FUZZY - ROBUST CONTROL OF A TWO-LINK ROBOT ARM

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Abstract: *Parameters of the robots are always changed due to the load being carried. Robust control is a method that considers the changes of control system performance related to the modification of system parameters. Stability and performance of the system can be well protected in case the change of system parameters does not affect the system. Even if there is several modified parameters, robust control system still provides the ability of control in a desired manner.*

In this work, parameters are made changeable and the upper limit of the uncertainty parameter is kept constant unlike other robust control studies. Control parameter is updated over time depending on the trigonometric functions. The values of the constant control parameters in trigonometric functions affect the performance of the system and it is quite difficult to find appropriate control parameter values. Logical fuzzy compensator is designed to find this parameter and investigated the effects on the tracking error of two-link robot.

Fuzzy Logic associated robust control methods developed using robust control has been compared through a computer simulation using the same trajectory and same model. Thanks to the designed fuzzy logic associated robust controller, robust control is improved and two-link robot's trajectory tracking error has been reduced to a very small value.

Keywords: Robot Manipulators, fuzzy logic control, robust control, adaptive control

1. Introduction

Slotine et al. [1] and Sciliano et al. [2] developed parameter estimation law for adaptive control of robot manipulators. After that, Spong [3] developed a robust control law based on the adaptive control law [1]. In robust control law [3], upper uncertainty bound is fixed and upper uncertainty limit is used as a control algorithm. Usage of the robust control creates a chattering with large tracking error in case of the uncertainty of parameter.

In order to reduce the tracking error, appropriate upper uncertainty bound limit should calculated well [4, 5, 6]. Burkan and Uzmay [7, 8], designed a new robust control rule for the robot face to uncertainty and stability of the system was guaranteed based on the Lypunov theory. As distinct from the earlier other earlier works, estimation of uncertainty bound is estimated as a function of exponential function depending on tracking error and robot kinematic. As a result, trajectory tracking error of the system is reduced. Fuzzy logic control has been studied by some researchers. The advantages of the fuzzy logic may be generated by fuzzy logic rules utilizing uncertain, time-varying, complex and simple solutions to bring the control of undefined systems, and experiences of the system control experts. Moreover it can be used for the nonlinear and unknown mathematical model systems. After Zadeh [9] and Mamdani [10], many research groups started to work on this particular topic and many studies were conducted in this area.

In this work, improvement of the robust control published in the earlier works [4, 5, 6] have been aimed. For this purpose, at first, an uncertainty estimation algorithm was designed for the robust controller [3] algorithm based on the Lyapunov theory, thus stability of uncertain system is guaranteed. In previous robust controllers [4, 5, 6], upper uncertainty bounds are updated in time. In this work, the parameters are adaptive while the uncertainty bound are kept constant. Control parameters are estimated depending on the trigonometric functions such as Cos and Sin functions. The values of the fixed control parameters in Cos and Sin functions significantly affect the system performance, however, it becomes too difficult to find the appropriate values. In order to find the appropriate control parameter, fuzzy logic based compensator was designed and the effects on the robot tracking errors were investigated. After computer simulations, it is seen that tracking performance of the system is improved.

2. Development Of A New Controller

The dynamic model of an n-link manipulator can be written as [3].

$$\begin{aligned} &\mathrm{M}(\theta)\ddot{\theta} + \mathrm{C}(\theta,\dot{\theta})\dot{\theta} + \mathrm{G}(\theta) = \tau \\ &\mathrm{Y}(\theta,\dot{\theta},\ddot{\theta})\pi = \tau \end{aligned} \tag{2.1}$$

where π is constant p-dimensional vector of robot parameters, θ denotes generalized coordinates, τ is the n-dimensional vector of applied torques (or forces), $M(\theta)$ is the nxn symmetric positive definite inertia matrix, $\mathrm{C}(\theta,\dot{\theta})\dot{\theta}$ is the n-dimensional vector of centripetal and Coriolis terms and $\mathrm{G}(\theta)$ is the n-dimensional vector of gravitational terms. The control parameters are defined as

$$\tilde{\theta}=\theta-\theta_d\,;\,\dot{\theta}_\mathrm{r} = \dot{\theta}_\mathrm{d} - \Lambda\tilde{\theta}\,;\,\sigma = \dot{\theta} - \dot{\theta}_\mathrm{r} = \dot{\tilde{\theta}} + \Lambda\tilde{\theta} \tag{2.2}$$

