DETERMINATION OF HEAT TRANSFER COEFICIENT BY INVERSE HEAT TRANSFER ANALYSIS WITH THERMOGRAPHIC EXPERIMENT

Определение коэффициента теплооддачи решением обратной задачи из термическово експеримента

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Abstract: *The objective of this paper is to determine temperature dependent heat transfer coefficient for the characterization of building porous materials in the range 20 – 60 º C, using the approach based on the solution of the inverse heat transfer problem and the results of measuring boundary conditions. The Levenberg –Marquardt iteration algorithm in software MATHEMATICA is applied to solve nonlinear system of algebraic equations resulting from the sensitivity matrix. The surface temperature distribution is measured using the infrared camera.*

KEY WORDS: HEAT TRANSPORT COEFFICIENT, INVERSE PROBLEM, THERMOGRAPHY, BUILDING MATERIAL

1.Introduction

The heat transfer coefficient - quantity controlling heat transfer process is given by Newton`s law of cooling of porous materials exposed to normal climatic conditions and can be determined using standard laboratory techniques. However, current research in determining the construction materials thermophysical properties indicates the unsuitable nature of standard measuring methods, Shin et al [1].
In order to determine the thermal characteristics of structural, thermal-protective and thermal-insulating materials as a function of temperature, the inverse heat transfer problem should be solved. New methodology combines accurate measurements of thermal quantities, which can be experimentally observed in real conditions and accurate data processing, based on the solution of inverse heat transfer problem. Performed inverse analysis involves the optimization procedures with the classical least squares calculations of the objective functions (differences of calculated from the perfect solution of the direct problem and measured temperatures).
In the present paper, the method for estimating heat transfer coefficient is carried out for building porous material. For such material the goal is to estimate the characteristic as temperature function by using results of measuring boundary conditions and surface temperature of body under consideration. The proposed procedure of determining heat transfer coefficient consists of heating a cylindrical geometry sample by a surface heating element and measuring (after reaching a steady-state) temperatures along side of the specimen using an infra-red camera. Temperature measurements are then processed through the inverse thermal modelling software, which identifies unknown heat transfer coefficient occurring in the model. It has to be stressed, that an infra-red camera used in the experiment allows to utilize a large amount of temperature measurement. This generally makes the approach fairly stable and accurate.

2. Preconditions and means for resolving the problem

2.1 Experimental setup

The sample is a part of vertical object made of the two same coaxial cylinders with surface heater in between them. In this paper only the results measured on the upper cylinder are processed. The sample is mounted vertically and heated from the bottom base by surface heater with constant electrical power. The sample is embedded in chamber with constant temperature. The experimental setup ensures axis-symmetric cooling conditions of 2D steady–state heat transport in the specimen. The surface temperature distribution of the specimen is measured by infrared camera, see Fig.1.

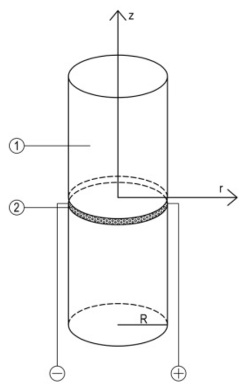

Fig. 1 *Setup of experiment 1 – heater, 2 – sample*

2.2 Mathematical model of 2D steady state heat transfer

The governing equation for above axis-symmetric steady-state thermal conditions, for material which thermal conductivity λ(T) is temperature dependent, can be written as

$$div[\lambda(T)\nabla T] = 0 \qquad (1)$$

Where $T(r,z)$ is temperature field of the sample, *r* and *z* are the radius and the height of the specimen, respectively.
Heat transfer along the surfaces of the specimen is defined by boundary conditions, which are expressed as follows

$$-\lambda(T)\frac{\partial T}{\partial z} = q \qquad r \le R \qquad \text{and} \qquad z = 0 \qquad (2)$$

$$-\lambda(T)\frac{\partial T}{\partial r}=h_v(T)(T-T_e) \qquad r=R \text{ and } \quad 0\le z\le H \qquad (3)$$

$$-\lambda(T)\frac{\partial T}{\partial r}=h_H(T)(T-T_e) \qquad r\le R \quad \text{and} \quad z=H \qquad (4)$$

Where T_e is the ambient temperature, q is the heating flux, $h_v(T)(T-T_e)$ and $h_H(T)(T-T_e)$ represents the heat exchanged with the ambient air, $h_v(T)$, $h_H(T)$ are the heat transfer coefficients dependent on temperature, which controls cooling process alongside and top surfaces, respectively.
Considering temperature range of interest in building applications is (20 – 60) º C.

