \qquad (6)$$

$$\sigma_0 = -0.03507\,P + 0.01057\,T + 1.00536\,V(r) - 2.11916 \qquad (7)$$

Here and similar to [16], the size of SiC particles P is assigned the value $1.7\mu m$. The prevailing temperature T is considered to be $700°K$. It is clear that creep parameters M and σ_0 are functions of the FGM disc radius r.

The structure under study is assumed to be an isotropic FGM disc made of 6061 Al-SiC constituents. In the analysis linear radial distributions of SiC particles are considered. The distribution of the volume fraction in the disc is designated as,

$$V(r) = 100 \times \left[0.4 - 0.3 \times \left(\frac{r-a}{b-a}\right)\right] \qquad (8)$$

In which, r is a typical radius of the disc. Moreover a and b are the inner and outer radii of the disc respectively. A profile of the disc and its parameters are shown in Fig. (1).

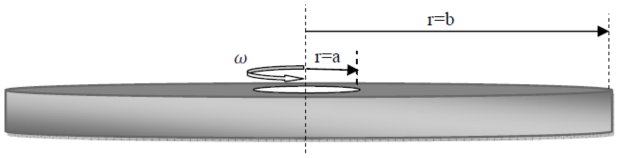

Fig. 1: *Studied disc and its parameters*

While any point of a heterogeneous FGM compound may seem like a unique substance, in fact it is a mixture of two or more phases. A mixture rule is a formula which shows how the property of a mixture is obtained from the dissimilar properties of its components. For instance, based on the level of volume fraction, a linear mixture rule for the density function, used in this study is,

$$\rho(r) = \rho_m + (\rho_d - \rho_m)\,V(r) \qquad (9)$$

In our application of Eq. (9) and similar to Gupta et.al. [16], $\rho_m = 2698.9\,kg/m^3$ is the density of the pure aluminum matrix and $\rho_d = 3210\,kg/m^3$ is the density of the dispersed SiC particles.

In an isotropic disc under the planar state of stress, the creep constitutive equations are [16]:

$$\dot{\varepsilon}_r = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}(2\sigma_r - \sigma_\theta) \qquad (10)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}(2\sigma_\theta - \sigma_r) \qquad (11)$$

$$\dot{\varepsilon}_z = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}(-\sigma_r - \sigma_\theta) \qquad (12)$$

Where $\dot{\varepsilon}_r, \dot{\varepsilon}_\theta, \dot{\varepsilon}_z, \sigma_r, \sigma_\theta, \sigma_z$ are the strain rates and stress components in radial, tangential and axial directions respectively. Besides $\dot{\bar{\varepsilon}}$ and $\bar{\sigma}$ are the effective strain rate and effective stress defined in Eqs. (4) and (5).

By replacing effective strain rate from Eq. (4) and effective stress from Eq. (5) into Eq. (10) and Eq. (11) one obtains,

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{\{M(r)[\bar{\sigma}-\sigma_0(r)]\}^8}{\sqrt{2}[\sigma_r^2+\sigma_\theta^2+(\sigma_r-\sigma_\theta)^2]^{1/2}}(2\sigma_r - \sigma_\theta) \qquad (13)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{\{M(r)[\bar{\sigma}-\sigma_0(r)]\}^8}{\sqrt{2}[\sigma_r^2+\sigma_\theta^2+(\sigma_r-\sigma_\theta)^2]^{1/2}}(2\sigma_\theta - \sigma_r) \qquad (14)$$

Defining a stress ratio as $x(r) = \frac{\sigma_r(r)}{\sigma_\theta(r)}$ and simplifying the above equations provides,

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{[2x(r)-1]\{M(r)[\bar{\sigma}-\sigma_0(r)]\}^8}{2[x(r)^2-x(r)+1]^{1/2}} \qquad (15)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{[2-x(r)]\{M(r)[\bar{\sigma}-\sigma_0(r)]\}^8}{2[x(r)^2-x(r)+1]^{1/2}} \qquad (16)$$

Dividing Eq. (15) by Eq. (16) and integrating the resulted quotient, consequences,

$$\int_a^r \frac{d\dot{u}_r}{\dot{u}_r}dr = \int_a^r \frac{1}{r}[\frac{2x(r)-1}{2-x(r)}]dr \qquad (17)$$

