ADAPTIVE CONTROL OF NONLINEAR TWO DIMENSIONAL AIRFOIL MODEL

Boğaç Bilgiç
Department of Mechanical Engineering, Engineering Faculty – Istanbul University, Turkey
bilgicbogac@gmail.com

Abstract: *In this paper, an adaptive control method is improved for airfoil model. By using energy method, the governing equations of the nonlinear 2-D airfoil model are obtained. As known, the system exhibits different behaviors at different speeds. For this purpose, flutter speeds are investigated and phase portraits of pitching are shown at critical speeds. Flutter for airfoils is such an enormous problem which have been considered. An adaptive control method is improved for minimizing the vibration at pre-flutter speed, flutter speed and post-flutter speed regimes. To show that the controller system works, controlled and uncontrolled airfoil model are simulated simultaneously. Results of these simulations are demonstrated graphically at the conclusion part.*

Keywords: AIRFOIL, ADAPTIVE CONTROL, FLUTTER.

1. Introduction

Nonlinear two-dimensional airfoil model has been used in much recent research because it is more simple and useful than others [1-4]. Through this model, three dimensional airfoils can be modelled as two dimensional and investigated non-linear phenomenon. Flutter is a dynamic instability phenomena resulting from the interaction between an elastic structure and the flow around the structure, for which the prevention technology is very important in the design of aircraft. In 2008, Zheng and Yang analyze the flutter speed using Hopf bifurcation theory [5]. And then, many active controllers have designed to prevent the flutter [6-8].

In this paper, firstly airfoil is modelled, and investigated the flutter speed. Then, new adaptive controller was represented and its achievements are shown graphically.

2. Modelling of Airfoil

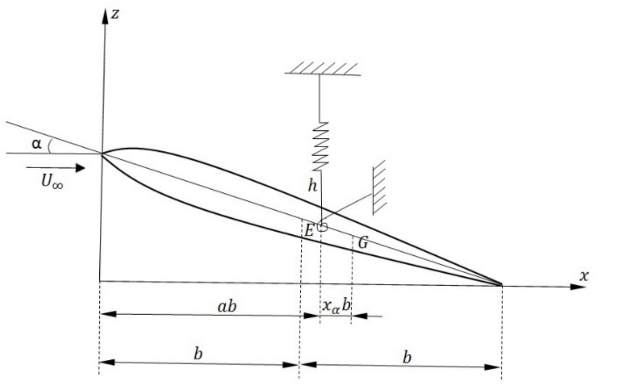

Fig. 1 *Sketch of two-degree-of-freedom airfoil.*

The sketch of a 2D lifting surface featuring plunging and pitching degrees of freedom, elastically constrained by a linear translational spring and nonlinear torsional spring respectively is shown in Fig.1. Based on the energy method, the governing equations of the lifting surface with differential cubic nonlinear stiffness in pitching direction can be expressed as [5]

$$m\ddot{h} + S_\propto \ddot{\alpha} + c_h \dot{h} + K_h h = L \tag{1}$$

$$S_\propto \ddot{h} + I_\propto \ddot{\alpha} + c_\alpha \dot{\alpha} + K_\alpha \alpha + eK_\alpha \alpha^3 = M \tag{2}$$

$$L = -\frac{\rho_\infty U_\infty b}{3M_\infty}\begin{pmatrix} 12\dot{h} + 12(1-a)b\,\dot{\propto} + 12U_\infty \propto \cdots \\ \ldots + M_\infty^2 U_\infty (1+k) \propto^3 \end{pmatrix} \tag{3}$$

$$M = \frac{\rho_\infty U_\infty b^2}{3M_\infty}\begin{pmatrix} 12(1-a)U_\infty \propto + 12(1-a)\dot{h} + \cdots \\ \ldots (16 - 24a + 12a^2)b\,\dot{\propto} + \cdots \\ \ldots M_\infty^2 U_\infty (1+k)(1-a) \propto^3 \end{pmatrix} \tag{4}$$