Based on the control parameters, the robust control law is defined as [3].

$$\begin{aligned} \tau_r &= \mathrm{M}_0(\theta)\ddot{\theta}_r + \mathrm{C}_0(\theta,\dot{\theta})\dot{\theta}_r + G(\theta) + Y(\theta,\dot{\theta},\theta_r,\dot{\theta}_r)u_p - K\sigma \\ &= Y(\theta,\dot{\theta},\theta_r,\dot{\theta}_r)(\pi_0 + u_1) - K\sigma \end{aligned} \tag{2.3}$$

Based on the control law (2.3) [3], let us define the control inputs u_2 in terms of the control vector (2.3) as

$$\begin{aligned} \tau &= \tau_0 + \mathrm{Y}(\theta,\dot{\theta},\dot{\theta}_\mathrm{r},\ddot{\theta}_\mathrm{r})(\mathrm{u}_1 + \mathrm{u}_2) \\ &= \mathrm{Y}(\theta,\dot{\theta},\dot{\theta}_\mathrm{r},\ddot{\theta}_\mathrm{r})(\pi_0 + \mathrm{u}_1 + \mathrm{u}_2) - \mathrm{K}\sigma \end{aligned} \tag{2.4}$$

where u_2 and u_1 is an additional control inputs that will be designed to achieve robustness to parametric uncertainty. Control parameters are defined as

$$\mathrm{u}_2 = -\hat{\pi} \tag{2.5}$$

$$\hat{\pi} = (\beta^2/\alpha)\cos\left(\int \alpha Y^T \sigma dt\right)\sin\left(\int \alpha Y^T \sigma dt\right) \tag{2.6}$$

$$u_1 = \begin{cases} -\rho \dfrac{Y^T\sigma}{\left\|Y^T\sigma\right\|} \quad If \quad \left\|Y^T\sigma\right\| > \varepsilon \\ -\dfrac{\rho}{\varepsilon} Y^T\sigma \quad If \quad \left\|Y^T\sigma\right\| \le \varepsilon \end{cases} \qquad (2.7)$$

Where ε > and β, α ЄR are adaptation gains. Substituting (2.4) into (2.1) and after some algebra yields

$$\begin{aligned} &\mathrm{M}(\theta)\dot{\sigma} + \mathrm{C}(\theta,\dot{\theta})\sigma + \mathrm{K}\sigma \\ &= \mathrm{Y}(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r)(\pi_0 - \pi) + u_1 + u_2) + \mathrm{K}\sigma \\ &= \mathrm{Y}(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r)(\tilde{\pi} + u_1 + u_2) + \mathrm{K}\sigma \end{aligned} \qquad (2.8)$$

It is assumed that there exists an unknown bound on parametric uncertainty such that.

$$\tilde{\pi} = (\pi_0 - \pi) \le \rho \qquad (2.9)$$

In previous studies [4], the $(\beta^2/\alpha)\cos(\int \alpha Y^T\sigma dt)\sin(\int \alpha Y^T\sigma dt)$ was used as estimation of uncertainty bound. In this work, the function $(\beta^2/\alpha)\cos(\int \alpha Y^T\sigma dt)\sin(\int \alpha Y^T\sigma dt)$ is used as a parameter estimation law. In order to derive the control law, the following Lyapunov function candidate is defined.

