THEORY OF CUMULATIVE FUEL CONSUMPTION AND EXAMPLE FOR ITS APPLICATION

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Abstract: *the theory of cumulative fuel consumption has been presented. The example of interurban bus research results have shown the way of getting to mathematical model of cumulative fuel consumption and the intensity of cumulative fuel consumption. The high value of prescience quotients is not a case- similar values were obtained in various different cases.*

KEY WORDS: *FUECONSUMPTION, THEORY, APPLICATION*

1. Introduction

The exploitative fuel consumption is described by the theory of cumulative fuel consumption. Its application enables the analysis of exploitative fuel consumption of both single vehicle as well as whole park machines, forecast of fuel consumption determination and also the comparison analysis of fuel consumption in vehicles powered by alternative sources of energy. This theory may also be one of elements of new tests' procedures defining fumes emission rules, which would count the exploitative conditions of vehicles. Such a theory has been presented. The example of interurban bus research results have shown the way of getting to mathematical model of cumulative fuel consumption and the intensity of cumulative fuel consumption.

2. Theory of cumulative fuel consumption.

The cumulative fuel consumption risen in relation to time *t* of engine exploitation may be determined as:

$$Q_{sp}(t)=\sum_{i=1}^{n(t)} v_i = n(t)\cdot\bar{v}(t) \tag{1}$$

Where: v_i - i-th quantum of consumed fuel (e.g. dose of fuel per one engine turn), $\bar{v}(t)$ - average amount of fuel quantum, consumed in relation to time *t*, $n(t)$ - number of fuel quanta consumed in relation to time *t*,

$Q_{sp}(t)$ - cumulative fuel consumption in relation to time *t*.

To assign cumulative fuel consumption in relation to time *t*, it is necessary to know the average quanta amount of fuel given per one engine turn and the number of fuel doses. If the fuel quantum is determined through the dose of fuel given per one engine turn (it is randomly variable value) and the engine does number of turns in relation to time *t*, in which given fuel equals *n(t)* then the cumulative fuel consumption is determined through product of average dose of fuel and its turns number.

Assuming that T_p is random variable indicating time between subsequent fuel doses, the distribution function of this variable is:

$$F_p(t) = P_p\ \{T_p < t\} \tag{2}$$

Where:

$P_p\ \{T_p < t\}$ - probability that T_p has lower values than *t*, *t* - Any time amount.

The density of random variable distribution, T_p is the derivative of distribution function:

$$\frac{dF_p(t)}{dt} = F_{p'}(t) = f_p(t) \tag{3}$$

The next assumption that $P_p\ \{t, t + \Delta t\}$ is a probability that in the time range Δt and thus, in time period (t, $t + \Delta t$) the fuel will not be given, and that the fuel was not given in time period (0, t), too.

Playing along with Baye's rule, it is got:

$$P_p\ \{t, t + \Delta t\} = \frac{P_p\ \{T_p \geq t + \Delta t\}}{P_p\{T_p \geq t\}} = \frac{R_p\ (t + \Delta t)}{R_p(t)} \tag{4}$$

Assuming that:

$$P_p\ \{T_p \geq t\} = 1 - P_p\ \{T_p < t\} = 1 - F_p(t) = R_p(t) \tag{5}$$

If $P_p^d\ \{ t,t + \Delta t \}= 1 - P_p\{ t,t + \Delta t \}$ is the probability of giving fuel dose in time range Δt, on the condition that such a giving has not taken place in time period (0, t), then it is got:

$$P_p^d\ \{ t,t + \Delta t \}= P_p\{ t \leq T_p \leq t + \Delta t \} \tag{6}$$

$$P_p^d\{ t,t + \Delta t \}= 1 - P_p\{ t,t + \Delta t \}= 1 - \frac{R_p(t + \Delta t)}{R_p(t)} \tag{7}$$

Through division of both sides byΔt, there is acquired the expression:

$$\frac{P_p^d\ \{ t,t + \Delta t \}}{\Delta t} = \frac{1}{\Delta t}\left[1 - \frac{R_p(t + \Delta t)}{R_p(t)}\right] = \tag{8}$$

$$= \frac{R_p(t) - R_p(t + \Delta t)}{\Delta t}\cdot\frac{1}{R_p(t)}$$

Which limit when $\Delta t \to 0$ is :

$$\lim_{\Delta t \to 0} \frac{P_p^d\ (t, t + \Delta t)}{\Delta t} = \lim_{\Delta t \to 0} - \frac{R_p(t + \Delta t) - R_p(t)}{\Delta t} \cdot \tag{9}$$

$$\cdot \frac{1}{R_p(t)} = - \frac{R_p'(t)}{R_p(t)}$$

The abovementioned limit shows the intensity of giving the fuel doses in moment t, which can be assigned as $\lambda_p(t)$.

