MHD THREE DIMENSIONAL STAGNATION-POINT FLOW OF A NEWTONIAN FLUID TOWARDS A UNIFORMLY HEATED AND MOVING PERMEABLE VERTICAL PLATE

Demir M. Ş. PhD., Prof. Dr. Barış S. PhD.
Faculty of Engineering, Department of Mechanical Engineering, Istanbul University, Avcilar 34320, Istanbul, Turkey
demirms@istanbul.edu.tr

Abstract: We investigate the three dimensional stagnation-point flow of an electrically conducting incompressible viscous fluid towards a moving vertical permeable plate in the presence of a transverse uniform magnetic field. The partial differential equations governing flow and heat transfer are reduced to a set of nonlinear ordinary differential equations by using the appropriate transformations. The MATLAB routine BVP4c is successfully applied to solve these nonlinear ordinary differential equations. The results are compared with those known from the literature and an excellent agreement is found. The effects of the suction/injection parameter on the velocity components, wall shear stress, temperature and heat transfer are discussed through the graphs and tables.

Keywords: CONVECTION, MAGNETOHYDRODYNAMICS(MHD), NEWTONIAN FLUID, STAGNATION-POINT FLOW

1. Introduction

Flow and heat transfer over a moving flat surface is a significant factor in several industrial and practical engineering applications such as, cooling of electronic components, crystal growth, geothermal systems, heat exchangers, nuclear reactors, metallurgical processes, and others.

The analysis of classical two-dimensional stagnation point flow toward a rigid horizontal plane dates back to the pioneering investigation by Hiemenz 1. The corresponding temperature distribution was reported by Goldstein 2. The axisymmetric case was investigated by Homann 3. Both two-dimensional and axisymmetric flows were extended to three dimensions by Howarth 4 and Davey 5. Stagnation point flows on moving plates were considered by Rott 6, Wang 7, Libby 8, and Weidman and Mahalingam 9. The most general solution of the Navier-Stokes equations and energy equation for non-axisymmetric three-dimensional stagnation-point flow and heat transfer on a flat plate was presented by Abbasi and Rahimi 10.

Convection in a boundary layer flow was first considered by Sparrow et al. 11. They obtained similarity solutions for the combined forced and free convection flow and heat transfer about a non-isothermal body subjected to a non-uniform free stream velocity. Due to their importance in various branches of science, engineering, and technology, convection flows have been extensively studied by several authors and discussed in some articles 12-18. Recently, Wang and Ng 17 presented the solutions for the two-dimensional and axisymmetric stagnation flow toward a heated vertical plate with Navier’s slip condition.

The effect of suction on the Hiemenz problem was first considered by Schlichting and Bussmann 13. The heat and mass transfer on a stretching sheet with suction or blowing was investigated by Gupta and Gupta 14. Hiemenz and Homann magnetic flows and heat transfer problems on a permeable surface were considered by Attia 20,21 in the presence of uniform suction or injection. The effect of uniform suction or injection on the two-dimensional stagnation point flow towards a stretching horizontal plate with heat generation was examined by Attia and Seddeek 22. Two-dimensional stagnation point flow and heat transfer problem towards a stretching vertical permeable sheet was studied by Ishak et al. 23.

Because of its increasing importance in recent years for its applications in MHD power generators, MHD pumps, MHD accelerators, and MHD flowmeters, flow and heat transfer problem of electrically conducting fluids has drawn the attentions of many investigators. The study of the hydromagnetic interaction of an electrically conducting viscous fluid with an applied magnetic field in stagnation-point flow was initiated by Neuringer and McIlroy 24. In their subsequent study 25, they considered the heat transfer aspect of the same problem. Ariel 26 reexamined the Hiemenz flow in hydromagnetics in general, and the numerical procedure given by Na 27 in particular. The problem of steady forced convection flow of an electrically conducting and heat generating/absorbing fluid near a stagnation point was solved numerically by Chamkha 28. In a following paper Chamkha 29 obtained non-similar solution with the finite difference method for the problem of mhd mixed convection flow along a semi-infinite vertical plate embedded in a uniform porous medium with heat generation and magnetic dissipation. Attia 30 concerned with the axisymmetric stagnation point flow towards a stretching surface in the presence of uniform magnetic field with heat generation. Abbasbandy and Hayat 31 developed the homotopy analysis solution for the problem considered by Chamkha 28. Recently, Borelli et al. 32 studied numerically the steady three-dimensional stagnation-point flow of an incompressible, homogenous, electrically conducting Newtonian fluid over a flat plate. In a more recent study Demir and Barış 33 obtained the numerical solutions for the two-dimensional and axisymmetric stagnation-point flow of an electrically conducting incompressible viscous fluid towards a moving vertical plate in the presence of a transverse uniform magnetic field using MATLAB routine BVP4c.