Where; m is mass of the wing, $S_\propto = mx_\propto b$ is the wing mass static moment about the elastic axis, $I_\propto = m(r_\propto b)^2$ is the inertial moment of the wing about the elastic axis, c_h and c_α are the linear plunging and pitching damping coefficients, K_h and $K_\propto$ are the plunging and pitching stiness coefficients, e is the non-dimensional nonlinear stiness coefficient, b is the wing's semi-chord length, h is the plunging displacement, $\propto$ is the pitching angle, L and M are the aerodynamic force and moment, M_∞ is the flight Mach number, ρ_∞ is the density of the air, U_∞ is the air speed, k is the isentropic gas coefficient and a is dimensionless elastic axis position measured from the leading edge.

Equations (3) and (4) are written in equations (1) and (2), and then should be taken the parameters as below,

$$\xi = h/b \tag{5}$$

$$\tau = U_\infty t/b \tag{6}$$

$$\omega_h = \sqrt{K_h/m} \tag{7}$$

$$\omega_\alpha = \sqrt{K_\alpha/I_\alpha} \tag{8}$$

$$\overline{\omega} = \omega_h/\omega_\alpha \tag{9}$$

$$\mu = m/(4\rho_\infty b^2) \tag{10}$$

$$V = U_\infty/(b\omega_\alpha) \tag{11}$$

the dimensionless equations of motion of the airfoil corresponding to Equations (12) and (13).

$$\ddot{\xi} + x_\alpha \ddot{\alpha} + 2\varsigma_h \frac{\overline{\omega}}{V}\dot{\xi} + \left(\frac{\overline{\omega}}{V}\right)^2 \xi = \cdots$$
$$\ldots - \frac{1}{\mu M_\infty}\left[\dot{\xi} + (1-a)\dot{\alpha} + \alpha + \frac{M_\infty^2}{12}(1+k)\alpha^3\right] \tag{12}$$

$$\frac{x_\alpha}{r_\alpha^2}\ddot{\xi} + \ddot{\alpha} + 2\varsigma_\alpha \frac{1}{V}\dot{\alpha} + \frac{1}{V^2}\alpha + \frac{e}{V^2}\alpha^3 = \cdots$$
$$\ldots - \frac{1}{\mu M_\infty r_\alpha^2}\begin{bmatrix} (1-a)\alpha + (1-a)\dot{\xi} \ldots \\ \ldots + \left(\frac{4}{3} - 2a + a^2\right)\dot{\alpha} \ldots \\ \ldots + \frac{M_\infty^2}{12}(1+k)(1-a)\alpha^3 \end{bmatrix} \tag{13}$$

Where $\varsigma_h = c_h/2\sqrt{K_h m}$ and $\varsigma_\alpha = c_\alpha/2\sqrt{K_\alpha I_\alpha}$. If equations (12) and (13) are arranged,

$$\ddot{\xi} = -x_\alpha \ddot{\alpha} - \left(2\varsigma_h \frac{\overline{\omega}}{V} + \frac{1}{\mu M_\infty}\right)\dot{\xi} - \left(\frac{\overline{\omega}}{V}\right)^2 \xi \ldots$$
$$\ldots - \frac{(1-\alpha)}{\mu M_\infty}\dot{\alpha} - \frac{1}{\mu M_\infty}\alpha - \frac{M_\infty(1+k)}{12\mu}\alpha^3 \tag{14}$$

$$\ddot{\alpha} = -\frac{x_\alpha}{r_\alpha^2}\ddot{\xi} + \left(\frac{\frac{4}{3} - 2a + a^2}{\mu M_\infty r_\alpha^2} - \frac{2\varsigma_\alpha}{V}\right)\alpha + \left(\frac{1-\dot{a}}{\mu M_\infty r_\alpha^2} - \frac{1}{V^2}\right)\alpha \ldots$$
$$\ldots + \left(\frac{M_\infty(1+k)(1-a)}{12\mu r_\alpha^2} - \frac{e}{V^2}\right)\alpha^3 + \frac{1-a}{\mu M_\infty r_\alpha^2}\dot{\xi} \tag{15}$$

are obtained.