2.3 Inverse determination of heat transfer coefficient

The objective is to determine the heat transfer coefficients with the assumption that everything in the direct heat transfer problem (1) – (4) is known except for one and some temperatures T_i^m, where $i=1,2,...,N_m$, which are measured at well defined locations alongside surface. The results of surface temperature measurements are assigned as necessary additional information to solve the inverse problem. In the inverse problem (1) – (4) it is necessary first of all to indicate the temperature range $[T_{\min}, T_{\max}]$ of the unknown function (heat transfer coefficient), which is general for experiment, and for which the inverse problem analysis has a unique solution. In the solution of the formulated direct problem, we employ a centric-difference scheme. As already mentioned, it is assumed that heat transfer coefficient varies with temperature. Hence the whole temperature range is divided into certain number of sub-ranges within which particular properties are modeled as piecewise linear. This practically means that one has to identify small number of parameters which represent heat transfer coefficient values at selected temperatures.
We want to find such a heat transfer coefficient $\mathbf{h}=(h_1,h_2,...,h_{N_h})$, that the objective function [2]

$$S(\mathbf{h})=\sum_{i=1}^{N_m}\frac{1}{\delta_T^2}\left(T_i^m-T_i^c(\mathbf{h})\right)^2+\sum_{j=1}^{N_h}\frac{1}{\delta_{h_j}^2}\left(h_j-h_j^0\right)^2 \qquad (5)$$

reaches the minimum among all admissible vectors **h**,
where $(h_1,h_2,...,h_{N_h})$ stand for the heat transfer coefficient values at selected temperatures (respectively locations alongside and top surfaces) T_j with $j=1,2,...,N_n$; $T_i^c(\mathbf{h})$ are calculated temperatures (at the positions where temperatures T_i^m are measured) with estimates for the unknown quantities **h**. The standard deviation δ_T is error associated with the temperature measurement. In processing the results of real experiments there are always errors, depending on a number of reasons. First of all, the errors in the experimentally measured data are both, random and systematic by nature. Random errors in input data are stipulated by a spread of thermal and electrical characteristics in measuring devices, by inaccuracy of their calibration, etc., as a rule, these errors show a large enough value. Systematic errors in the input data are usually connected with inaccuracy in determining the coordinates of positions of T_i^m, with displacement of the specimen during filming survey etc. The second group of errors – the errors of finite–difference approximation of differential operator in initial problem and round-of errors in computer. Besides, there are errors because of uncertainties in the a-priori assigned characteristics of mathematical model (1) – (4), which are determined either through calculations or from a solution of the corresponding inverse problem. The deviation δ_{h_j} is an interval within each of the parameters h_j is allowed to vary around a priori (i.e., guessed) parameter h_j^0. The function $S(\mathbf{h})$ becomes the standard least-squares method when the δ_j are set to infinity, and h_j will be fixed to the value h_j^0. The values h_j in each interval, $\lfloor T_j, T_{j+1}\rfloor$ are linearly interpolated. The minimizing procedure, in which components of the vector **h** are updated, is based on the concept of sensitivity coefficient [3]. In order to minimize $S(\mathbf{h})$, one writes

$$\frac{\partial S}{\partial h_j}=\sum_{i=1}^{N_m}\frac{-2}{\delta_T^2}\left(T_i^m-T_i^c(\mathbf{h})\right)^2\cdot Z_{ij}+\frac{2}{\delta h_j^2}\left(h_j-h_j^0\right) \qquad (6)$$

where Z_{ij} is the sensitivity coefficient.

$$Z_{ij}=\frac{\partial T_i^c(\mathbf{h})}{\partial h_j}=\frac{T_i^c\left(h_1,...,h_j,...,h_{N_h}\right)-T_i^c\left(h_1,...,h_j,..,h_{N_h}\right)}{\delta h_j} \qquad (7)$$

where δh_j is an a priori variation of the parameter h_j. In the iterative procedure, the calculated temperatures $T_i^c(\mathbf{h}^{k+1})$ at iteration $(k+1)$ is linearized as follows

$$T_i^c(\mathbf{h}^{k+1})=T_i^c(\mathbf{h}^k)+\sum_{r=1}^{N_h}Z_{ir}.\Delta h_r \qquad (8)$$