The purpose of the present paper is to study the hydromagnetic viscous flow and heat transfer in the vicinity of a stagnation-point on a moving permeable vertical plate. A study of the problem under discussion was conducted by Demir and Barış 33 in the absence of suction or injection. The governing equations are transformed into a system of nonlinear ordinary differential equations by means of similarity variables. The resulting equations, together with their corresponding boundary conditions, are solved through the MATLAB routine BVP4c. Particular cases of our results are compared with existing results of Wang and Ng 17 and Demir and Barış 33. An excellent agreement of the present results with existing results has been shown.

2. Formulation of Problem

We consider the three dimensional stagnation-point flow of an incompressible viscous fluid towards a moving permeable vertical plate. The flow is illustrated in Figure 1. The velocity components corresponding to the x-, y- and z-directions are respectively denoted by \( u \), \( v \) and \( w \). Far from the plate, as \( z \) tends to infinity, the velocity components are denoted by \( u_\infty \), \( v_\infty \) and \( w_\infty \). \( a \) is a physical constant, depending on the velocity in potential motion, \( T_\infty \) the temperature of the plate, \( T_s \) the temperature far from the surface, \( g \) the gravitational acceleration, \( V_o \) the constant plate velocity in the y-direction and \( B_o \) the external uniform magnetic field applied in the z-direction.

The fluid is electrically conducting in a transverse magnetic field. The induced magnetic field is neglected under the assumption of a small magnetic Reynolds number. We assume that the flow is steady and laminar, all the fluid properties are constant and the plate is electrically non-conducting. The effects of viscous dissipation, Ohmic heating and Hall current are not included in the analysis, since they are generally small in the stagnation-point region. Also,
the radiant heating is neglected. In addition, the imposed and induced electrical fields are assumed to be negligible.

Following Demir and Barış33, the three-dimensional Navier-Stokes equations with the Boussinesq term, the energy equation and the boundary conditions are written as:

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}, \]

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}, \]

\[ \rho c_v \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \kappa \nabla^2 \mathbf{T} + Q_{in}, \]

where \( \rho \) is the density, \( p \) the pressure, \( \mu \) the dynamic viscosity, \( T \) the temperature, \( c_v \) the specific heat at constant pressure, \( k \) the thermal conductivity, \( Q_{in} \) the volumetric rate of heat generation, \( \beta \) the coefficient of thermal expansion and \( \mathbf{f} \) is the imposed magnetic field, where \( \sigma_0 \) is the electrical conductivity and \( \mathbf{B} = (0,0,B_0) \) is the transverse uniform magnetic field applied to the fluid. Following Wang and Ng17 and Demir and Barış33, we shall seek a solution of the form

\[ u = m(z) + x f'(z), \]

\[ v = h(z) + y f'(z), \]

\[ w = -2 f(z), \]

\[ T = T_0 + (T_e - T_0) \theta(z). \]

where the prime denotes the differentiation with respect to \( z \).

Substituting Eq. (6) into Eqs. (1)-(4), eliminating the pressure term from these equations, and non-dimensionalizing the resulting equations using

\[ \eta = \sqrt{\frac{\rho a}{\mu}}, \quad F(\eta) = \sqrt{\frac{\rho}{\mu a}} f(z), \]

\[ H(\eta) = \frac{1}{V_0} h(z), \quad M(\eta) = \frac{1}{V_0} m(z) \]

we arrive at the following equations

\[ F'' + 2FF' - (F')^2 + H(1 - F') + 1 = 0 \]

\[ H'' + 2FH' - F'H - H = 0 \]

\[ M'' + 2FM' - MF' + M + \lambda \theta = 0 \]

where the prime denotes the differentiation with respect to \( \eta \). \( H \) is the non-dimensional magnetic parameter, \( \lambda \) is the non-dimensional convection parameter, \( Pr \) is the Prandtl number, \( \alpha \) is the non-dimensional heat generation parameter and \( s \) is the suction/injection parameter and they are defined as:

\[ Ha = \frac{\sigma_0 B_0^2}{\rho a}, \quad \lambda = \frac{\beta \rho (T_e - T_0)}{a \sqrt{\mu a}}, \quad Pr = \frac{\mu c_v}{k}, \quad \alpha = \frac{Q_{in}}{\rho c_v a}, \quad s = \frac{W_0}{2} \frac{\rho}{V_0} \frac{1}{\sqrt{\mu a}} \]

It should be noted that \( s > 0 \) is for mass suction and \( s < 0 \) is for mass injection51,27. The dimensionless expressions for the velocity components, shear stress on the plate in the \( x \) and \( y \) directions and the heat transfer rate per unit area on the plate are given through the following equations:

\[ \frac{u}{\eta} = \frac{1}{x} \sqrt{\frac{\mu}{\rho a}} M(\eta) + F'(\eta), \quad v = H(\eta) + \frac{\alpha V}{V_0} F(\eta), \]

\[ \frac{w}{\eta} = -F(\eta) \]

\[ r_{xx} = -\frac{1}{\mu a} \frac{T_{xx}}{x} = -\frac{1}{x} \frac{\mu}{\rho a} M'(0) - F''(0), \]

\[ r_{yy} = -\frac{1}{V_0} \frac{r_{xx}}{t} = -H'(0) - \frac{\alpha V}{V_0} F''(0) \]

\[ q_s = -\frac{\partial T}{\partial z} \bigg|_{z=0} = -k \frac{\rho a}{\mu} (T_e - T_0) \theta'(0). \]

