SCHEMATISATION THE STUDY PROCESS OF MULTIBODY ROAD VEHICLE MODELS DESCRIBING RIDE COMFORT - PART I. METHODS OF CLASSICAL MECHANICS

СХЕМАТИЗАЦИЯ ПРОЦЕСА НА ИЗСЛЕДВАНЕ НА МНОГОМАСОВИ МОДЕЛИ НА ПЪТНИ ПРЕВОЗНИ СРЕДСТВА ОПИСВАЩИ ПЛАВНОСТТА НА ДВИЖЕНИЕ - ЧАСТ I. МЕТОДИ НА КЛАСИЧЕСКАТА МЕХАНИКА

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Abstract: *The objective of this study is investigation possibilities to obtain main parameters of road vehicle ride comfort using formal matrix approach. The automobile is modeled as a dynamic system made of masses interconnected by springs and dampers with linear characteristics. The procedure is applied about four to eight wheel models. The engine, transmission, passengers and loads are presented like a simple masses suspended on a single springs. In the paper are written the main principles of calculation procedure for formal modeling typical vehicle dynamic system.*

KEYWORDS: *VEHICLE DYNAMICS, DEGREE OF FREEDOM, DIFFERENTIAL EQUATIONS, MATRICES*

1. Introduction

The road vehicles are complex mechanical systems, which consist large number of masses with different types of connections between them. For studying the laws of motion of these masses it use the term "degree of freedom" (DOF) which is mean the number of independent movements of the individual elements of the system. Typically, when modelling the ride comfort the elements of the vehicle are consider as elastic suspended rigid bodies (Fig. 1).

The movement of mechanical systems is described by second order systems of differential equations. The number of equations is equal to the number of degrees of freedom.

The complexity in describing the vibration of the vehicle, i.e. included in the considered mechanical system of more or less degrees of freedom depends on the nature of the task to be solved. For example in approximate engineering research, the models are mainly plane and do not include more than 5-6 degrees of freedom for a single car. For road train vehicles models the degrees of freedom reach to 8-10.

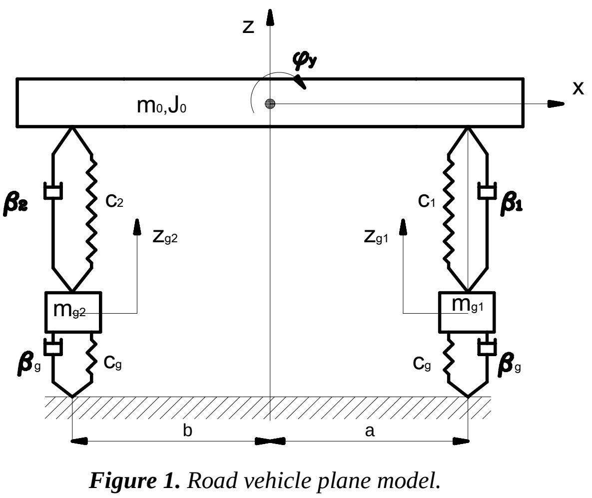

Figure 1. *Road vehicle plane model.*

2. Vehicle modelling

In vehicle ride comfort theory usually divide the masses into two main groups - suspended and non suspended. Suspended masses include all elements whose weight is transmitted through the main elastic elements (leaf springs, coil springs, torsion springs, etc.) of the specific suspension system. To this group belong - body, frame, mounted on them unites (e.g. power unit, seats with passengers and the driver, etc.) and part of the suspension elements which have common links with the frame or body. Group of the non suspended masses are considered by the authors in two ways: as the sum of the individual mass of the elements in it, disregarding elastic connections between them or as a system with many degrees of freedom, taking into account relevant elastic, damping and kinematic relations.

Non suspended masses group includes all masses under the main elastic element and for it made the same consideration as presented above.

Analysis taking into account the number of masses and elastic joints between them leads to a significant increase in the number of degrees of freedom to describe the system of the car. For example, if the center of gravity of the vehicle body (suspended masses) locates the beginning of the coordinate system $Oxyz$ for it will generate three displacements in the coordinate axes and three rotations around them. If the same system we consider the influence of non suspended masses the number of DOF will increase substantially.

In the dynamic behavior study of the units attached to the frame or body of the car and also the impact on the driver, passengers and cargo in the first approximation the most common schemes are two - with one point suspension and four-point suspension (Fig. 2).