$$\mathrm{V} = \frac{1}{2}\sigma^T\mathrm{M}(\theta)\sigma + \frac{1}{2}\tilde{\theta}^T\mathrm{B}\tilde{\theta} + \frac{1}{2}\hat{\pi}^T\Phi^2\hat{\pi}\ ; \quad \mathrm{V} \ge 0 \qquad (2.10)$$

where $\mathrm{B} \in \mathrm{R}^{nxn}$ is a positive diagonal matrix, Φ is chosen as a pxp dimensional diagonal matrix changing in time. The time derivative of V along the system (2.10) is

$$\begin{aligned} \dot{\mathrm{V}} &= \sigma^T\mathrm{M}(\theta)\dot{\sigma} + \sigma^T\frac{1}{2}\dot{\mathrm{M}}(\theta)\sigma + \tilde{\theta}^T\mathrm{B}\dot{\tilde{\theta}} \\ &+ \hat{\pi}^T\Phi^2\dot{\hat{\pi}} + \hat{\pi}^T\Phi\dot{\Phi}\hat{\pi} \end{aligned} \qquad (2.11)$$

then

$$\begin{aligned} \dot{\mathrm{V}} &= \sigma^T[\frac{1}{2}\dot{\mathrm{M}}(\theta) - \mathrm{C}(\theta,\dot{\theta})]\sigma - \sigma^T\mathrm{K}\sigma + \tilde{\theta}\mathrm{B}\dot{\tilde{\theta}} \\ &+ \mathrm{Y}(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r)(\tilde{\pi} + u_1 + u_2) + \hat{\pi}^T\Phi^2\dot{\hat{\pi}} + \hat{\pi}^T\Phi\dot{\Phi}\hat{\pi} \end{aligned} \qquad (2.12)$$

Taking $\mathrm{B} = 2\Lambda\mathrm{K}$, using the property $\sigma^T[\dot{\mathrm{M}}(q) - 2\mathrm{C}(q,\dot{q})]\sigma = 0 \ \forall\sigma \in \mathrm{R}^n$ and Equation (2.12) becomes

$$\begin{aligned} \dot{\mathrm{V}} &= -\dot{\tilde{\theta}}^T\mathrm{K}\dot{\tilde{\theta}} - \tilde{\theta}^T\Lambda\mathrm{K}\Lambda\tilde{\theta} + \sigma^T\mathrm{Y}(u_1 + u_2) + \sigma^T\mathrm{Y}\tilde{\pi} \\ &+ \hat{\pi}^T\Phi^2\dot{\hat{\pi}} + \hat{\pi}^T\Phi\dot{\Phi}\hat{\pi} \end{aligned} \qquad (2.13)$$

The time varying function Φ is defined as [4].

$$\Phi = diag(\frac{1}{\beta_i \mathrm{Cos}(\alpha_i \int \mathrm{Y}^T\sigma dt)_i}) \qquad (2.14)$$

$$\hat{\pi}_i = (\beta_i^2/\alpha_i)\mathrm{Sin}(\alpha_i \int \mathrm{Y}^T\sigma dt)_i \mathrm{Cos}(\alpha_i \int \mathrm{Y}^T\sigma dt)_i \qquad (2.15)$$

Substituting Eq. (2.14) and (2.15) into Eq.(2.13), the last term in Eq. (2.13) will be

$$\hat{\pi}^T\Phi^2\dot{\hat{\pi}} + \hat{\pi}^T\Phi\dot{\Phi}\hat{\pi} = \hat{\pi}^T\mathrm{Y}^T\sigma \qquad (2.16)$$

As a result, the time derivative of the Lyapunov function is written as.

$$\begin{aligned} \dot{\mathrm{V}} &= -\dot{\tilde{\theta}}^T\mathrm{K}\dot{\tilde{\theta}} - \tilde{\theta}^T\Lambda^T\mathrm{K}\Lambda\tilde{\theta} + \sigma^T\mathrm{Y}(u_1 + u_2) \\ &+ \sigma^T\mathrm{Y}\tilde{\pi} + \sigma^T\mathrm{Y}\hat{\pi} \end{aligned} \qquad (2.17)$$

Control parameters are defined in Equation (2.11) such that $u_1 = -\hat{\pi}$. Substitution the control parameters $u_1 = -\hat{\pi}$, and the following Equation is obtained.

$$\dot{\mathrm{V}} = -\dot{\tilde{\theta}}^T\mathrm{K}\dot{\tilde{\theta}} - \tilde{\theta}^T\Lambda^T\mathrm{K}\Lambda\tilde{\theta} + \sigma^T\mathrm{Y}u(t)_2 + \sigma^T\mathrm{Y}\tilde{\pi} \qquad (2.18)$$

Eq. (2.18) is the same as would be robust control law in [3] and the rest of the proof is given in [3].