$$\lambda_p(t) = - \frac{R_p(t)}{R_p(t)} \tag{10}$$

The number of fuel quanta given to the engine in moment t is assigned by the paradigm:

$$n(t) = \int_0^t \lambda_p(t) dt = \int_0^t - \frac{R'_p(t)}{R_p(t)} = -[\ln R_p(t) + C] \Big|_0^t = \tag{11}$$

$$= [\ln \frac{1}{R_p(t)} - C] \Big|_0^t$$

According to the fact, that in the moment $t=0$ the fuel dose is not given to the engine and from the $R_p(t = 0) = 1 - F_p(t = 0) = 1 - 0 = 1$definition, there is acquired the following relationship:

$$0 = \ln \frac{1}{R_p(t = 0)} - C = \ln 1 - C = -C \tag{12}$$

From this relationship $C=0$ is acquired, hence the number of doses given in relation to time t is formulated with relationship:

$$n(t) = \ln \frac{1}{R_p(t)} \tag{13}$$

For the assignment of number of fuel doses, it is essential to know the random distribution variable T_p, and hence, the distribution of time range between subsequent fuel doses.

The amount of particular fuel quanta ν_i is random and can be described by two dimensional static distribution of density $f(\nu, t)$.

The average amount of fuel quanta given to the engine in time$(0,t)$ can be assigned as:

$$\overline{\nu}(t) = \int_{\nu_{\min}}^{\nu_{\max}} \int_0^t \nu \cdot t \cdot f(\nu, t)\, d\nu\, dt \tag{14}$$

Values ν_{max} *and* ν_{min} determine the maximum and minimum values of fuel quanta, which are related with dosage quality of a given engine.

According to the abovementioned determinations, the following relationship indicating cumulative fuel consumptions can be written:

$$Q_{sp}(t) = \overline{\nu}(t) \cdot n(t) = \overline{\nu}(t) \ln \frac{1}{R_p(t)} = \overline{\nu}(t) \ln \frac{1}{1 - F_p(t)} \tag{15}$$

During engine exploitation, the $\overline{\nu}(t)$ and $R_p(t)$ are unknown. Both amounts may be assigned in exploitation studies from histogram built of measured values and matching it to one of known statistic distribution. Such studies have not been made yet, that is why the value $F_p(t)$ has been assumed on the basis of the following conditions:

- Intervals between giving the particular fuel doses have static distribution of the same kind. It may be assumed that, these intervals can be described by the use of Poisson's type distribution of distribution function $F(t) = 1 - e^{-\lambda t}$
- The accumulation is not performed, which means that in time range $dt \to 0$ only one dose of fuel is given.

Taking into consideration the fact that engine elements indulge in degradation, the amount of fuel dose can be described by time function. In the first approximation, it can be assumed that the value of this amount is a certain constant multiplied by quotient reliant on time. It can be described as:

$$\overline{\nu}(t) = \overline{\nu} \cdot t^a .$$

On the basis of the abovementioned assumptions, and taking into consideration already described relationship indicating cumulative fuel consumption, the following relationship can be written:

$$Q_{sp}(t) = \overline{\nu}(t) \cdot n(t) = \overline{\nu}(t) \cdot \ln \frac{1}{1 - F(t)} = \overline{\nu} t^a \cdot \ln \frac{1}{1 - (1 - e^{-\lambda t})} = \tag{16}$$

$$= \overline{\nu} t^a \cdot \ln \frac{1}{\exp(-\lambda t)} = \overline{\nu} t^a \cdot \ln \exp(\lambda t) = \overline{\nu} t^a\, \lambda\, t$$

From the assumption that $\overline{\nu} \equiv const$, and $\lambda \equiv const$, the relationship is received: $\overline{\nu} \cdot \lambda = c = const$ (17)