If equation (15) is written in equation (14) and then equation (14) is written equation (15) and they are arranged,

$$\ddot{\xi} = -\left(\frac{1}{\left(1-\frac{x_\alpha^2}{r_\alpha^2}\right)}\right) * \ldots$$

$$\ldots \left\{ \begin{array}{c} \left[2\varsigma_h \frac{\bar{\omega}}{V} + \frac{1}{\mu M_\infty} + \frac{(1-a)x_\propto}{\mu M_\infty r_\alpha^2}\right]\dot{\xi} + \cdots \\ \ldots\left[\left(\frac{\bar{\omega}}{V}\right)^2\right]\xi + \cdots \\ \ldots\left[\frac{(1-a)}{\mu M_\infty} + \frac{\left(\frac{4}{3}-2a+a^2\right)x_\propto}{\mu M_\infty r_\alpha^2} - \frac{2\varsigma_\alpha x_\propto}{V}\right]\dot{\alpha} + \cdots \\ \ldots\left[\frac{1}{\mu M_\infty} + \frac{(1-a)x_\propto}{\mu M_\infty r_\alpha^2} - \frac{x_\propto}{V^2}\right]\alpha + \cdots \\ \ldots\left[\frac{M_\infty(1+k)}{12\mu} + \frac{M_\infty(1+k)(1-a)x_\propto}{12\mu r_\alpha^2} - \frac{e x_\propto}{V^2}\right]\alpha^3 \end{array}\right\} \quad (16)$$

$$\ddot{\alpha} = \left(\frac{1}{\left(1-\frac{x_\alpha^2}{r_\alpha^2}\right)}\right) * \ldots$$

$$\ldots \left\{ \begin{array}{c} \left[\frac{1-a+x_\propto}{\mu M_\infty r_\alpha^2} + \frac{2\varsigma_h \bar{\omega} x_\propto}{r_\alpha^2 V}\right]\dot{\xi} + \cdots \\ \ldots\left[\frac{\bar{\omega}^2 x_\propto}{r_\alpha^2 V^2}\right]\xi + \cdots \\ \ldots\left[\frac{\frac{4}{3}-2a+a^2+(1-a)x_\propto}{\mu M_\infty r_\alpha^2} - \frac{2\varsigma_\alpha}{V}\right]\dot{\alpha} + \cdots \\ \ldots\left[\frac{1-a+x_\propto}{\mu M_\infty r_\alpha^2} - \frac{1}{V^2}\right]\alpha + \cdots \\ \ldots\left[\frac{M_\infty(1+k)(1-a+x_\propto)}{12\mu r_\alpha^2} - \frac{e}{V^2}\right]\alpha^3 \end{array}\right\} \quad (17)$$

are acquired.

Let $x_1 = \xi$, $x_2 = \alpha$, $x_3 = \dot{\xi}$ and $x_4 = \dot{\alpha}$, equations (16) and (17) can be written as the lower order differential form

$$\dot{x}_1 = x_3 \quad (18)$$

$$\dot{x}_2 = x_4 \quad (19)$$

$$\dot{x}_3 = -\left(\frac{1}{\left(1-\frac{x_\alpha^2}{r_\alpha^2}\right)}\right) * \ldots$$

