GEOMETRICAL SYNTHESIS OF FINE-MODULE RATCHET TOOTHING

ГЕОМЕТРИЧЕСКИЙ СИНТЕЗ МЕЛКОМОДУЛЬНОГО ХРАПОВОГО ЗАЦЕПЛЕНИЯ

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Abstract: Geometrical synthesis of fine-module ratchet toothing in which the contacting surfaces of the teeth formed by the straight segments are considered. The proposed profile of fine-module ratchet teeth allows increasing the load capacity and manufacturability production. The formulas for determination of the main geometrical parameters of the proposed fine-module ratchet teeth are obtained.

Keywords: RATCHET TOOTHING, SYNTHESIS OF TEETH PROFILE, SHAPING TEETH, ONE-WAY CLUTCH

1. Introduction

Ratchet toothing is used in a variety of mechanisms for load transfer by normal forces. For example, in the non-friction eccentric one-way clutches can used fine-module ratchet teeth with the module \( m_t = 0.3…1.0 \) mm [1-4].

When designing of non-friction eccentric one-way clutches the question of synthesis (calculating geometrical parameters) of ratchet tooth profile is important, because they are the main working elements of these mechanisms.

The profile of teeth of non-friction eccentric one-way clutches is selected to meet two main criteria: manufacturability production and the working capacity of toothed in the transmission of large loads.

Load capacity of non-friction eccentric one-way clutches can be improved by using fine-module ratchet teeth with the new profile, which provide their contact in engagement on the surface.

Modern methods used for the synthesis of profiles of involute and ratchet teeth can not be applied to proposed teeth of non-friction eccentric one-way clutches due to the peculiarities of their geometry [5-6].

2. Determining the geometrical parameters of fine-module ratchet teeth.

Let us assume initial data for calculating the geometrical parameters of profile of fine-module ratchet teeth: \( r_{f1}, r_{f2} \) – radii of the addendum circle of external and internal ratchet teeth; \( \gamma_1 \) – gradient angle of the front edge of ratchet teeth.

Theoretical height of ratchet teeth is assumed equal to the module \( m_t \), and is expression as \( H_t = r_{f2} - r_{f1} \).

The circular and angular pitch of ratchet teeth is defined as \( p_t = \pi m_t \) and \( \tau = 180 m_t / r_{f1} \).

The theoretical profile of ratchet tooth (Fig. 1) is determined by the position of points \( A, B, C \) in the coordinate system \( xOy \).

The coordinates of the points \( A \) and \( C \) are determined by the formulas:

\[
X_A = 0; \quad Y_A = r_{f1}; \\
X_C = r_{f1} \sin \tau; \quad Y_C = r_{f1} \cos \tau.
\]

The coordinates of point \( B \) correspond to the coordinates of the intersection point of the straight line \( AB \) and the circle of radius \( r_{f2} \).

Thus, coordinates of point \( B \) can be express system of equations:

\[
\begin{align*}
Y_B &= X_B \tan(90 - \gamma_1) + r_{f1}; \\
Y_B^2 + X_B^2 &= r_{f2}^2.
\end{align*}
\]

The system of equations (1) can be solve in the following order:

\[
Y_B^2 = X_B^2 \tan^2 \gamma_1 + 2r_{f1} X_B \tan \gamma_1 + r_{f1}^2;
\]

\[
X_B^2 (1 + (\cos^2 \gamma_1 / \sin^2 \gamma_1)) + 2r_{f1} X_B \tan \gamma_1 + r_{f1}^2 = r_{f2}^2;
\]

\[
X_B^2 / \sin^2 \gamma_1 + 2r_{f1} X_B \tan \gamma_1 - (r_{f2}^2 - r_{f1}^2) = 0;
\]

\[
-2r_{f1} \tan \gamma_1 + \sqrt{4r_{f1}^2 \cos^2 \gamma_1 + 4(r_{f2}^2 - r_{f1}^2)} /
\]

\[
2 - (1 / \sin^2 \gamma_1); \]

\[
X_B = (\sqrt{r_{f2}^2 - r_{f1}^2} \sin \gamma_1 - r_{f1} \cos \gamma_1) \sin \gamma_1;
\]

\[
Y_B = (\sqrt{r_{f2}^2 - r_{f1}^2} \sin \gamma_1 - r_{f1} \cos \gamma_1) \cos \gamma_1 + r_{f1}.
\]

Fig.1 Geometrical parameters of fine-module ratchet toothing.