It is quite easy to verify, that substituting (7) into (8) the system of linear equations is obtained as follows

$$\sum_{r=1}^{N_h}\left(\sum_{i=1}^{N_m}\frac{Z_{ij}}{\delta_T^2}+\frac{\delta_{jr}}{\delta_{h_j}^2}\right)\cdot\Delta h_r=$$

$$\sum_{i=1}^{N_m}\frac{1}{\delta_T^2}\left(T_i^m-T_i^c(\mathbf{h^k})\right)\cdot Z_{ij}-\frac{1}{\delta_{h_j}^2}\left(h_j^k-h_j^0\right) \qquad (9)$$

where δ_{jr} is the Kroneker symbol. The increments of the parameters are found at each iterations as the solution of the system (9).
The identification procedure of the quantities $h_1,h_2,...,h_{N_m}$ is as follows. The components of vector **h** are initiated to some values $\mathbf{h}^0$ and the temperatures $T_{ij}^c(\mathbf{h}^0)$ are calculated from the direct problem. (In this work the centric - difference approximation is utilized). Each of the parameters h_j^0 is varied by δh_j, temperatures are calculated again and sensitivity coefficients are deduced (each time the vector **h** is updated) using (7). $N_m\times N_m$ system equations (9) are solved to obtain the increments, Δh_j and h_j values are updated: $h_j^{k+1}=h_j^k+\Delta h_j$, the calculation proceeds with the next

iteration. The procedure is finished, if the maximum relative variation of the parameters $\left|\frac{\Delta h_j}{h_j}\right|$ is smaller than the desired tolerance. This presented modified the least-squares technique is implemented as the main program calling the direct CDM heat flow code [2]. The effectiveness of the used iteration process is measured by the residual

$$RMS = \sqrt{\frac{1}{N_m}\sum_{i=1}^{N_m}\left(T_i^m - T_i^c(\mathbf{h})\right)^2} \qquad (10)$$

3. Results

In this work, the temperature dependent heat transfer coefficient was determined from experimental set-up with steady-state thermal conditions, as mentioned in the section 2. The sandstone sample has radius *0,025 m*, height *0,08m* , temperature dependent thermal conductivity is *(0,82+0,015.T)* $W.m^{-1}.K^{-1}$ for temperature interval *20 – 60 ºC*, the heat flux supplied by heater is *348* $W.m^{-2}$, the temperature in the chamber is *20 ºC* . The surface temperature distribution was carried out by NEC TH7102MX infrared camera. Such images guarantee that measurement errors generally do not exceed *0,2K*. An infrared picture of the specimen surface is shown in Fig.2.

Figure 3 demonstrates results obtained for heat transfer coefficient alongside the specimen as temperature dependence.

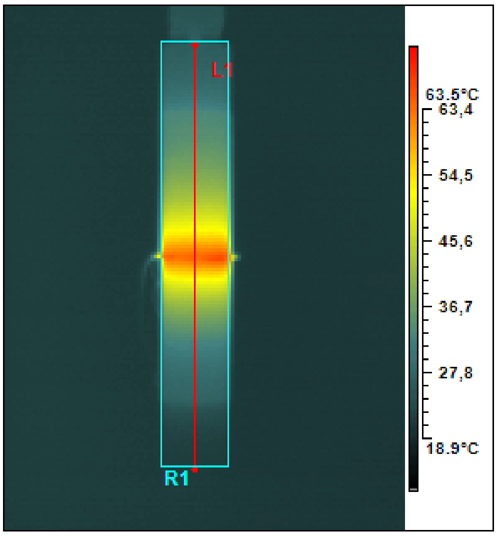

Fig. 2 *Termography of surface temperature field*

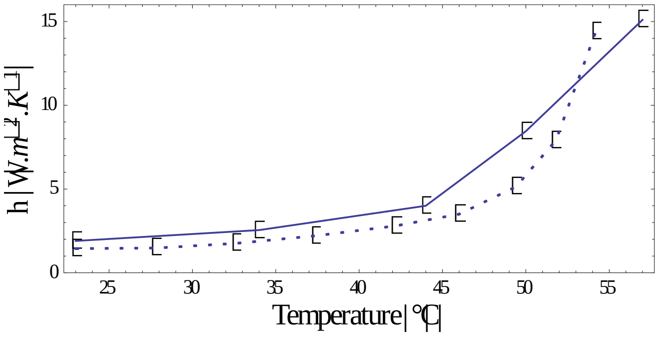

Fig. 3. *Temperature dependence of heat transfer coefficient from sample sides for different number of subintervals*

4. Conclusions

In this paper the procedure of the determining of heat transfer coefficient of concrete materials, based on steady-state temperature measurements using an infrared camera is proposed. Collected temperature measurements are processed through the inverse thermal modeling software which utilizes an appropriate model of heat transfer phenomena and identifies heat transfer coefficient as temperature function. This work presents the inverse analysis which is a primary step of designing an experiment to measure selected thermal properties materials.

Acknowledgment

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