$$\ldots \left\{ \begin{array}{c} \left[2\varsigma_h \frac{\bar{\omega}}{V} + \frac{1}{\mu M_\infty} + \frac{(1-a)x_\propto}{\mu M_\infty r_\alpha^2}\right]x_3 + \cdots \\ \ldots\left[\left(\frac{\bar{\omega}}{V}\right)^2\right]x_1 + \cdots \\ \ldots\left[\frac{(1-a)}{\mu M_\infty} + \frac{\left(\frac{4}{3}-2a+a^2\right)x_\propto}{\mu M_\infty r_\alpha^2} - \frac{2\varsigma_\alpha x_\propto}{V}\right]x_4 + \cdots \\ \ldots\left[\frac{1}{\mu M_\infty} + \frac{(1-a)x_\propto}{\mu M_\infty r_\alpha^2} - \frac{x_\propto}{V^2}\right]x_2 + \cdots \\ \ldots\left[\frac{M_\infty(1+k)}{12\mu} + \frac{M_\infty(1+k)(1-a)x_\propto}{12\mu r_\alpha^2} - \frac{e x_\propto}{V^2}\right]{x_2}^3 \end{array}\right\} \quad (20)$$

$$\dot{x}_4 = \left(\frac{1}{\left(1-\frac{x_\alpha^2}{r_\alpha^2}\right)}\right) * \ldots$$

$$\ldots \left\{ \begin{array}{c} \left[\frac{1-a+x_\propto}{\mu M_\infty r_\alpha^2} + \frac{2\varsigma_h \bar{\omega} x_\propto}{r_\alpha^2 V}\right]x_3 + \cdots \\ \ldots\left[\frac{\bar{\omega}^2 x_\propto}{r_\alpha^2 V^2}\right]x_1 + \cdots \\ \ldots\left[\frac{\frac{4}{3}-2a+a^2+(1-a)x_\propto}{\mu M_\infty r_\alpha^2} - \frac{2\varsigma_\alpha}{V}\right]x_4 + \cdots \\ \ldots\left[\frac{1-a+x_\propto}{\mu M_\infty r_\alpha^2} - \frac{1}{V^2}\right]x_2 + \cdots \\ \ldots\left[\frac{M_\infty(1+k)(1-a+x_\propto)}{12\mu r_\alpha^2} - \frac{e}{V^2}\right]{x_2}^3 \end{array}\right\} \quad (21)$$

The parameters used in this work are chosen as $\mu = 50$, $\bar{\omega} = 1$, $x_\propto = 0.25$, $r_\alpha^2 = 0.5$, $\alpha = 0.5$, $\varsigma_h = \varsigma_\alpha = 0.1$, $e = 20$, $M_\infty = 6$, $a = 0.5$ and $k = 1.4$. Initial conditions are $X(0) = [0 \;\; 0.001 \;\; 0 \;\; 0]$.

From figure 2, we can see that there are complicated responses of the airfoil model, such as convergence (see Figure 2(a)), limit cycle oscillation (Figure 2(b)), divergence (Figure 2(c)) and even chaos (Figure 2(d)). In particular, divergent and chaotic motions of the airfoil will give the aircraft a serious flight safety problem. To prevent the flutter, next, the adaptive control algorithm method will be applied to design an active controller.