3. Fuzzy-Robust Controller

Fuzzy logic controller has two inputs and two outputs. As input values, the first and second manipulator of the orbital tracking error values (e1, e2), as the value of output torque to be applied to the joints used to calculate the values of α and β values were obtained.

Input values (e1, e2): figure 3.1, figure 3.2

Output values (α, β): figure 3.3, figure 3.4

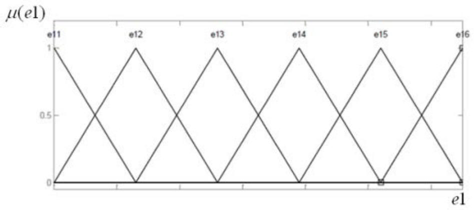

Figure 3.1 Membership function for tracking error of first manipulator

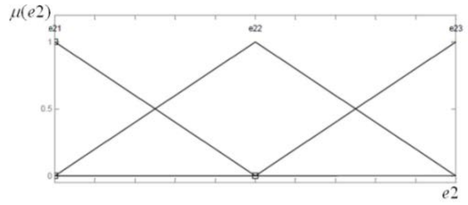

Figure 3.2 Membership function for tracking error of second manipulator

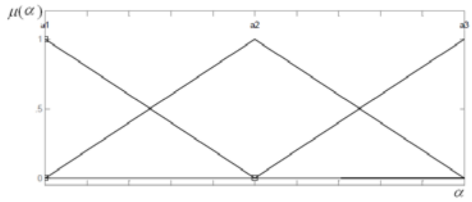

Figure 3.3 Membership function of α

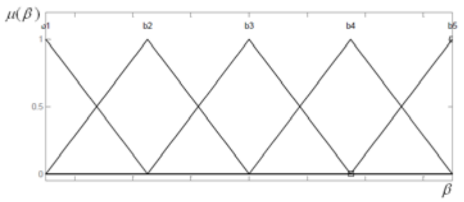

Figure 3.4 Membership function of β

Found by trial and error limit values as follows:

e1 :	[-0.8 ; 0.5]
e2 :	[-0.3 ; 0.7]
α :	[0.5 ; 1.5]
β :	[1 ; 15]

Table 1: Rule table for fuzzy logic controller.

e2 \ e1 (α / β)	e11	e11	e13	e14	e15	e16
e21	α1 β5	α3 β5	α2 β2	α2 β5	α2 β5	α2 β5
e22	α2 β4	α2 β4	α3 β3	α3 β5	α1 β4	α3 β4
e23	α3 β3	α1 β1	α1 β4	α1 β5	α3 β3	α3 β1

Using (If ... and ... Then ...) structure of the rule table is created and is shown at Table 1.

Defuzzification method is used in the Centroid. Coefficients α and β in order to improve the performance of error found by fuzzy logic and was added to the system in Figure 3.5

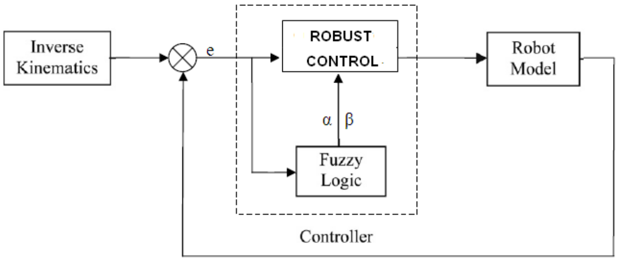

Figure 3.5 Block diagram of the proposed fuzzy-robust controller.

4. Simulation Results

In order to investigate the controller performance, the proposed control law is applied to a two-linked manipulators.