So,

$$\dot{u}_r = \dot{u}_{ra}\exp(\int_a^r \frac{f(r)}{r}\,dr) \qquad (18)$$

Where $\dot{u}_{ra}$ is the radial displacement rate in the inner radius of the disc and $f(r)$ is,

$$f(r) = \frac{2x(r)-1}{2-x(r)} \qquad (19)$$

Dividing both sides of Eq. (18) by r, we obtain:

$$\frac{\dot{u}_r}{r}=\frac{\dot{u}_{ra}}{r}\exp\left(\int_a^r\frac{f(r)}{r}dr\right)=\frac{[2-x(r)]\{M(r)[\bar{\sigma}-\sigma_0(r)]\}^8}{2[x(r)^2-x(r)+1]^{1/2}} \qquad (20)$$

After substituting $\bar{\sigma}$ from Eq. (5) into Eq. (20) and simplifying the answer, the tangential stress in each arbitrary radius of the disc is obtained as,

$$\sigma_\theta(r)=\frac{(\dot{u}_{ra})^{1/8}}{M(r)}\lambda_1(r)+\lambda_2(r) \qquad (21)$$

Where in this equation,

$$\lambda_1(r)=\frac{\lambda(r)}{[x(r)^2-x(r)+1]^{1/2}} \qquad (22)$$

$$\lambda_2(r)=\frac{\sigma_0(r)}{[x(r)^2-x(r)+1]^{1/2}} \qquad (23)$$

and

$$\lambda(r)=[\frac{2}{r}\frac{[x(r)^2-x(r)+1]^{1/2}}{[2-x(r)]}\exp(\int_a^r\frac{f(r)}{r}dr)]^{1/8} \qquad (24)$$

Now, as in Fig. (2) considering an element of rotating disc which rotate by an angular velocity ω, which is considered to be 180 rad/s in this analysis, the equilibrium of a disc element implies,

$$\frac{d}{dr}(r\sigma_r)-\sigma_\theta+\rho(r)r^2\omega^2=0$$
(25)

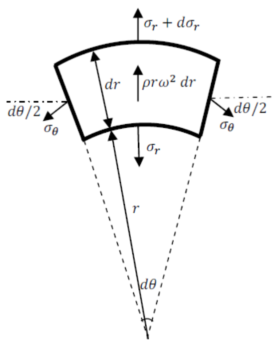

Fig. 2: *Schematic pictures of a disc and a disc element*

The boundary conditions of the rotating disc are,

$$\sigma_r(r=a)=0 \qquad (26)$$

$$\sigma_r(r=b)=0 \qquad (27)$$

Integrating the equilibrium equation between $r=a$ and $r=b$ results in,

$$\int_a^b\sigma_\theta dr=\omega^2\int_a^b\rho(r)r^2dr \qquad (28)$$

To obtain $\dot{u}_{ra}$ (i.e., the radial displacement rate at the inner radius of the disc), we can integrate Eq. (21) from $r=a$ to $r=b$. It provides,

$$(\dot{u}_{ra})^{1/8}=\frac{\int_a^b M(r)\sigma_\theta(r)dr-\int_a^b M(r)\lambda_2(r)dr}{\int_a^b\lambda_1(r)dr} \qquad (29)$$

To solve this problem, the computational procedure presented henceforward is employed. In this practice primarily in each step a uniform distribution of temperature and definite amount of angular velocity is assumed. These quantities can be selected independently. In the first step we can assume that the distribution of tangential stress is uniform. In this case we can write $\sigma_\theta(r)=(\sigma_\theta)_{ave}$, in which $(\sigma_\theta)_{ave}$ is obtained from the following equation.

$$(\sigma_\theta)_{ave}=\frac{1}{(b-a)}\int_a^b\sigma_\theta\,dr \qquad (30)$$

Then we can obtain σ_r from the equilibrium equation as:

$$\sigma_r=\frac{1}{r}\int_a^r\sigma_\theta\,dr-\frac{\omega^2}{r}\int_a^r\rho(r)r^2\,dr \qquad (31)$$

When σ_r is obtained, we can found the ratio of σ_r and σ_θ which has already been defined as $x(r)$. Now the radial distributions of $f(r)$, $\lambda(r)$, $\lambda_1(r)$, $\lambda_2(r)$, $\bar{\sigma}_r$ functions and the amount of $\dot{u}_{ra}$ can be found. Afterwards Eq. (21) is used to obtain a new estimation of σ_θ field. This loop is repeated until a desirable convergence for the tangential stress distribution in the radial direction is achieved. So we can calculate the strain creep rate and stress fields in the steady conditions of uniform ambient temperature throughout the rotating disc.