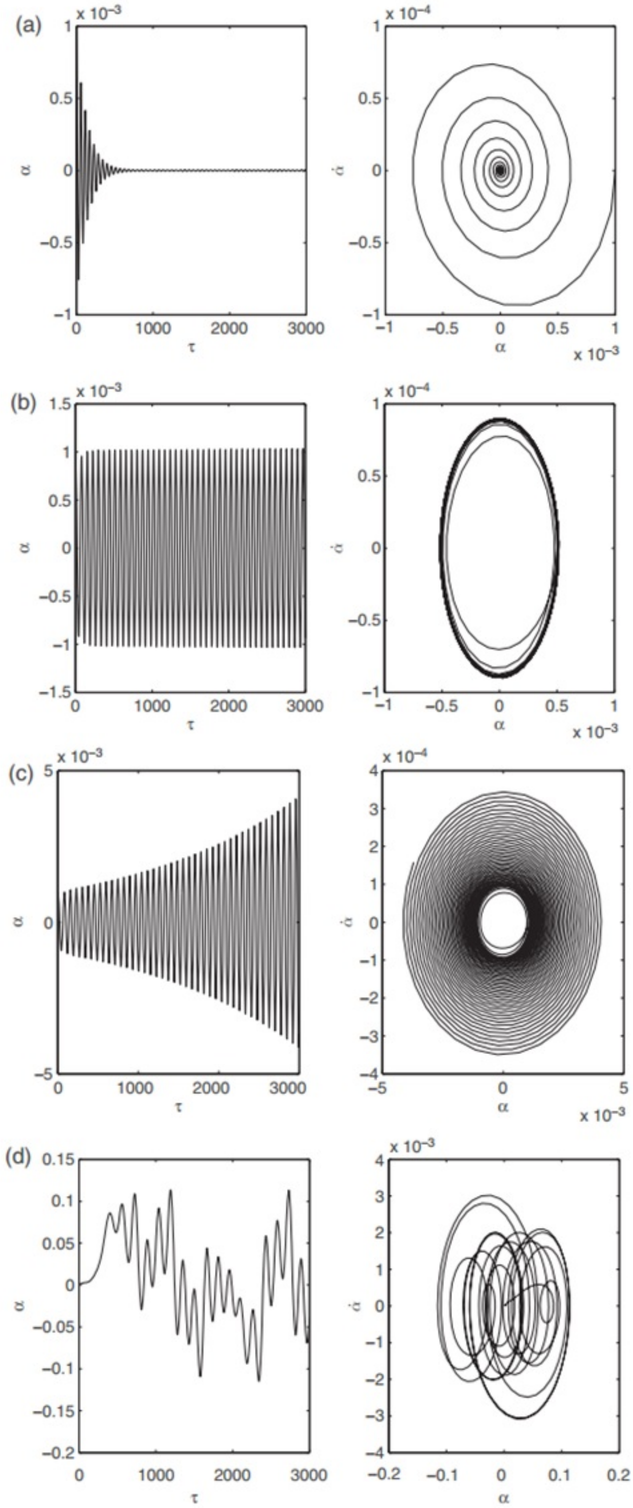

Fig. 2 *Response plots at different V. (a) V=8.132; (b) V=10.388; (c) V=10.572; (d) V=17.934 [6]*

3. Design of Adaptive Controller

System can be described

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u \qquad (22)$$

where

$$\boldsymbol{g}(\boldsymbol{x}) = [0 \ \ 0 \ \ 1 \ \ 0]^T \qquad (23)$$

Adaptive control is the control method used by a controller which must adapt to a controlled system with parameters which vary, or are initially uncertain.

The behavior of the airfoil varies greatly according to the speed. Because of this, adapting control rule is created. PID controller and only P controller in different conditions can be used. Errors $e_\xi = e_1 = x_{1ref} - x_1 = \xi_{ref} - \xi$ and $e_{\dot{\xi}} = e_3 = x_{3ref} - x_3 = \dot{\xi}_{ref} - \dot{\xi}$ determine which controller can be used.

$$u = \begin{cases} u_p & if \quad |e_1| < e_{1c} \; and \; |e_3| > e_{3c} \\ u_{pid} & otherwise \end{cases} \qquad (24)$$

Where, e_{1c} and e_{3c} are critical error values.

$$u_{pid}(\tau) = K_p e_1(\tau) + K_i \int_0^\tau e_1(s)ds + K_d \dot{e}_1(\tau) \qquad (25)$$

$$u_p(\tau) = K_p e_1(\tau) \qquad (26)$$

where K_p, K_i, and K_d all non-negative, denote the coefficients for the proportional, integral, and derivative terms, respectively.