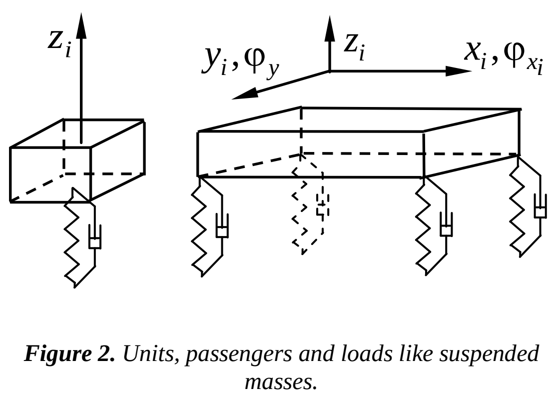

Figure 2. *Units, passengers and loads like suspended masses.*

Schemes are used depending on the nature of the study. For example passenger seats can present with one spring (left on Fig. 2) and power unit with four springs as on the right on Fig. 2.

The large number of degrees of freedom, describing the movements of the groups of masses of the vehicle is usually reduced in consideration of the influence of the different movements on the ride comfort of the vehicle. For example, in the plain models for basic movements affecting ride comfort is considered moving on Oz axis and rotation of the body around Oy axis. In spatial models are added the rotation around Ox axis (roll). Translation along the axis Oy and the rotation around Oz axis mainly influence on the vehicle stability. Translation on Ox axis is taken into account only in road train dynamics studies.

When considering plane or spatial multibody systems should to reduce the spring stiffness of the main elastic element to the wheel axis which is axis of displacement of the non suspended masses in the multibody models. Spring stiffness reducing is very important in the arm suspensions. In them the axis of action of the main elastic element do not coincides with the axis of movement of the non suspended masses (Fig.3).

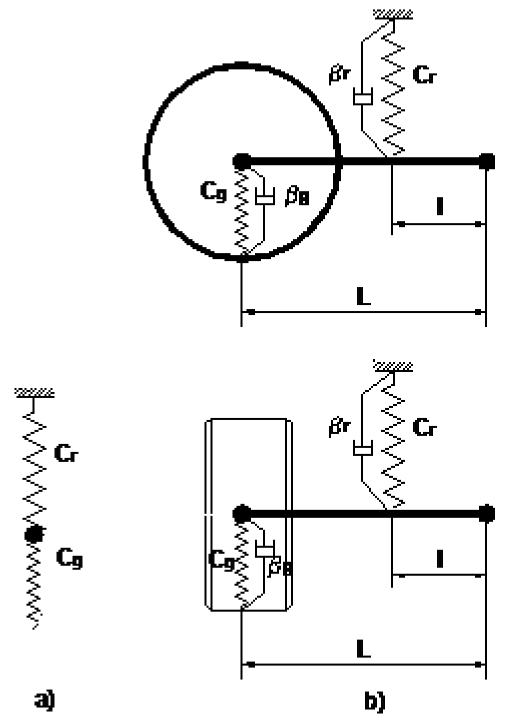

Figure 3. *Spring stiffness reduction to the wheel axis.*

For mechanical systems from Fig. 3 reduced spring stiffness is:

$$c_\Sigma = c_g c_r / (c_g + c_r), \text{ for Fig. 3, a)}, \qquad (1)$$
$$c_\Sigma = c_g c_r l^2 / (c_g L^2 + c_r l^2), \text{ for Fig. 3, b)}.$$

Based on the above considerations and analyzes are given differential equations of motion of the suspended and non suspended masses to a class of mechanical systems used in the examination of vehicle ride comfort.