Hence the simple relationship of cumulative consumption in a time function:

$$Q_{sp}(t) = c \cdot t^a \cdot t = ct^{a+1} \tag{18}$$

The intensity of cumulative fuel consumption is assigned by the derivative of the abovementioned expression:

$$\frac{dQ_{sp}}{dt} = c(a+1)t^a \tag{19}$$

For assigning of mathematical model of cumulative fuel consumption, it is necessary to know the a and c quotients of the equation (22):

After logarithmic calculation of both sides of this equation, the first degree polynomial is received:

$$\ln Q_{sp}(t) = \ln (c\, t^{a+1}) = \ln c + (a+1) \ln t\ ,$$

Substituting:

$\ln Q_{sp}(t) = y;\quad \ln c = b_o;\quad (a+1) = b_1;\quad \ln t = x;$

The lineal equation is received:

$$y = b_o + b_1 x \tag{20}$$

Studies of cumulative fuel consumption are made in discrete way after several exploitation periods. After researches, two data vectors are received:

$\boldsymbol{Q_{sp}} = [Q_{sp}(t_1), Q_{sp}(t_2), Q_{sp}(t_3),..., Q_{sp}(t_i),..., Q_{sp}(t_j)]^T$ (28)

$T = [t_1, t_2, t_3,...,t_i,...t_j]^T$ (21)

Assuming further that:

$$X = \begin{bmatrix} 1 & \ln t_1 \\ 1 & \ln t_2 \\ 1 & \ln t_3 \\ . & . \\ . & . \\ . & . \\ 1 & \ln t_j \end{bmatrix} \quad (22)$$

b_o and b_1 constants may be assigned by e.g. the use of the smallest squares method, with using the relationship:

$$\begin{bmatrix} b_o \\ b_1 \end{bmatrix} = \boldsymbol{b} = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{Y} \quad (23)$$

Assigned in this way, the model values of cumulative fuel consumption may differ from the measured values. It is essential to evaluate those variations e.g. with the use of variation analysis.

3. *Example of cumulative fuel consumption application.*

The example of cumulative fuel consumption application has been presented with the use of mileage data and exploitative fuel consumption in the interurban bus. The received results are presented in the table 1.

Table 3.1. *Measurement and model calculation results of cumulative fuel consumption and intensity of cumulative fuel consumption for interurban bus.*