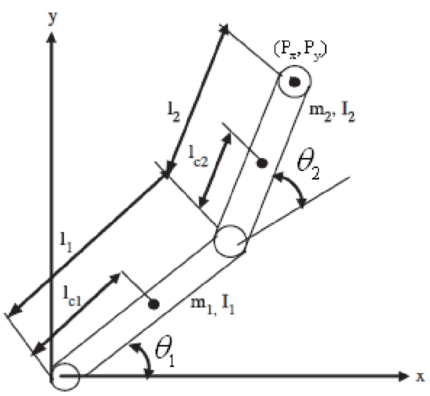

Figure 4.1 Two dimensional planer manipulators [3].

Robot parameters are given as follows.

$$\begin{aligned} \pi_1 &= m_1 l_{c1}^2 + m_2 l_1^2 + I_1 \\ \pi_2 &= m_2 l_{c2}^2 + I_2 \\ \pi_3 &= m_2 l_1 l_{c2} \\ \pi_4 &= m_1 l_{c1} \\ \pi_5 &= m_2 l_1 \\ \pi_6 &= m_2 l_{c2} \end{aligned} \tag{4.1}$$

$$M(\theta) = \begin{bmatrix} \pi_1 + \pi_2 + 2\pi_3 \cos(\theta_2) & \pi_2 + \pi_3 \cos(\theta_2) \\ \pi_2 + \pi_3 \cos(\theta_2) & \pi_2 \end{bmatrix} \tag{4.2}$$

$$C(\theta, \dot{\theta}) = \begin{bmatrix} -\pi_3 \sin(\theta_2)\dot{\theta}_2 & -\pi_3 \sin(\theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \\ \pi_3 \sin(\theta_2)\dot{\theta}_1 & 0 \end{bmatrix} \tag{4.3}$$

$$G = \begin{bmatrix} g(\pi_4 + \pi_5)\cos(\theta_1) + g\pi_6 \cos(\theta_1 + \theta_2) \\ g\pi_6 \cos(\theta_1 + \theta_2) \end{bmatrix} \tag{4.4}$$

Table 2: Parameters of the unloaded arm [3].

m1	m2	l1	l2	lc1	lc2	I1	I2
10	5	1	1	0.5	0.5	10/12	5/12

Table 3: π_i for the unloaded arm [3].

π1	π2	π3	π4	π5	π6
8.33	1.67	2.5	5	5	2.5

Table 4: Nominal parameter vector π_0 [3].

π_{01}	π_{02}	π_{03}	π_{04}	π_{05}	π_{06}
13.33	8.96	8.75	5	10	8.75

Table 5: Uncertainty bound [3].

ρ1	ρ2	ρ3	ρ4	ρ5	ρ6
5	7.29	6. 25	0	5	6.25

For the simulation, the trajectory is chosen as 0.5cos (0.5πt)-0.5 for each joint.

Proposed and known robust controller [3] was applied to the same model under the same conditions with the same trajectory and obtained results are given in Figures 4.2-4.3.

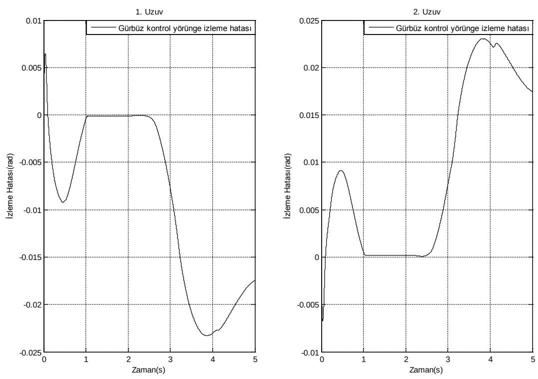

Figure 4.2 Tracking error for known robust controller [3] with the parameters Λ=dia(50 50), K=diag(15 15)

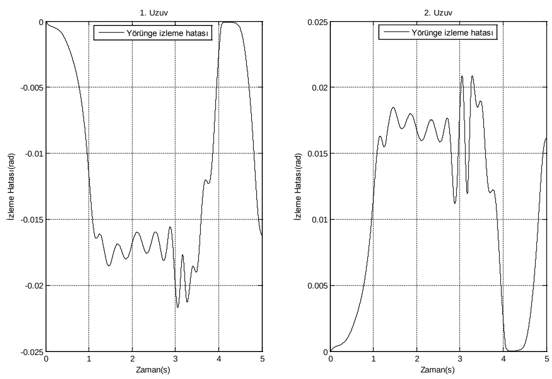

Figure 4.3 Tracking error for proposed robust controller with the parameters. Λ=diag(50 50), K=diag(15 15), α= 1.5, β=1.7