3. Results and discussion

The present paper is an extension of the investigation by the authors33, in which the suction/injection cases were not considered. Therefore, we aim in this section to study the effects of the suction/injection parameter on the behavior of the fluid velocity, the temperature distribution, the shear stresses and the heat transfer rate on the plate in the case of permeable sheet. The solution was obtained numerically by the MATLAB routine BVP4c. We refer the reader to the book by Shampine et al.34 for details about how to solve boundary value problems with bvp4c. We set the relative and absolute tolerance equal to \( 10^{-7} \) and the corresponding differential
equations were integrated from $\eta = 0$ to $\eta = \eta_\infty$, where $\eta_\infty$ is a sufficiently large number. In practice, setting $\eta_\infty$ as low as 12 yields satisfactory accuracy for the problem under discussion. As a test of the accuracy of the solution, the values of $F''(0)$, $M'(0)$ and $\theta'(0)$ are compared with corresponding numerical values reported by Wang and Ng\(^7\) and Demir and Barış\(^3\). As shown in Table 1 an excellent agreement is found for $Ha = 0$, $\alpha = 0$, $\gamma = 1$, $Pr = 0.7$ and $s = 0$.

Table 1: Comparison of the numerical results with those of Wang and Ng (2013)

<table>
<thead>
<tr>
<th>Pr</th>
<th>$F''(0)$</th>
<th>$M'(0)$</th>
<th>$\theta'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>1.31194</td>
<td>0.4955</td>
<td>-0.6654</td>
</tr>
</tbody>
</table>

Table 1: Comparison of the numerical results with those of Wang and Ng (2013)

Figure 2 shows velocity component in the $x$- direction. It is clear from Figure 2 that in the case of mass suction ($s > 0$) the velocity component in the $x$- direction increases. On the other hand, we observe that the velocity component in the $x$- direction decreases for the case of mass injection ($s < 0$).

The velocity component in the $y$- direction is presented in Figure 3. From the figure, we observe that the velocity component in the $y$- direction increases for the case of mass injection ($s < 0$) while decreases for the case of mass suction ($s > 0$). Furthermore, the tangential velocity components approach the free stream values more quickly in the case of mass suction ($s > 0$). In other words, the velocity boundary layer is thicker in the case of mass injection ($s < 0$).

Figure 4 displays the effect of suction/injection parameter on the velocity component in the $z$- direction. The figure shows that the magnitude of velocity component in the $z$- direction decreases in the case of mass injection ($s < 0$) while increases for the case of mass suction ($s > 0$).

Figure 5 illustrates the temperature profiles for different values of the suction/injection parameter. The numerical results show that the effect of increasing values of suction/injection parameter results in a decrease of the thermal boundary layer thickness. It is apparent from Figure 5 that the temperature of the fluid decreases in the case of mass suction ($s > 0$) while increases in the case of mass injection ($s < 0$).

The values of tangential shear stresses and heat transfer rate on the plate are tabulated in Table 2 for different values of the suction/injection parameter. We conclude from this table that the magnitudes of tangential shear stresses and heat transfer rate increase with the increase in the suction/injection parameter.
Table 2. Shear stress and heat transfer rate on the plate (Ha = 1, Pr = 0.7, α = 0.2, β = 0.5, $\sqrt{a_s/\rho_s} = 0.4$, $a_y/v'_y = 0.4$)

<table>
<thead>
<tr>
<th>s</th>
<th>$\tau_x$</th>
<th>$\tau_y$</th>
<th>$-\theta(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-0.892956</td>
<td>0.199941</td>
<td>-0.000230</td>
</tr>
<tr>
<td>-0.5</td>
<td>-1.241543</td>
<td>0.382210</td>
<td>0.214127</td>
</tr>
<tr>
<td>0</td>
<td>-1.752019</td>
<td>0.685989</td>
<td>0.608957</td>
</tr>
<tr>
<td>0.5</td>
<td>-2.355983</td>
<td>1.093726</td>
<td>1.129628</td>
</tr>
<tr>
<td>1</td>
<td>-3.085162</td>
<td>1.571353</td>
<td>1.721611</td>
</tr>
</tbody>
</table>

Conclusions

In this paper, we investigated the effect of the suction/injection parameter on the hydromagnetic three-dimensional stagnation-point flow against a moving vertical plate that is described by systems of coupled nonlinear ordinary differential equations. The results were obtained using the popular MATLAB routine BVP4c. The graphical and tabular presentation of the results revealed the effects of the relevant parameters on the velocity components, temperature distribution, tangential shear stresses and heat transfer.

References

33. M.S. Demir and S. Barış, “Mhd stagnation flow of a Newtonian fluid towards a uniformly heated and moving vertical plate”, Journal of Applied Fluid Mechanics, (accepted for publication)