Using of the solution of system (1) and taking into account \( r_{f2} = r_{f1} + H_t \) we can obtain the expressions for determining the position of a point \( B \) in the coordinate system \( xOy \):

\[
\begin{align*}
X_B &= (\sqrt{r_{f1}^2 \cos^2 \gamma_1 + 2r_{f1} H_t + H_t^2 - r_{f1} \cos \gamma_1}) \sin \gamma_1; \\
Y_B &= (\sqrt{r_{f1}^2 \cos^2 \gamma_1 + 2r_{f1} H_t + H_t^2 - r_{f1} \cos \gamma_1}) \cos \gamma_1 + r_{f1}.
\end{align*}
\]

The theoretical length of the front edge of ratchet teeth \( L_{th} \) can be express as

\[
L_{th} = \sqrt{X_B^2 + (Y_B - Y_A)^2} ;
\]

\[
X_B = (\sqrt{r_{f1}^2 \cos^2 \gamma_1 + 2r_{f1} H_t + H_t^2 - r_{f1} \cos \gamma_1}) \sin \gamma_1 +
\]

\[
(\sqrt{r_{f1}^2 \cos^2 \gamma_1 + 2r_{f1} H_t + H_t^2 - r_{f1} \cos \gamma_1}) \cos \gamma_1 + r_{f1}^2; \]

\[
Y_B = (\sqrt{r_{f1}^2 \cos^2 \gamma_1 + 2r_{f1} H_t + H_t^2 - r_{f1} \cos \gamma_1}) \cos \gamma_1 + r_{f1}.
\]
Finally the formula for determining the taper angle of ratchet teeth can be write in the form
\[ \gamma_5 = \arccos \left( \frac{L_{11} + r_f \left[ \cos \gamma_1 - \cos (\tau - \gamma_1) \right]}{L_{11}} \right). \]  

In the calculation of ratchet tothing one should consider that in addition to the straight segments of the front edge profile of the external \( L_{11} \) and internal \( L_{21} \) ratchet teeth there will be some curvilinear fillet surfaces. The contact of teeth on curved surfaces should be excluded. The length of the straight segments depend on the geometrical parameters of the tooothing and the cutting tools for their manufacturing.

The length of the contacting slot of the front edges of the external and internal ratchet teeth can be defined as
\[ L_{11} = l_{11} + l_{21} - L_{11}. \]  

The radius of the addendum circle of external ratchet teeth is determined from the triangle \( OAE \) on the law of cosines as
\[ r_{a1} = \sqrt{r_{f1}^2 + l_{11}^2 - 2 r_{f1} l_{11} \cos (180 - \gamma_1)}. \]  

The radius of the addendum circle of internal ratchet teeth is determined from the triangle \( OAD \) on the law of cosines as
\[ r_{a2} = \sqrt{r_{f1}^2 + (l_{11} - l_{21})^2 - 2 r_{f1} (l_{11} - l_{21}) \cos (180 - \gamma_1)}. \]  

The depth of external and internal ratchet teeth is expressed by formulas: \( h_1 = r_{a1} - r_{f1} \) (12) and \( h_2 = r_{f2} - r_{a2} \). (13). Then working depth of ratchet teeth can be defined as \( h_1 = r_{a1} - r_{a2} \). (14).

3. Analysis of shaping methods of external fine-module ratchet teeth

Modern production methods of teeth are diverse and include more than 50 types [5-10]. One of the most efficient and widespread methods for shaping of different teeth profiles on cylindrical surfaces is tooth-cutting by continuously indexing method. Let us analyze the methods used for shaping fine-module ratchet teeth of the described profile on the basis of a procedure for the generating of working surfaces by continuously indexing method [6-7].

Shaping external teeth by the continuously indexing method is possible in two ways – shaper cutter or rack cutter [6, 8-10]. For the preparation of design models let us assume: \( r_{f1} \) - radii of addendum and addendum circles of teeth of edge tool; \( r \) and \( r_{11} \) – radii of blank circles, passing through the dedendum of external and internal the shaped teeth \( (r = r_{11} \) and \( r_{11} = r_{f1} \)).

Shaper cutter with the centre \( O_{L1} \) and blank with the centre \( O \) (Fig. 2) are shown in reversed motion with respect to each other with motionless blank. During forming the point \( M \) the shaper cutter is in the position \( O_{L1} \) and a point on the shaper cutter with at the moment is coinciding with the point \( M \). Because which is owned by its cutting edge.

There is a point \( M_0 \) (Fig. 2), which is forming by the tooth point of the shaper cutter, located on the circle of radius \( r_{11} \).

All points located on a segment \( AM_0 \) don’t have conjugate points on the shaper cutter, because when are formed outside the circles of radius \( r_{f1} \) and \( r_{11} \). Consequently, the shape of the cutting teeth will be created: at the segment \( [BM_0] = l_{11} \) by straight line and at the segment \( AM_0 \) by transition curve formed by the tooth point of shaper cutter.