Year	Month	Mileage	Cumulated fuel consumption	Cumulated fuel consumption (model)	Deviation		Intensity of cumulited fuel consumption
1	2	3	4	5	6		7
		km	dm^3	dm^3	dm^3	%	dm^3/km
2008	3	8 475	2 398	2 566	-168	-6,55	0,296
	4	22 119	6 473	6 564	-91	-1,39	0,291
	5	30 352	8 902	8 948	-46	-0,52	0,289
	6	40 360	11 942	11 828	114	0,96	0,287
	7	50 178	14 917	14 639	278	1,9	0,286
	8	56 079	16 724	16 322	402	2,46	0,285
	9	61 310	18 268	17 811	457	2,56	0,284
	10	71 338	21 065	20 659	406	1,97	0,284
	11	80 732	23 604	23 319	285	1,22	0,283
	12	91 266	26 610	26 294	316	1,2	0,282
2009	1	100 275	29 145	28 833	312	1,08	0,282
	2	109 375	32 043	31 393	650	2,07	0,281
	3	119 139	34 675	34 134	541	1,59	0,281
	4	129 620	37 495	37 071	424	1,14	0,28
	5	144 326	41 513	41 185	328	0,8	0,279
	6	155 790	44 560	44 385	175	0,39	0,279
	7	166 083	47 359	47 254	105	0,22	0,279
	8	176 856	50 349	50 253	96	0,19	0,278
	9	187 697	53 189	53 267	-78	-0,15	0,278
	10	197 834	55 895	56 083	-188	-0,33	0,278
	11	208 124	58 639	58 937	-298	-0,51	0,277
	12	217 564	61 316	61 553	-237	-0,39	0,277
2010	1	220 896	62 313	62 476	-163	-0,26	0,277
	2	231 005	65 277	65 274	3	0,01	0,277
	3	243 371	68 552	68 693	-141	-0,21	0,276
	4	254 045	71 345	71 642	-297	-0,41	0,276
	5	264 875	74 263	74 631	-368	-0,49	0,276
	6	276 369	77 356	77 800	-444	-0,57	0,276
	7	283 886	79 555	79 871	-316	-0,4	0,275
	8	295 767	83 092	83 143	-51	-0,06	0,275
	9	310 180	87 450	87 108	342	0,39	0,275
	10	322 336	90 727	90 449	278	0,31	0,275
	11	334 192	93 800	93 705	95	0,1	0,275
	12	345 974	96 928	96 938	-10	-0,01	0,274
2011	1	357 589	100 183	100 123	60	0,06	0,274
	2	370 388	103 633	103 631	2	0,01	0,274
	3	382 100	106 701	106 838	-137	-0,13	0,274
	4	383 657	107 094	107 264	-170	-0,16	0,274
	5	383 657	107 094	107 264	-170	-0,16	0,274
	6	393 544	109 723	109 970	-247	-0,22	0,274
	7	401 939	112 063	112 267	-204	-0,18	0,273
	8	405 952	113 172	113 364	-192	-0,17	0,273
	9	414 268	115 413	115 637	-224	-0,19	0,273
	10	424 574	118 126	118 453	-327	-0,28	0,273
	11	434 051	120 631	121 041	-410	-0,34	0,273
	12	443 657	123 253	123 663	-410	-0,33	0,273
2012	1	451 783	125 532	125 881	-349	-0,28	0,273
	2	459 481	127 679	127 980	-301	-0,24	0,273
	3	467 009	129 764	130 033	-269	-0,21	0,273
	4	473 923	131 494	131 917	-423	-0,32	0,273
	5	475 807	132 118	132 431	-313	-0,24	0,273
	6	481 673	133 716	134 029	-313	-0,23	0,272
	7	487 251	135 290	135 549	-259	-0,19	0,272
	8	492 990	136 853	137 112	-259	-0,19	0,272
	9	501 186	139 149	139 343	-194	-0,14	0,272
	10	511 135	141 744	142 051	-307	-0,22	0,272
	11	522 669	144 704	145 188	-484	-0,33	0,272
	12	536 734	148 365	149 013	-648	-0,43	0,272
2013	1	546 919	151 240	151 781	-541	-0,36	0,272
	2	552 672	152 851	153 344	-493	-0,32	0,272
	3	560 556	154 914	155 485	-571	-0,37	0,272
	4	569 504	157 131	157 915	-784	-0,5	0,271
	5	579 744	159 705	160 694	-989	-0,62	0,271

Having bus mileage and fuel consumption related to this mileage at the disposal, quotients b_0 and b_1 of the equation (31) were assigned. In the calculations the 3rd column (table 1) of mileage presented in ln(km) with vector of value 1 create matrix **X** , whereas the 4th column (table 1) of fuel consumption presented in ln(dm^3) creates matrix **Y**.

The values of model prediction quotients are included in table 3.2.

Table 3.2. *The values of model prediction quotients*

Regression statistics	
R multiple	0,999913
R square	0,999825
Matched square R	0,999823
Standard mistake	0,01179
Observations	64

The model matching quotients to measured values are very high (close to unity). The low variations of values measured during exploitation to values assigned on the basis of model (column 6 table 1) are the result of that fact. Besides the first result, the percentage variation fluctuates in range of 3%. The graphic illustration of analyzed data is presented in fig. 3.1

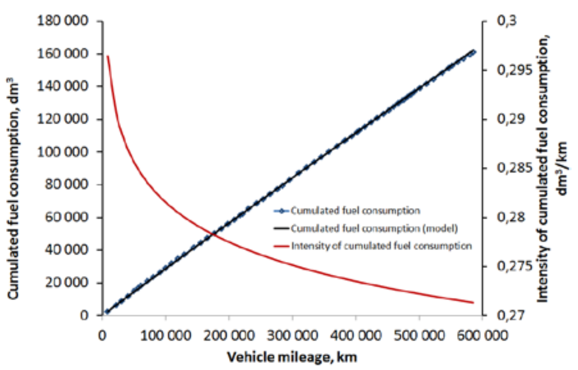

Fig. 3.2. *Graphic illustration of measurement results and cumulative fuel consumption calculations and its intensity of the researched bus.*

4. Conclusions

The theory of cumulative fuel consumption has been presented. The example of interurban bus research results have shown the way of getting to mathematical model of cumulative fuel consumption and the intensity of cumulative fuel consumption. The high value of prescience quotients is not a case- similar values were obtained in various different cases. Conversance of mathematical model of cumulative fuel consumption allows to carry out comprehensive analysis of this significant exploitative parameter.

5. Literature

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