ELECTRIC POWER SYSTEMS MODELING AND EDUCATION: SHORT TERM UNIT COMMITMENT AND ECONOMIC DISPATCH MODELING FRAMEWORK

M.Sc. Trashlieva V. [1], M.Sc. Radeva T. PhD.[1]
Department of Electrical Power Engineering – Technical University of Sofia, Bulgaria
vesselina.trashlieva@gmail.com

Abstract: *In this paper we try to build a framework in optimization model building for students without previous mathematical optimization training and experience. While modeling is a matter of art, approach and compromise between complexity and closeness to reality we need to spend some time on the basics of optimization modeling such as cost function, constraints, variables and coefficients. There is a huge variety of optimization problems solved in Electric Power Systems. The most complex of all is considered to be the Unit Commitment problem where at each iteration an Economic Dispatch is performed.*

Keywords: OPTIMIZATION MODELING, ELECTRIC POWER SYSTEMS OPTIMIZATION

1. Introduction

Students mastering Electric Power Systems (EPS) often have no previous mathematical modeling and optimization experience but are required to solve certain optimizations problems for EPS operation. Such problems are the Unit commitment (UC) problem and the Economic dispatch (ED) problem. The UC is an optimization problem used to determine the operation schedule of the generating units at every hour interval with varying loads under different constraints and environments[1]. At each hour the output power of each generator is determined via a solution to an ED problem. An ED is the problem for determining the power output of each power plant, and power output of each generating unit within a power plant, which will minimize the overall cost of fuel needed to serve the system load [2]. In an optimization problem the goal to achieve might be a single or a multiple criteria formulation, or multiple criteria may be combined together in a single cost function. There are also two big groups of optimization problems concerning the implicit existence of constraints: constrained and unconstrained optimization problems. In EPS modeling constraints are often present describing the process dynamics and the physics of the systems' components. Using Lagrange multipliers a constrained problem may be transformed into an unconstrained one leading to a general nonlinear unconstrained optimization problem. The latter is very sensitive to initial conditions while nonlinear functions seem to have more than one local extreme values and most of the methods for solving nonlinear problems find a local not a global extreme. The inconvenience of providing initial values for the search method and the fact that the risk of finding a local extreme not the global one make the Linear Programming approach a leading one in EPS optimization modeling. Linear (affine) functions have only one minima and maxima that are global and provides for the omission of second order Karush-Kuhn-Tucker conditions acquisition. Certain binary modeling techniques provide for the modeling of a set of important nonlinearities that often appear in practice while the resulting problem is still linear. Cost function and constraints may be linear or nonlinear functions, variables under consideration may be continuous, integer or binary, bounded or unbounded.

So all these matters and topics presented in the last paragraph determine the opulence of approaches and methods for optimization problems solving. This same richness requires a systematic approach in the processes and elements presentation in the mathematical model [3-6].

2. EPS model elements

When optimizing the performance of the EPS the respective power levels of the different power plants and loads are considered. To present the amount of active power produced by a certain power plant a continuous variable is used. These are optimization variables with general lower and upper bounds resulting from the real plant's capacity. If different states in the power plant's operation are possible integer and binary variables come at hand to represent the different operation states. Such states are for example "*on*" and "*off*" or a role in frequency control reserve: "*primary*", "*secondary*" and "*tertiary*". Generally the set of all possible states is a closed one, meaning it includes a finite number of elements. In practice, binary variables have proven themselves as the most convenient for power plants' states representation leaving the space for general integer variables for other purposes. When the set of all possible states includes more than two elements it can be divided in more than one binary sets. For example, a pump hydro-power station can be idle (non-operational because of a fault, a planned repair or simply because its power is not needed in the power balance), can work as a load or as a generator. These three states can be presented with three numbers: -1 for pump, 0 for idle, 1 for generator. Two of these states coded with non-zero numbers (-1 and 1) can be further represented with binary variables with logical *ones* for *true* and zeroes otherwise. It is obvious that with such approach of breaking the initial set into few binary sets there is an overlap with the two binary sets' false values and the zero state in the initial set. So such breaking technique must be used very cautiously in order not to achieve a redundancy of binary variables that make problem solving more difficult.

Branch and bound and branch and cut are the contemporary techniques for solving mixed-integer optimization problems while the relaxed solution might be achieved via different optimization methods. No matter of the algorithm used, branching is an exhaustive process so the number of the integer and binary variables make the problem solving more complex and time spending according to the number of possible integer states. This is the reason that a trend towards integer and binary variables number as well as the number of the possible integer values reduction exists in order to make the branching easier and faster.