The studies have been carried out under the following assumptions and limitations:

- The studies relating to wheeled vehicle having 8 wheels or 4 axles (two front and two rear);
- There is longitudinal symmetry plane of the mass, elastic and damping characteristics of the system;
- Non suspended masses are concentrated in the axis of the wheels and perform only vertical movements. The centers of gravity of each of these elements are positioned on the axes parallel to the axis Oz;
- Do not take into account the influence of the drive moments of the powertrain on the wheels;
- Suspended masses are divided into "basic" (frame, body) and "additional" (power units, passenger seats, etc.) elastic suspended on the basic;
- In the mass centers of the main and additional suspended masses are placed the centers of coordinate systems. Coordinate system of the main suspended mass is spatial $Oxyz$. Here is accounted displacement on Oz axis, rotation around Oy axis and rotation around Ox axis (roll). The additional suspended masses are divided into two main types - suspended with one or with four springs to the main mass (Fig. 2). For masses with one spring only their displacement on axis parallel to the axis Oz is account, but those with four springs includes rotation around axis parallel to Ox and Oy;
- The location of the additional suspended masses on the base mass is determined by distances e_{Xi} and offset e_{Xi}. The sign of the last is determined by the direction of the measurement to the center of the coordinate system;
- Given is that the movements of the system are small and the elastic and damping properties are linear.

Figure 4 shows the general scheme of multi-axle vehicle. In the axes of the wheels are centered the parts of the non suspended masses. Also there are shown the possible displacements of these masses. In the center of gravity of the main suspended mass is concentrated spatial coordinate system $Oxyz$. On the main suspended mass is shown an additional suspended mass at a distance e_{X1} and e_{Y1} at the center of gravity of the main one (vehicle body). In its center of gravity is centered coordinate axis Oz_1.

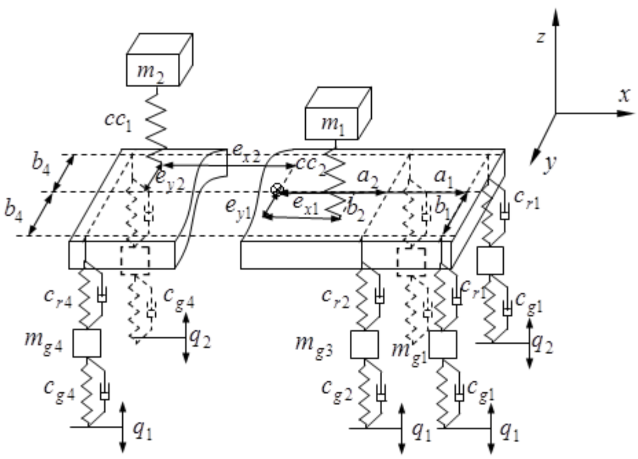

Figure 4. *Spatial structural scheme of a multi-axle vehicle.*

For shown mechanical system are received the potential (U) and kinetic (E) energy and the Rayleigh's functions (R). After applying the Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial \Pi}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q_i ,$$

The received differential equations of motion can give in matrix form:

$$M\ddot{q} + B\dot{q} + Cq = F(t) \qquad (2)$$

Further to the system (2) applies procedures for its normalization and alignment and in lower order. It takes on the appearance:

$$\dot{y} + Ly = Y, \qquad (3)$$

where $L = \begin{bmatrix} M^{-1}B & M^{-1}C \\ -E & 0 \end{bmatrix}$, a $Y = \begin{bmatrix} M^{-1}F(t) \\ 0 \end{bmatrix}$.

From the matrix can get natural frequencies of the mechanical system and using Runge-Kutta method to receive the laws of motion.

From the above it follows that if the matrices M, C and B (inertial, elastic, damping) are known then searching for a solution for the system (2) is formal and is a matter of skills related to use of computer programs.

The aim of this work is to present an approach which to formalize (automated) process of seeking a system of differential equations (formation of matrices M, C and B) describing the behavior of one class of mechanical systems.

If the solutions of the equations for the system on Figure 4 could be made in the following sequence:

- four wheel scheme - six wheel scheme - eight wheel scheme;
- without additional suspended masses - with one additional suspended mass - with two or more additional suspended masses, etc., then using the principle of mathematical induction is noticeable that inertia, elastic and damping matrix form their members in a certain order and they are subject to relatively symmetrical relations. To be an analysis of how to fill in individual cells of the matrix will be examined inertia and elastic ones. General appearance of the elastic matrix has the form:

(4)

$[c_{11}]$	$[c_{12}]$	$[c_{13}]$
$[c_{21}]$	$[c_{22}]$	$[c_{23}]$
$[c_{31}]$	$[c_{32}]$	$[c_{33}]$

Here $[c_{i,j}]$ are submatrices each forming its members depending on the complexity of the mechanical system (number of mounts number of additional suspended masses).

