BOUSINESQ'S PROBLAM IN THEORY OF ELASTICITY AND ULTRASONIC

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Abstract : *The evaluation of yield stress - σ_S is frequently encounter in material testing. In this method there is necessity of manufacture of test-tube from tested material or detail and made tension test. There is destructive method. For many details there is not acceptable. In material testing there is interest to non-destructive evaluation of yield stress σ_S for the materials and details. In this paper is lock at possibility for non-destructive evaluation of yield stress σ_S by means measure Brinel,s hardness – HB or velocities of propagation of longitudinal and transversal ultrasonic waves - V_L and V_T in tested materials and details.*

KEY WORDS: *NON-DESTRUCTIVE EVALUATION OF YIELD STRESS, VELOCITIES OF PROPAGATION OF ULTRASONIC WAVES*

1. INTRODUCTION

The evaluation of the technological characteristics yield stress - σ_S is frequently encounter in material testing [1]. In this method there is necessity of manufacture of test-tube from tested material or detail and made tension test. There is destructive method. For many details there is not acceptable.

In material testing there is interest to non-destructive evaluation of yield stress σ_S for the materials and details.

In this paper is lock at possibility for non-destructive evaluation of yield stress σ_S by means measure Brinel,s hardness – *HB* or velocities of propagation of longitudinal and transversal ultrasonic waves - V_L and V_T in tested materials and details.

2. CONTACT BOUSSINESQ'S PROBLEM

The contact Boussinesq's problem in theory of elasticity [2,3,4] in follow. Let the surface $x_1\ x_2$ is loaded with force *F* by sphere with diameter D (figr.1). The stresses $\{\sigma_{ij}; i, j = 1,2,3\}$ and displacements $\{u_i; i = 1,2,3\}$ is necessity to determined.

In particular, displacement $u_3(0)$, in contact point, is $u_3(0) = \frac{1}{4.\pi}\left(\frac{1}{\mu}.\frac{\lambda+2.\mu}{\lambda+\mu}\right)\frac{F}{d/2}$ [1], where λ, μ - Lame's coefficients [1,6].

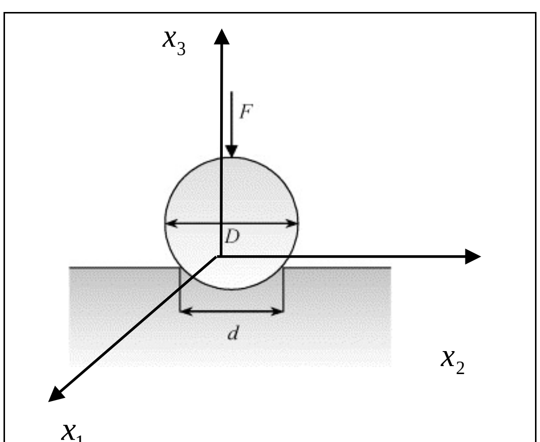

Fig.1. *Contact problem*

In Brinel's testing [4] the conditions there are [1]:
$\frac{P}{D^2}$ = 30; D = 10mm; $\frac{d}{D} = \frac{2}{5}$. Therefore
$u_3(0) = \frac{375}{\pi}\left(\frac{1}{\mu}.\frac{\lambda+2.\mu}{\lambda+\mu}\right)$.

From the stresses $\{\sigma_{ij}; i, j = 1,2,3\}$, there is the [2,3]

$$\tau_{\max} = \frac{1}{2}\ \varphi(\nu)\ HM, (\ Bousinesq) \qquad (1)$$

$$\varphi(\nu) = \frac{1}{2}(1-2.\nu) + \frac{2}{9}(1+\nu).[2.(1+\nu)]^{1/2}$$

HM is Maier's hardness. $HM = \frac{F}{S}$, where *F* – load, *S* – area of circle of impress in area ($x_1\ x_2$), $S = \frac{\pi.d^2}{4}$. The relationship between *HM* and Brinel's hardness *HB* is $\frac{HB}{HM} = 1 - \Psi$, where Ψ - steppe of deformation in impress [4]. In general $\Psi = \Psi\left(\frac{d}{D}\right)$ and therefore *HB* < *HM*. If $\frac{d}{D} < \frac{2}{5}$ (Ψ < 3 %)q then deferent between *HB* and *HM* is ~ 3 %. Therefore

$$HM \approx HB \qquad (2)$$

If introduce the sphere in area ($x_1\ x_2$) there is elastic-plastic contact (fig.1). There is Treska's condition [5]

$$\tau_{\max} = \tau_S = \frac{1}{2}\ \sigma_S \qquad (3)$$

In this case the relationship (1) reduce to

$$\sigma_S \approx \varphi(\nu)\ HB, (\ Bousinesq) \qquad (4)$$

In experimental verification he yield stress σ_S calculated by (2) is not acceptable. There is necessity from condition for agreement of experimental data with (2). To introduce coefficient K_{AGR} . In this case the (2) reduce to

$$\sigma_S \approx K_{AGR}\ \varphi(\nu)\ HB,(\ Bousinesq) \qquad (5)$$

For steel with %C ~ 0.15 the data from mechanical test are: HB^{MT} = 1430 *MPa*, σ_S^{MT} = 255 *MPa*, ν = 0.28 [7] т.е. $\varphi(0.28)$ = 0.67. Therefore

$$K_{AGR} = \frac{1}{3.75} \approx \frac{1}{4}.$$

3. BRINEL'S HARDNESS

In material science Brinel's hardness *HB* is define by [1,4]

$$HB = \frac{P}{\pi.D.u_3(0)} \qquad (6)$$

Put the test condition and the displacement $u_3(0)$ in (6) and Brinel's hardness is

$$HB \approx \frac{4}{5}\left(\mu.\frac{\lambda+\mu}{\lambda+2\mu}\right) \qquad (7)$$