General integer variables are mostly used in EPS modeling when there are more than one system elements with similar features. In certain cases different power plants might be grouped according to common or similar characteristics: type or costs for fuel, similar working ranges, etc. This is another manifest of the latter approach for an overall variables reduction that provides for the usage of a single variable for the whole group of plants while the participation of each element of a group is a matter of a different optimization problem (ED). This ED might be nonlinear and will not include binary or integer variables, so its solving is generally more easy. Practitioners in EPS optimization modeling often use such approach in order to reduce the number of variables.

In EPS optimization modeling the variables are:

- integer and continuous according to their feasible sets;

- real and artificial according to the nature of physical processes they represent: real variables model real powers, loads and volumes, while artificial variables model states and alternatives;

- optimization and dual according to the stage of analysis;

3. Loads modelling

Some loads are controllable and others are not. From the systems' point of view controllable loads *help* for the power balance while the fixed loads *determine* a part of power balance. The work of the uncontrollable (fixed) loads might be forecasted and they participate in power balance constraints as forecasted values on the right-hand sides.

The controllable loads are as important as the generators are especially in the current renewable power generators penetration growth. Base power plants are not capable of flexible load following in both up and down directions. Wind and solar power stations are considered as stochastic uncontrollable base power plants so the power they might inject into the EPS is also considered as a forecasted value and unfortunately inject most of their power in low load hours. In high load hours peak power plants as hydro-power stations help for preserving the power balance. In other low load hours, the power of the base loads such as nuclear power stations need to be decreased and sometimes for wind and solar power stations even to be rejected if there is not enough load.

In detailed modeling terms controllable loads are considered in the following groups:

- controllable (dispatch) over time
- controllable over power level
- controllable over time and power level
- loads with interruptible or non-interruptible work cycle

For example a pump hydro-power station is a load that is dispatchable over time because pumps can be turned on when needed. A smart household appliance might be considered as a dispatchable over time load with a non-interruptible power cycle if it can not be stopped once started until the cycle is completed. In general, controllable loads modeling require the introduction of additional binary and integer variables.

4. Power plants operation modelling

According to the flexibility and the load following abilities generators are divided into two big groups: those of base and peak power plants. As mentioned above, there is one more aspect of power plants work namely if the output power level is controllable or stochastic. Wind and solar power stations are considered uncontrollable therefore their power generation is forecasted before the building and solving of the UC or ED problem [2]. These forecasted values participate as right hand sides along with the forecasts of the uncontrollable loads. All thermal, nuclear and hydro-power plants are considered controllable. The function representing the relation between spent amounts of fuels and produced power is generally non-linear because the electric power production process is nonlinear. These functions are often captured in practice with direct observations and in order to achieve a functional expression the appropriate function approximation techniques must be applied. Such experimental data is approximated with polynomials of order N=2 or 3:

$$R(P) = \sum_{n=0}^{N} a_n P^n \qquad (1)$$

Higher powers are suitable when higher precision is required or when inflexion points are more than 3. If the model has to be linear, a piece-wise approximation might also be used:

$$R(P) = \sum_{l=1}^{L} b_l P_l \qquad (2)$$

In the latter expression $P = \sum_{l=1}^{L} P_l$ is the produced power in all linear intervals $l = 1{:}L$, b_l is the fuel consumption growth in each consecutive interval l, and P_l is the power in the interval. So in a ED or UC optimization problem the number of continuous variables for each power plant that is modeled depends on the number of observation intervals, i.e. the value of L.

Table 1: *Observed fuel consumption of a thermal power plant within its working range P_{min}, P_{max}*

Interval l	$P_{l,min}$	$P_{l,max}$	b_l
	MW	**MW**	***tons***
	P_{min}	-	23,88
1	60	70	26,68
2	70	80	29,54
3	80	90	32,44
4	90	95	34,03
5	95	100	35,63
7	100	105	37,85
8	105	110	40,31
9	110	P_{max}	45,87

Given the data that is experimentally carried out for the fuel consumption of a power plant (Table 1, Figure 1), a piece-wise fuel consumption function may be used. In this case the number of variables increases with the increase of the number of observations.

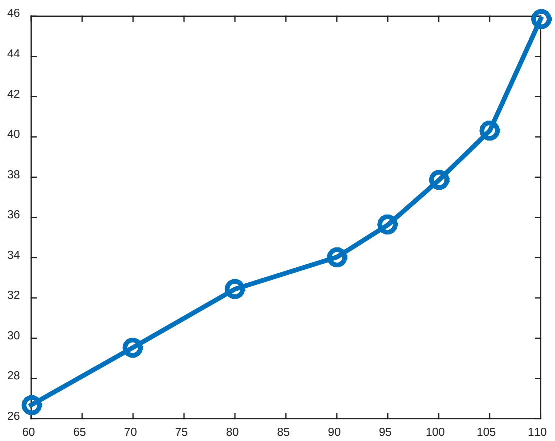

Fig. 1 *Observed fuel consumption of a thermal power plant within its working range P_{min}, P_{max} graphical representation*

