PROBABILITY STRESS CONDITIONS IN MACHINE ELEMENTS

ВЕРОЯТНОСТНИ ЯКОСТНИ УСЛОВИЯ ЗА МАШИННИ ЕЛЕМЕНТИ

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Abstract: *Ordinary in the theory of machine elements a deterministic stress conditions are used. In general case there are relationships between "acceptance stress" and "working stress". In practice the stresses are random values. Therefore the stress conditions have got probabilistic formulation.*
In this paper the probabilistic formulation of stress conditions for machine elements are presented.
KEY WORDS: *PROBABILITY, STRENGTH, MACHINE DESIGN*

1.Introduction

In the theory of machine elements a deterministic stress conditions are used [1]. In general case there are relationships between "acceptance stress" and "working stress". In practice the stresses are random values. Therefore the stress conditions have got probabilistic formulation[2].

This paper the probabilistic formulation of stress conditions for machine elements are presented.

2. Probability stress condition for body free of imperfections

In general case the formulation of deterministic stress condition is $\sigma_{ACC} - \sigma_{WORK} \geq 0$ [1]. The probabilistic of stress condition is [2]

$$Pr\,(\,\sigma_{ACC} - \sigma_{WORK} \geq 0\,) \qquad (1)$$

where $\sigma_{WORK}\left(\{\sigma_{ij}\};ij=1,2,3\right)$, $\sigma_{ACC}\left(\sigma_{YS};\sigma_{UTS};...\right)$, $\sigma_{ij};ij=1,2,3$ - tensor of mechanical stress, $\sigma_{YS};\sigma_{UTS}$ - yield stress limit and ultimate tensile stress respectively. The condition (1) is generalization to concept of "reliability". The reliability R for safety work for machine design free of imperfections is define by the probability $Pr\,\{\,S - M \geq 0\,\}$ [3]

$$R = Pr\,\{\,M - S \geq 0\,\} \qquad (2)$$

where for a construction clement the mechanical characteristics is start diving by the vector $M \equiv M(\sigma_{YS};\sigma_{UTS};E;G;\nu)$, where $E;G;\nu$ are elastic modulus [4] and stress-deformation state is the vector $S \equiv S\left(\{\sigma_{ij}\};\{\varepsilon_{ij}\};ij=1,2,3\right)$, where $\left(\{\sigma_{ij}\};\{\varepsilon_{ij}\};ij=1,2,3\right)$- tensors of stress and deformation respectively.
The relationship (2) is transform to

$$R = \int_{-\infty}^{\infty} p_M(M)\left[\int_S^{\infty} p_M(S)dS\right]dM \qquad (3)$$

If in (3) the densities of distribution - $p_M(M)$ и $p_S(S)$ are with Weibull's distributions, then

$$p_W(x) = \frac{\beta_X}{\vartheta_X}\,\frac{(x-X)^{\beta_X-1}}{\vartheta_X}\,\exp\left[-\left(\frac{x-X}{\vartheta_X}\right)^{\beta_X}\right]$$ with

parameters $\beta_X;\vartheta_X$, $X = M(\sigma_{YS};\sigma_{UTS};E;G;\nu)$ and $X = S\left(\{\sigma_{ij}\};\{\varepsilon_{ij}\};ij=1,2,3\right)$, then relationship (3) transform to

$$R = 1 - \int_0^{\infty} e^{-y}\exp\{-h(y)\}dy \qquad (4)$$

where $h(y) = \left[\frac{\vartheta_M}{\vartheta_S}y^{1/\beta_M} + \left(\frac{M-S}{\vartheta_S}\right)\right]^{\beta_S}$ [3].

3. Probability stress condition for body with imperfections

In this case the probabilistic of stress condition is

$$Pr\,(\,K_{IC} - K_I \geq 0\,) \qquad (5)$$

where K_I - stress intensity factor, K_{IC} – critical coefficient of stress intensity if load, imperfection geometry is know. The values K_I and K_{IC} are know from fracture mechanics [4]. In overt type the probabilistic of stress condition (5) is

$$R = \int_{-\infty}^{\infty} p_{K_{IC}}(K_{IC})I(K_{IC})dK_{IC} \qquad (6)$$

where $I(K_{IC}) = \int_S^{\infty} p_{K_I}(K_I)dK_I$.

In (6) the densities of distributions $p_{K_I}(K_I)$ and $p_{K_{IC}}(K_{IC})$ are known. If $p_{K_I}(K_I)$ and $p_{K_{IC}}(K_{IC})$ are not known, then the numeric characteristics: mathematical expectation $\mathbf{E}(K_I)$,$\mathbf{E}(K_{IC})$ and dispersion $\mathbf{D}(K_I)$, $\mathbf{D}(K_{IC})$ [8] are used. In this case the probabilistic of stress condition is Chebishev's inequality [2]

$$\left\{\left|K_I - K_I^{(E)}\right| \geq K_{IC}^{(E)}\right\} \leq \left(K_{IC}^{(E)}\right)^{-2} K_I^{(D)} \qquad (7)$$

where $K_I^{(E)} = \mathbf{E}(K_I)$, $K_I^{(D)} = \mathbf{D}(K_I)$, $K_{IC}^{(E)} = \mathbf{E}(K_{IC})$;

$[\mathbf{E}(K_{IC})]^2 = \frac{1}{\sqrt{3}}\,\mathbf{E}(\bar{D}).\,\xi(E,\nu)\,\eta(\sigma_S,\psi)$;

$$\xi(E,\nu) \approx \mathbf{E}(E)\{1 + [\mathbf{E}(\nu)]^2\};$$
$$\eta(\sigma_S,\psi) \approx \mathbf{E}(\sigma_S)\{\mathbf{E}(\psi) - 0.5[\mathbf{E}(\psi)]^2\}.$$

The evaluation of $(\overline{D})$, $\mathbf{E}(E)$, $\mathbf{E}(\nu)$, $\mathbf{E}(\sigma_S)$, $\mathbf{E}(\psi)$ is look at [8].