When selecting the type and size of the cutting tool for shaping external teeth it is necessary to provide a greater length of the straight segment of the tooth profile in relation to its theoretical length.

\[ L_{11} = \sqrt{\left( r_f \gamma_1 \right)^2 + 2 r_f (H_t - H_i^2) - r_f \cos \gamma_1} \left( \sin^2 \gamma_1 + \cos^2 \gamma_1 \right). \]  

Taking into account \( H_i = m_z \) can write
\[ L_{11} = \sqrt{\left( r_f \gamma_1 \right)^2 + 2 r_f (m_z - m_i) \cos \gamma_1}. \]  

(2)

The theoretical length of the back edge of ratchet teeth \( L_{21} \) can be express as
\[ L_{21} = \sqrt{\left( X_B - X_C \right)^2 + (Y_B - Y_C)^2}; \]

\[ L_{21} = \sqrt{\left( \left( r_f \gamma_1 \cos \gamma_1 \right)^2 + 2 r_f (H_t - H_i^2) \sin \gamma_1 - r_f \sin \tau + r_f \cos \gamma_1 \right)^2 + \left( r_f \cos \gamma_1 \cos \gamma_1 + r_f \sin \tau \cos \tau \right)^2}; \]

\[ L_{21} = \sqrt{\left( L_{11} \sin \gamma_1 - r_f \sin \tau \cos \tau \right)^2 + \left( \left( L_{11} \sin \gamma_1 + r_f \left( 1 - \cos \tau \right) \right)^2 \right.}; \]

Finally, we obtain
\[ L_{21} = \sqrt{L_{11}^2 + 2 r_f \left( 1 - \cos \tau \right) + 2 r_f L_{11} \cos (\tau - \gamma_1)}. \]  

(3)

Next we will found expressions for the angles characterising a theoretical profile of ratchet teeth.

Let us define the gradient angle \( \gamma_2 \) of the back edge of ratchet teeth to the radial line. From the triangle \( OBC \) using the law of cosines we will have
\[ OB^2 = OC^2 + BC^2 - 2 \left( OC \cdot BC \cos (180 - \gamma_2) \right), \]

\[ \cos (180 - \gamma_2) = \frac{r_f^2 + L_{21}^2 - r_f^2}{2 r_f L_{11}}. \]

Then we can write the formula for determining the gradient angle in the form
\[ \gamma_2 = 180 - \arccos \left( \frac{L_{11}^2 - 2 r_f (m_z - m_i)}{2 r_f L_{11}} \right). \]  

(4)

To determine the angles \( \gamma_3 \) and \( \gamma_4 \) of the coordinates of the point \( B \) we can use the law of sines for triangles \( OAB \) and \( OBC \):
\[ \frac{r_f}{\sin \gamma_3} = \frac{r_f}{\sin (180 - \gamma_2)} \quad \text{and} \quad \frac{r_f}{\sin \gamma_4} = \frac{r_f}{\sin (180 - \gamma_2)}; \]

Whence: \( \gamma_3 = \arcsin \left( \frac{r_f \sin \gamma_1}{r_f} \right) \); \( \gamma_4 = \arcsin \left( \frac{r_f \sin \gamma_2}{r_f} \right) \).

(5)

(6)

Let define the taper angle of ratchet teeth \( \gamma_5 \). From the triangle \( OAC \) on the law of cosines we will have:
\[ \frac{AC^2 = OA^2 + OC^2 - 2 \cdot OA \cdot OC \cos \tau}{\frac{AC^2 = 2 r_f^2 (1 - \cos \tau)}{\frac{AC^2}{2}}} \]

From the triangle \( ABC \) on the law of cosines we will have:
\[ \frac{AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cos \gamma_5}{\frac{AC^2}{2}} \]

Then \( \cos \gamma_5 = \frac{L_{11}^2 + L_{21}^2 - 2 r_f \left( 1 - \cos \tau \right)}{2 L_{11} L_{21}}. \]  

(7)

Substituting the relation (3) into equation (7) we obtain the formula
\[ \cos \gamma_5 = \frac{L_{11} + r_f \left[ \cos \gamma_1 - \cos (\tau - \gamma_1) \right]}{L_{11}}. \]
Fig. 3 shows that the desire to increase the length of the straight segment $l_{11}$ for the same tooth leads to the need to significantly increase the radius of the shaper cutter. Therefore, for shaping the external teeth is the most appropriate the use of rack cutter which $\eta_2 = \infty$.