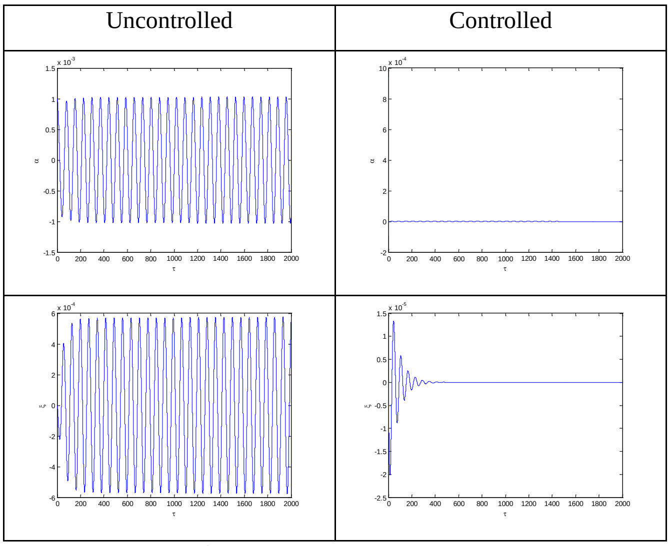

Fig. 3 *Achievement of controller at V=10.388*

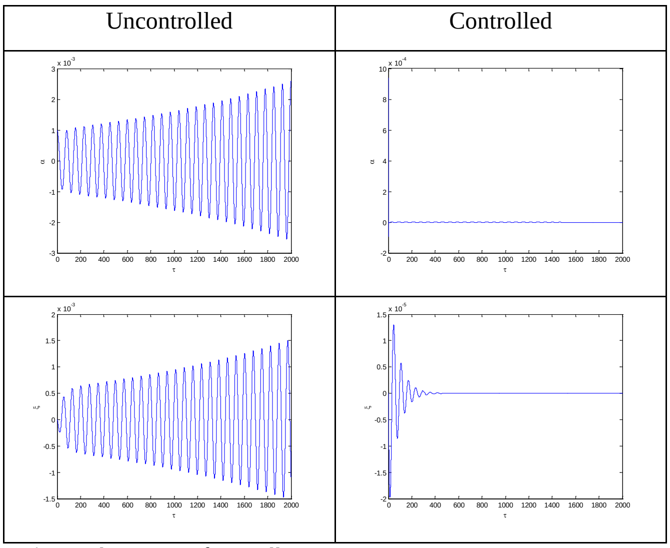

Fig. 4 *Achievement of controller at V=10.572*

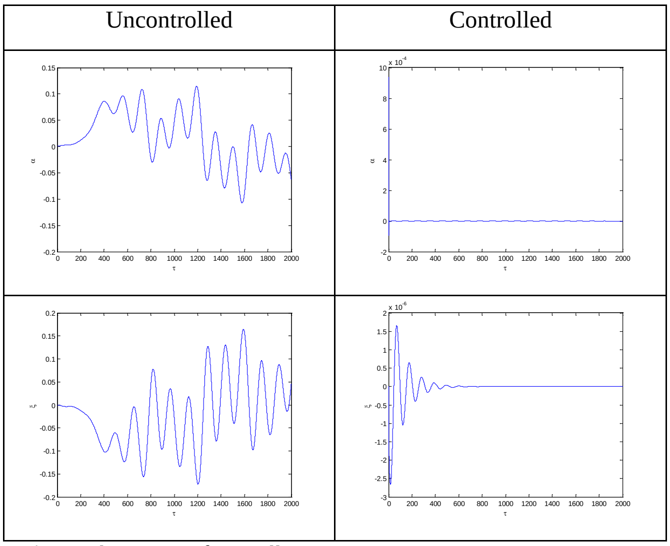

Fig. 5 *Achievement of controller at V=17.934*

4. Conclusion

In this work, an adaptive controller for non-linear two dimensional airfoil model is proposed. Nonlinear characteristics of flutter with different flow speed are analyzed. The simulated results show that the design controller can effectively suppress airfoil flutter, which provides a valuable reference to solve the active control problem of airfoil flutter.

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