A single continuous optimization variable may be used when the fuel consumption curve is nonlinear (1). Each point of this polynomial representation of a nonlinear function has its coordinates (P_n, b_n). In this case an appropriate curve fitting procedure has to used that minimizes the square of the difference between the real observation (P_l, b_l) and the evaluated one (P_n, b_n) in order to evaluate the values of the coefficients a_n bringing the polynomial approximation (1) closer the observed values:

$$\min \sum [F(P_l) - R(P_n)]^2 \qquad (3)$$

Because the cost function of the latter unconstrained optimization problem is nonlinear the appropriate initial conditions a_0 have to be provided for the problem optimization (3). As mentioned above the number n (i.e. the polynomial order N) of the coefficients whose value has to be optimized depends on the number of the inflexions of the initial data shown on *Fig. 1*. The procedure of curve fitting is performed only once whereas the observation data is stationary meaning that the values in the observations onto the fuel consumption do not change over time.

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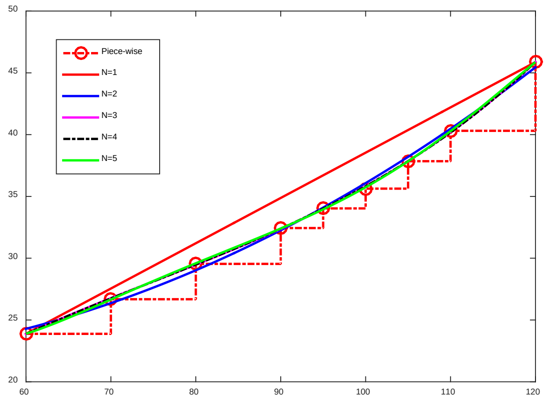

Fig. 2 *Polynomial approximations of different orders*

5. Costs structure and criteria

In optimization problems an optimal solution is found that fits best to certain criteria. In optimal power planning the most commonly used criteria is the minimal productions costs one. This is a variation of the minimal fuel costs (minimal raw materials consumption) criteria, that are determined by the consumption curve of each power plant. The fuel consumption curve multiplied by the price for the raw material (fuel) gives the fuel costs. These functions are generally nonlinear. Usually quadratic or polynomial approximations are used for the nonlinear case. In US problem on/off costs must be included. So two types of costs might be included in a cost function: costs that depend on the power plant's power level and costs that do not. This general idea for costs segregation introduces additional binary variables in the optimization models. More terms concerning different economic and financial relations may be also modeled as well as ecological criteria. All this makes the variety of cost functions wide enough to build different combined (multi criteria) goals.

6. Constraints

For the purpose of building a general idea and a modelling framework two large groups of constraints formulated in the optimization tasks in the EPS can be formulated. These are the system's conditions stating the balance requirement for each interval in the optimization horizon and the requirements for securing the reserves and the constraints arising from the production capacities. The second set of constraints may be further divided into subgroups according to the type of power plants (thermal, hydro), and a separate group of equality constraints responsible for water balances. Often in the optimization problems the working ranges of the plants are modelled by the simple bounds. Constraints that include efficiency coefficient η of the different cycles or mutually exclusive alternatives in the operation of units capable to work in reverse mode (pumps and turbines) form a separate subgroup:

minimize *Total Costs* subject to:

Balance constraints: *Production = Load + Losses*

Efficiency coefficient: *Accumulation.* η = *Generation*

Alternatives: *While true ≤ Max variable value*

If false: Variable = 0

Water balance: *Waters used by pumps and turbines in different reservoirs and time intervals should equal or less than given constraints*

Simple bounds: *Min value ≤ Variable ≤ Max value*

Integer constraints: *Some variables are integer and ome variables are binary* {0,1}

On the other part constraints generally are represented mathematically with equations and inequalities. In optimization equality constraints are called *tight* or *binding* because every change in the right-hand side of an equality constraints leads to inevitable changes in the total costs. When there is an inequality constraints that is satisfied in the optimal solution as an equality this constraint is also *binding* because there is no *reserve* for changes in its right-hand side. Constraints also may be explicit or implicit in the formulation with implicit constraints being generally the possible sets of the values of all integer variables and the simple bounds arising from the working ranges while the latter may be easily included in the models constraints while the integer ones require further mathematical knowledge and work.

7. Conclusion

The presented topics above give general idea on EPS optimization modeling basics and possible points of view. A certain classification of power plants, loads and costs criteria is given. Some optimization modeling milestones are mentioned as well as few techniques to model size reduction are given.

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