Relationship between Lame's coefficients - λ, μ and velocities of propagation of longitudinal and transversal ultrasonic waves - V_L and V_T [ASTM E 494-92] is [6]

$$\lambda + 2.\mu = \rho V_L^2\ ;\ \mu = \rho V_T^2 \qquad (8)$$

There is relationship between Lame's coefficients – λ, μ and Young's modulus – E and Poisson's ratio - ν

$$\lambda = a_\lambda(\nu)\,E\ ;\ \mu = a_\mu(\nu)\,E \qquad (9)$$

where $a_\lambda(\nu) = \frac{\nu}{(1+\nu)(1-2.\nu)}$, $a_\mu(\nu) = \frac{1}{2.(1+\nu)}$. As for carbon steels the Poisson's ratio is written in a relatively narrow range, namely $0.23 \le \nu \le 0.32$, it can be considered an average value of $\upsilon \approx 0.28$. In this sense, in the parameters $a_\lambda(\nu)$ and $a_\mu(\nu)$ from (9) are constants, i.e. $a_\lambda(\nu) \equiv a_\lambda$ and $a_\mu(\nu) \equiv a_\mu$.

Density distributions p(λ) and p(μ) of the random values λ and μ are obtained from the theorem of density functional distribution of dependent random value (Probability Theory) and applied to the dependencies:

$$p(\lambda) = \frac{1}{a_\lambda}\ p(E)\ ;\ p(\mu) = \frac{1}{a_\mu}\ p(E) \qquad (10)$$

where $p(E) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(E-a)^2}{2\sigma^2}\right)$

The mathematical expectation and the dispersion of the random values λ and μ are given in:

$$\mathbf{M}(\lambda) = a_\lambda\ \mathbf{M}(E)\ ;\ \mathbf{M}(\mu) = a_\mu\ \mathbf{M}(E) \qquad (11)$$

$$\mathbf{D}(\lambda) = a_\lambda^2\ \mathbf{D}(E)\ ;\ \mathbf{D}(\mu) = a_\mu^2\ \mathbf{D}(E) \qquad (12)$$

Density distribution $p_{HB}(HB)$ is calculated by

$$p_{HB}(HB) = \frac{1}{2}\ (HB)^{-1/2}\ p_\mu\left((HB)^{1/2}\right) \qquad (13)$$

4. EQUIPMENT AND MEASURING IN NDE

The following technical tools (SONATEST, England, PANAMETRICS, USA) are used for measurements velocities of ultrasonic wave propagation through investigated materials and details:

- Digital ultrasonic flaw detector SITESCAN 150S (SONATEST, England). The measurements are carried out by means option "measurement of time propagation of ultrasonic wave" with accuracy 0.01 μs.
- Calibration block – CBV with V_L= 5.93 mm/μs (SONATEST, England).
- Transducers with X-cut of piezo-electric element (for longitudinal waves, fig.4. and Y-cut of piezo-electric element (for transversal waves, fig.5. and frequency 5 MHz.

Fig.4. *Transducers with X-cut of piezo-electric element*

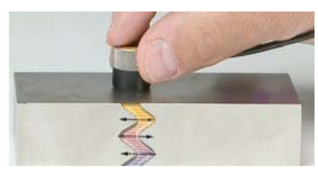

Fig.5. *Transducers with Y-cut of piezo-electric element*

A digital micrometer for thickness measure (Digimatic Micrometer, Range 0-30 mm, Accuracy 0.0001 mm (f. Mitutoyo – Japan) is used also.

The values of the "unknown" velocity of longitudinal end transversal ultrasonic wave $V_X \equiv (V_L; V_T)$ in $mm/\mu s$ is calculate by

$$V_X = \frac{l_X}{T_X/2} \qquad (14)$$

where $T_X \equiv (T_L; T_T)$, μs are the values of the "unknown" time of propagation of longitudinal and transversal ultrasonic wave, l_X - is the thickness of the studied specimen in mm.

Presentation of velocity measurement of V_L and V_T.

The relative error for velocity $\frac{\Delta V_{L,T}}{V_{L,T}}$ is

$$\frac{\Delta V_{L,T}}{V_{L,T}} = 2.\left(\frac{\Delta l}{l} + \frac{\Delta t_{L,T}}{t_{L,T}}\right) \qquad (15)$$

where $\Delta V_{L,T}; \Delta l; \Delta t_{L,T}$ - absolute errors respectively for velocity, thickness, time. Confidence interval for measure velocity is

$$\overline{V_{L,T}} \pm \left(1 + \frac{1}{n}\right) T(n;\alpha) S_{V_{L,T}} \qquad (16)$$

where $\overline{V_{L,T}}$ и $S_{V_{L,T}}$ are respectively mean and standard deviation for measure velocity, n – number of readings, $T(n;\alpha)$ - Student's distribution in probability $\Pr = 1 - \alpha$.

5. CONCLUSION

The solution of Boussinesq's problem in theory of elasticity is analyzing and obtained the relationship $\sigma_S \approx \frac{1}{4}\ \varphi(\nu)$ *HB*, who is agreement with experimental data. The functions $\nu(V_L;V_T)$ and $HB(V_L;V_T)$ are known.

For non-destructive evaluation of yield stress σ_S by means measure of velocities of propagation of longitudinal and transversal ultrasonic waves - V_L and V_T in testing materials and details is derivate.

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