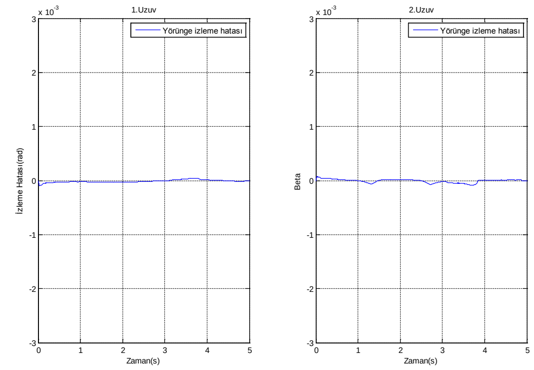

Figure 4.4 Tracking errors for the proposed fuzzy-robust controller with the parameters. Λ=diag(50 50), K=diag(15 15)

As seen from the Figure 4.4, the tracking error was reduced. By using the fuzzy compensators, α and β values are updated in time and the most appropriate values of α and β are updated. As a result, tracking error was reduced. The time history of the α and β are given in Figure 4.5

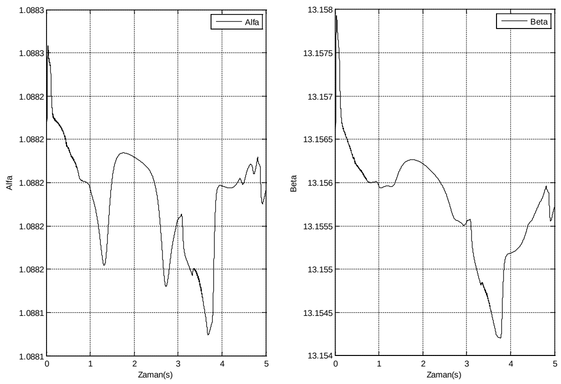

Figure 4.5 α and β values change over time

According to trajectory tracking error, the best values of alpha and beta is obtained by fuzzy logic and is entered the system.

As a result, very small tracking error is obtained. Combination results of the proposed fuzzy-robust and known robust controller are given in Figure 4.6.As seen from Figure 4.6, the proposed fuzzy-robust controller gives much better results.

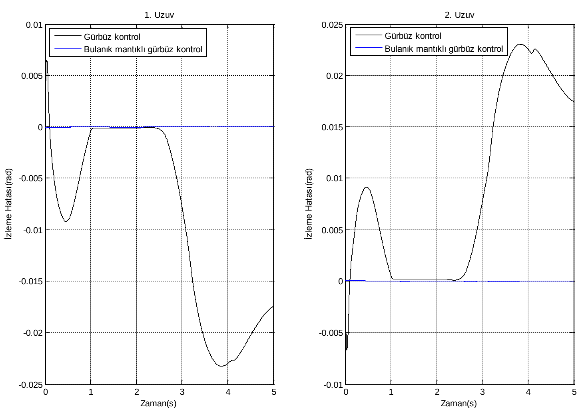

Figure 4.6 Tracking error for known robust controller [3] and the proposed fuzzy-robust controller with the parameters Λ=diag(50 50), K=diag(15 15).

4. Conclusion

For comparisons, same model and same trajectory were used for computer simulations. To investigate the effect of control parameters, α and β the values of K and Λ are kept constant while α and β are changed.

From the obtained results, it is seen that the tracking error of robust control is large. This tracking error is significantly reduced by the suggested adaptive control algorithm. It is observed that tracking error is changed according to the values of α and β however, these values are fixed. Selection of these values is quite difficult, and also the tracking error is not exactly enhanced due to these fixed values, α and β. Therefore, fuzzy logic based robust control algorithm is developed to improve this control law. The obtained results are given in Figure 4.6. As can be seen from the figures, the tracking error is reduced to very small levels because of the fuzzy logic based controller.

5. References

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