Elastic matrix C for the simplest case (four supports, one unsprung mass and case when not recognize the influence of the unsprung masses) has the form:

$$\begin{bmatrix} 2(c_1 + c_2) & 0 & 2(c_1 a_1 - c_2 a_2) \\ 0 & 2(c_1 b_1^2 + c_2 b_2^2) & 0 \\ 2(c_1 a_1 - c_2 a_2) & 0 & 2(c_1 a_1^2 + c_2 a_2^2) \end{bmatrix} \qquad (5)$$

where c_1, c_2 are the reduced stiffness coefficients according to 3, a) under the front and rear suspension points.

Here matrix $[c_{2,2}]$ corresponds to the sub matrix (4). In a more complex model begin the formation of a sub matrix $[c_{i,j}]$.

For sub matrices $[c_{i,j}]$ in the case of two additional suspended masses and without impact of non suspended masses can be write:

$$[c_{11}] = \begin{bmatrix} cc_1 & 0 \\ 0 & cc_2 \end{bmatrix}, \qquad (6)$$

$$[c_{21}] = -\begin{bmatrix} -cc_1 & -cc_2 \\ cc_1 e_{Y1} & cc_2 e_{Y2} \\ -cc_1 e_{X1} & -cc_2 e_{X2} \end{bmatrix}$$

$$[c_{12}] = \begin{bmatrix} -cc_1 & cc_1 e_{Y1} & -cc_1 e_{X1} \\ -cc_2 & cc_2 e_{Y2} & -cc_2 e_{X2} \end{bmatrix}$$

For sub-matrix $[c_{21}]$ and $[c_{12}]$ is in fact $[c_{21}] = [c_{12}]^T$.

To members of sub matrices $[c_{33}]$ add members to influence the additional over suspended masses.

$$[c_{22}] = \begin{bmatrix} 2(c_1 + c_2) + cc_1 + cc_2 \; . & . & . & . & . & . \\ -(cc_1 e_{Y1} + cc_2 e_{Y2}) & cc_1 e_{Y1} + cc_2 e_{Y2} + \\ 2(c_1 a_1 - c_2 a_2) + cc_1 e_{X1} + cc_2 e_{X2} \; . & . & . \end{bmatrix}$$

If do not consider the influence of non suspended masses and submatrices $[c_{13}], [c_{31}], [c_{32}]$ and $[c_{33}]$ do not receive values for its elements, i.e. these submatrices are not involved in forming the matrix of elasticity C.

By increasing the number of axes of two to four (from four to eight wheels, see Figure 4), and considering the effects of the non suspended masses, the sub matrices $[c_{11}], [c_{12}], [c_{21}]$ retain its members unchanged, i.e. they are not affected by the inclusion of the non suspended masses and the corresponding new degrees of freedom. Sub matrices have a type:

$$\begin{bmatrix} 2(c_1 + c_2 + c_3 + c_4) + cc_1 + cc_2 \ldots\ldots \\ -(cc_1 e_{Y1} + cc_2 e_{Y2}) \quad cc_1 e_{Y1} + cc_2 e_{Y2} \ldots\ldots \\ 2(c_1 a_1 + c_3 a_3 - c_2 a_2 - c_4 a_4) + cc_1 e_{X1} + cc_2 e_{X2} \end{bmatrix} \qquad (7)$$

where are represented only members by the first column.

Sub matrices $[c_{31}]$ and $[c_{13}]$ have only zero members and $[c_{31}] = [c_{13}]^T$.

Sub matrix $[c_{33}]$ like sub-matrix $[c_{11}]$ is square and has members only on the main diagonal. It is presented as:

$$\begin{bmatrix} c_1 + c_{g1} & 0 & 0 & 0\ldots\ldots \\ 0 & c_1 + c_{g1} & 0 & 0\ldots\ldots \\ 0 & 0 & c_3 + c_{g3} & 0 \;\ldots\ldots \\ 0 & 0 & 0 & c_3 + c_{g3}\ldots \\ 0 & 0 & 0 & 0\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{bmatrix} \qquad (8)$$

Sub matrices $[c_{32}]$ and $[c_{23}]$ have a type:

$$[c_{32}]=\begin{bmatrix} -c_1 & c_1b_1\ldots\ldots \\ -c_1 & -c_1b_1\ldots\ldots \\ -c_3 & c_3b_3\ldots\ldots\ldots \\ -c_3 & -c_3b_3\ldots. \\ -c_2 & \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \end{bmatrix}, \quad (9)$$

$$[c_{23}]=\begin{bmatrix} -c_1 & -c_1 & -c_3 & -c_3 & -c_2\ldots. \\ c_1b_1 & -c_1b_1 & c_3b_3 & -c_3b_3\ldots\ldots & \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & & \end{bmatrix}$$

For sub matrices $[c_{32}]$ and $[c_{23}]$ can be written $[c_{32}]=[c_{23}]^T$.