3. Results and discussions

Based on the analytical techniques introduced in the previous section and using a self-developed computer code some calculations are performed to find the radial distributions of the stress and strain rates for FGM rotating disc. But there is a challenge in this condition which ignored in the literature. In a rotating disc the stress field is varied in different sections it means despite of the constant temperature the phase of creep can be changed in various areas of rotating disc so the creep mechanism changed in different section of rotating disc.

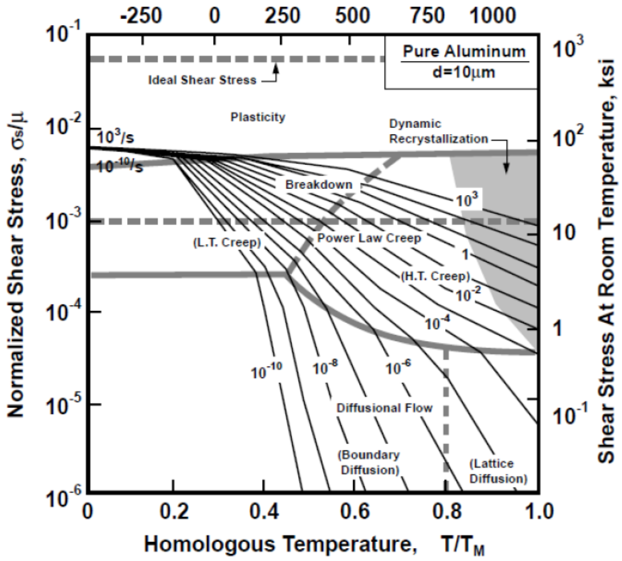

Fig. 3: *Creep mechanism map of Aluminum*

Mechanical behavior of disc is depended to stress fields, beside the stress field completely dependent on the material behavior, so an analytical and repetitive cycle is required. Referring the creep mechanism map (please see Fig. (3)) it is obvious that in 0.75 of melting temperature of aluminum, if the stress ratio σ_e/G is lesser than the 5×10^{-4} the diffusion mechanism is governed and if the stress ratio is higher than the 5×10^{-4} the governing mechanism is dislocation climb. The creep mechanism change is effective on stress power component means n in Eq. (4). For diffusion mechanism, the stress exponent is 3 and for the dislocation mechanism 5 is considered.

Fig. (4) and Fig. (5) show the radial and tangential stress distribution in disc with and without considering the creep mechanism change. Based on these figures, the effect of creep phase transform on the stresses is negligible. Also the relatively higher levels of tangential stress occur near the inner radius, while lower amounts are seen near the outer radius of the disc. The radial stress at inner and outer radius reached to zero because of the free-free boundary condition which is considered.