Fig. 4 shows the position of a rack cutter and blanks when the tooth point of rack cutter is forming extreme point $M_0$ on the straight segment of the tooth edges $l_{11}$.

Using the triangle $OBS$ we can written

$$\frac{r_1}{\sin(90 + \psi_1)} = \frac{r_1 - H_t}{\cos^2 \psi_1},$$

where $H_t = PN$ - theoretical depth of the rack cutter tooth.

Using the triangle $ONK$ we can written

$$\frac{r_1}{\sin(90 + \psi_1)} = \frac{r_1 - H_t}{\cos^2 \psi_1},$$

where $H_t = PN$ - theoretical depth of the rack cutter tooth.

After the transformation of the formula (16) and taking into account $H_t = m_i$, the length of the straight segment on the front edge of the external ratchet tooth can be expressed as

$$l_{11} = \eta \left(2 - \sin^2 \gamma_3 - 2m - \sin \gamma_1 \sqrt{\eta^2 \sin^2 \gamma_3 + 4\eta H_t} - 2H_t \right).$$

4. Analysis of shaping methods of internal fine-module ratchet teeth

Internal teeth can be shaped by the continuously indexing method only by a shaper cutter [6, 8-10]. Fig. 5 shows the shaping the front edge of the internal ratchet tooth in reversed motion.

Each point $M$ of the shaped profile will match conjugated point on the profile of shaper cutter tooth. To determine the position of the last known point let us use the property of engagement – common normal at the meshing point of conjugated profiles to pass through a pitch point [5-7].

In the selected point $M$ on straight line $AB$ we can draw a normal line to the profile of shaped tooth until crossing with a circle of radius $r$ in point $N$. When continuously indexing the point $N$ at a certain moment will take the position $N'$ and will become a tool pitch point.

The same time the point $M$ will take the position $M'$ and become common point for shaper cutter and forming profile. It this moment the point $M$ is formed. If one repeats the plotting of all points on the straight line $AB$, one will get a set of points $M'$, which determine the profile of the shaper cutter tooth.

There is a critical angle of the tooth front edge $\gamma_1^*$ (Fig. 6) when the normal to the tooth theoretical profile $AB$, is drawn at its extreme point $B$, touches the circle of radius $r$ in point $N$.  

$$\frac{r_1 - l_{11}/\cos \gamma_3}{\sin \psi_1} = \frac{r_1}{\sin(90 + \gamma_3)},$$

where $\sin \psi_1 = \sqrt{1 - \cos^2 \psi_1}$ or

$$\sin \psi_1 = \sqrt[2]{\frac{r_1(2 - \sin^2 \gamma_3 - \sin \gamma_1 \sqrt{\eta^2 \sin^2 \gamma_3 + 4\eta H_t} - 2H_t)}{2\eta}}.\ (17)$$
If the angle $\gamma_1 > \gamma_1^*$, normal line to the profile at any point of the length $AB$ will cross a circle of radius $r$.

If the angle $\gamma_1 < \gamma_1^*$, then there will be the point $M_0$ on the length $AB$ restricting the possible straight segment of the tooth profile $l_{21}$, as shown on Fig. 7.

Let us $l_0 = \|AM_0\|$. To connect the points $O$ and $N$, and draw a normal to the length $ON$ from the point $A$. Get a rectangle $ADNM$ for which $\|DN\| = \|AM_0\| = l_0$. Using the triangle $OAD$ we can write $r \cos \gamma_1 = r - l_0$.

The length of the straight segment of the internal ratchet tooth can be express $l_{21} = AB - l_0 = L_1 - r(1 - \cos \gamma_1)$. (20)

5. Conclusion

The selection of shaping method of fine-module ratchet teeth is important, because it determines the manufacturing capability and economic feasibility of their production.

For shaping ratchet teeth to use continuously indexing method it is recommended. One can select a shaper cutter or rack cutter as the cutting tool. Shaper cutter is a universal cutting tool because it allows shaping both external and internal fine-module ratchet teeth.

For shaping the external ratchet teeth the use of rack cutter is the most appropriate. In this case can be formed the greatest possible length of the straight segment on the front edge of the ratchet tooth.

Shaping the internal ratchet teeth has a significant differences compared with the shaping the external teeth.

First, the size of shaper cutter for their shaping is limited by the condition $\gamma_1 < r$.

Second, normal line drawn to any point of the profile will cross with a circle of radius $r$ not all angles $\gamma_1$. Therefore, the position of the extreme point $M_0$ on the straight segment of the front edge $l_{21}$ in some cases is determined by the size of ratchet toothing, and in the other shaper cutter.

6. References