4.Evaluation of K_I and K_{IC}

For evaluation of K_I look at plate with crack (fig.1.).

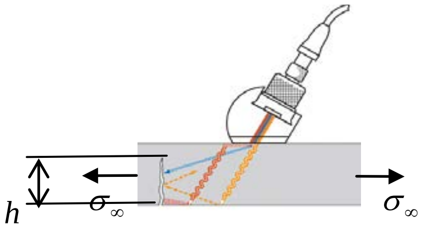

fig.1. Depth of crack evaluation– *h*, by means ultrasonic measures [8] with transducer type CDS (30^0-70^0-70^0), PANAMERTICS, САЩ Measurement by EN 583-1, EN 583-5.

In this case [4]

$$K_I = \sigma_\infty \sqrt{\pi.h}\; F(\lambda) \qquad (8)$$

where $h = \frac{1}{2} V_T^X \Delta\tau \cos\theta$ - crack length (fig.1.), θ - reflection angle, $\Delta\tau = \tau_1 - \tau_2$ - time derivation in propagation of longitudinal and transversal ultrasonic waves with velocity V_T^X ; $F(\lambda) = \sum_{k=0}^{4} (-1)^K a_K \lambda^K$; $\lambda = \frac{h}{b} < 0.7$;

a_0 =1.12, a_1 =0.23, a_2 =10.6, a_3 =21.7, a_4 =30.4.

The value K_{IC} is material characteristics. For K_{IC} there is relationship with mechanical characteristics $(E,\nu,\sigma_{YS};\sigma_{UTS})$ and structural characteristic $\overline{D}$ of the material [5].

$$(K_{IC})^2 = \frac{1}{\sqrt{3}}.\overline{D}.\sigma_{YS}.\xi.\zeta \qquad (9)$$

where $\xi = \left[\frac{E}{1-\nu^2}\right]$;

$$\zeta = \left[\ln\left(1 + \frac{420}{2.\sigma_{UTS} + \sigma_{YS}}\right)\right].$$

The average grain size $\overline{D}$ is obtain by [8]

$$\left[\frac{4.\pi^2}{1125}\frac{V_T^4}{V_L^3}\left(\frac{2}{V_L^5} + \frac{3}{V_T^5}\right).f\right](\overline{D})^3 - \alpha_L = 0 \qquad (10)$$

where $V_L;V_T;\alpha_L;f$ are respectively velocity, attenuation and frequency in ultrasonic propagation [8].

The values $V_L;V_T;\alpha_L$ are measure by means digital ultrasonic flow detectors. The methods for measure of $V_L;V_T;\alpha_L$ in according ASTM E 494:2010 are presented.

The yield stress limit – σ_{YS} by Holl – Petch's model [9] is obtained

$$\sigma_S = \sigma_0 + K_y(\overline{D})^{-1/2} \qquad (11)$$

The material constants $\sigma_0;K_y$ are evaluation by means experiment [8,9]. The elastic modulus [8] are obtained by means

$$\nu = \frac{0.5-(V_T/V_L)^2}{1-(V_T/V_L)^2} \qquad (12)$$

$$E = \left(\frac{3-4(V_T/V_L)^2}{1-(V_T/V_L)^2}\right)\rho V_T^2 \qquad (13)$$

where ρ - material density.

The relationship between ultimate tensile strength σ_{UTS} and Brinel's hardness is [6]

$$\sigma_B = \frac{1}{3} HB \qquad (14)$$

where *HB* – Brinel's hardness.
The value of *HB* is obtain by means Leeb method of hardness testing, according ASTM A 956:2012

5.Conclusion

The probabilistic of stress condition are written for body free of imperfections. A body with imperfections is look. In this case probabilistic of stress condition is written by Chebishev's inequality. The method for evaluation of the value K_I and K_{IC} and average grain size $\overline{D}$ by means ultrasonic measure are shown.

Reference

1. Lefterov L., &, Machine elements, Tehnika, Sofia, 1994 (In Bulgarian).
2. Svetlickii V.A., Statistical mechanics and theory reliability, MGTU im.N.E.Baumana, Moskow, 2002 (In Russian).
3. Alexandrovskaia D.N., &, Safety and reliability technical systems, Logos, Moskow, 2008 (In Russian).
4. Hellan K., Introduction to fracture mechanics, McGraw-Hill Book Company, New York,1984.
5. Andreykiv A.E., Three-dimensional problems in crack theory, Naukova dumka, Kiev, 1982 (In Russian).
6. Markovec M.P., Non-samples methods to evaluation of mechanical properties of metals, MEI, Moskow, 1983 (In Russian).
7. Physical acoustics, vol. IV, B, Edition by U. Mezon, Fizmatlit, Moskow, 1963 (In Russian), Papadakis E., Ultrasound attenuation dependent from scattering in polycrystalline materials.
8. Popov l., Probabilistic methods in evaluation of mechanical properties in ferro-carbon alloys (Mon0graphy). Edited by Institute of mechanics – Bulgarian Academy of Science, Series "Mechanics", April 2014 (ISSN 1314-3034), (In Bulgarian).
9. Trefilov V.I., &, Deformation at strengthen and destruction of polycrystalline materials, Naukova dumka, Kiev, 1987 (In Russian).