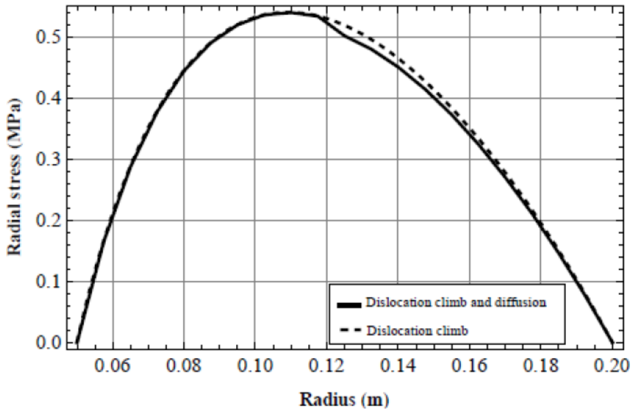

Fig. 4: *Radial stress distribution*

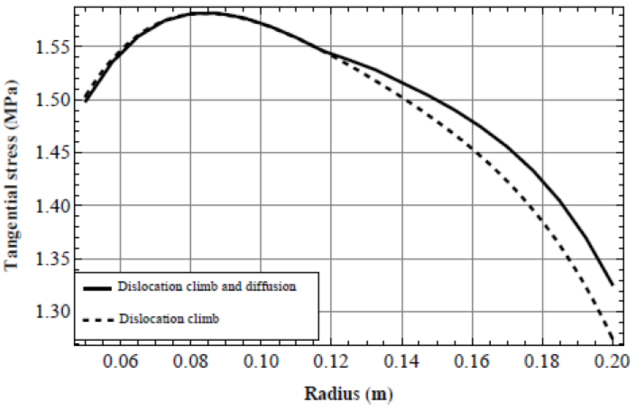

Fig. 5: Tangential stress distribution

Fig. (6) *presents the radial displacement rate distribution of FGM rotating disc with two distinct creep behavior model and without considering this changing. The figure shows that the change of creep mechanism is completely affected on radial displacement rate.*

Radial and tangential creep rates are investigated in Figs. (7) and (8). As it can be seen nearly in the middle of the disc which the creep mechanism is changed from dislocation climb to diffusion mechanism, the creep rate distributions also have tremendous changes. It is obvious that ignoring this changing mechanism lead to significant errors. Radial and tangential creep rates have a decreasing trend via radial direction and the maximum values of both rates occurs at inner radiuses.

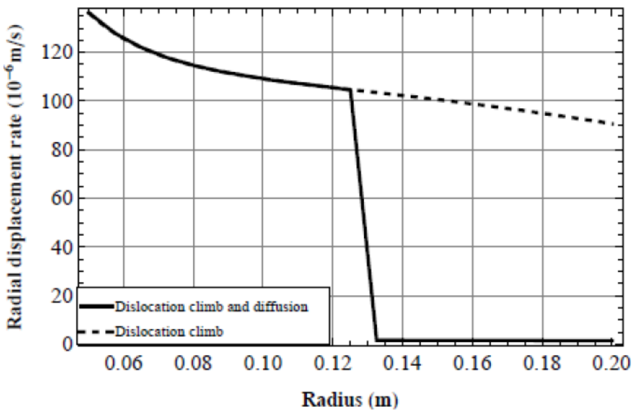

Fig. 6: *Radial displacement rate*

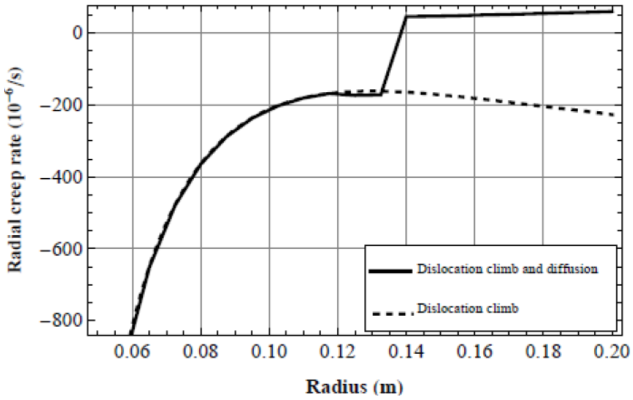

Fig. 7: *Radial creep rate distribution*

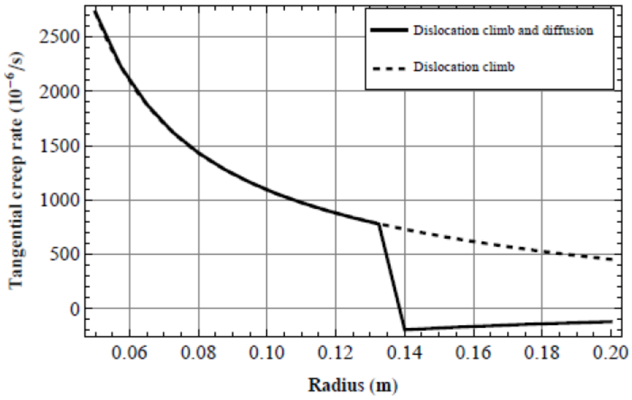

Fig. 8: *Tangential creep rate distribution*

4. Conclusion

In high rotational speeds the creep relaxation phenomena can affect the performance of FGM rotating disc. Creep behavior is depended to stress and temperature fields, time, loading history, temperature history and microstructure type of materials. Therefore, through the various parameters which are effective on creep behavior, it is usual to exist two distinct creep mechanism in a single substance and results show that creep response is very sensitive to the governing creep mechanism in FGM rotating disc and also considering only one governing mechanism in all parts of rotating disc results the large errors.

5. References

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