Analysis of the members and the rank of the sub matrices indicate symmetry and proportionality in their members in the complexity of the models. In the shown case additional over suspended masses have single spring suspension for simplifying the examples. In accepted degrees of freedom for the main suspended mass (3 DOF) sub-matrix always has dimension 3x3.

Sub matrices $[c_{11}],[c_{12}],[c_{21}],[c_{31}]$ and $[c_{13}]$ absent in the case of consideration the suspended mass as united all additional suspended masses. In the case under consideration Figure 4, have one main, two additional suspended masses by single spring suspension and eight non suspended masses and for the dimension of sub matrices may be written:

$$[c_{11}]=[n,n],[c_{12}]=[n,3],[c_{21}]=[3,n],[c_{22}]=[3,3]$$
$$[c_{31}]=[m,n],[c_{32}]=[m,3],[c_{33}]=[m,m]$$
$$[c_{33}]=[n,m].$$

About shown dimension of sub matrices $[c_{i,j}]$ can make the following clarifications. Here m is the number of non suspended masses. When the vehicle has eight wheels $m=8$. The index n reflects taken into account single additional over suspended masses. For this case it is $n=2$. Repeated three dimensionality in some of the sub matrix shows the number of degrees of freedom for the main suspended mass (displacement on Oz and rotations on Ox and Oy).

The analysis shows that dimensionality of the elastic matrix C is $C \quad [n+p+m,n+p+m]$. Here $"p"$ is the number of degrees of freedom of the main suspended mass.

Inertia matrix M for the estimated mechanical system on Figure 4 can be written as (4):

(10)

$[m_{11}]$	$[m_{12}]$	$[m_{13}]$
$[m_{21}]$	$[m_{22}]$	$[m_{23}]$
$[m_{31}]$	$[m_{32}]$	$[m_{33}]$

Here sub matrix $[m_{22}]$ like sub matrix $[c_{22}]$ represents inertia matrix M for the case when not recognize the influence of the non suspended mass and suspended mass are treated as alone. Its dimension for three degrees of freedom for the main suspended mass is $[m_{22}]=[3,3]$. For other sub matrices have the same dimensions like sub matrices $[c_{i,j}]$.

Inertial sub matrix $[m_{11}]$ reflects the impact of the additional suspended masses, the sub matrix $[m_{22}]$ reflects the influence of the main suspended masses, $[m_{33}]$ reflects the influence of non suspended masses. Other inertial sub matrices have only zero members in the considering case.

The dimension of the inertia matrix M has the type $M \quad [n+p+m,n+p+m]$ and formed in the manner of an elastic one.

The matrix reflecting the damping B develops in the manner shown for the matrix of elasticity C.

3. Conclusion

The present review allows formalizing the modelling of one class dynamical transport system complying with the above mentioned restrictive conditions without going through the stage of compulsory solution of differential equations of motion. This is possible on the basis of the proven relations who exist in the matrix and the regularity in increasing their members.

Organization for automated formation of M, C and B can be carried out on the basis of the presetting (e.g. in the interactive mode) to the dimension of sub matrices $[m_{i,j}],[c_{i,j}]$ and $[b_{i,j}]$. Dimensionality is determined by the number of additional suspended masses $"n"$ number of non suspended masses $"m"$ and the number of degrees of freedom assigned to the main suspended mass $"p"$. Likewise pre-set the mass, elastic and damping characteristics of the system, as well as the coordinates of the points of suspension.

After receiving the required information on the specific values is done filling the cells with elements of sub matrices. In the final stage of the procedure is done rearranging the elements of sub matrices $[m_{i,j}],[c_{i,j}]$ and $[b_{i,j}]$, as elements of matrices M, C and B. It gives possibility further to apply standard methods (3) on the system for the preparation of its parameters.

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