

# ANALYSIS OF A MECHANISM FOR SHEET CUTTING

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**Abstract:** Significant number of mechanisms for sheet cutting are already designed and produced. In spite of the fact that all of these mechanisms are suitable for cutting, all of them have some advantages and disadvantages. The aim of the paper is to present a new type of cutting mechanism. It is a crank mechanism with the cutting tool. The geometric and kinematic properties of the mechanism are analyzed. As the result the constraints of the mechanism are defined. The force and torque distribution of the elements of the mechanism is investigated. Based on the known cutting force the necessary input torque of the input element of the mechanism is calculated. In the paper a numerical example is considered. Based on the obtained results the discussion about properties of such mechanisms is given. Besides, the priority for application of such mechanism in comparison to the already known is motivated.

**Keywords:** CRANK MECHANISM, DRIVING TORQUE, KINETO-STATIC ANALYSIS, CUTTING FORCE

## 1. Introduction

Nowadays, there is significant number of mechanisms which are used for cutting sheets (see for example [1] and [2]). The most of them are designed to satisfy some specific conditions of cutting which depend on the properties of the sheet: its mechanic characteristics, type of the material of the sheet, dimensions of the sheet, its geometry, etc. Besides, the space in which the machine has to be put is very often a limitation for applying of some already produced mechanisms. Nevertheless, the economic efficiency of the mechanism has also to be included into consideration. The designers have a heavy task to satisfy all the conditions and requirements to design or take the already produced mechanism which will be the optimal one.

Our task is to give the design of the mechanism for cutting polymer sheets which will be incorporated into the system for packaging in alimentary industry and pharmacy. The sheet has small thickness but high rigidity. The cutting process has to be in straight line. The mechanism has to work continually and every working cycle has to cut the new sheet. The mechanism has to be simple but efficient in application and to have low price. In the paper [3] a mechanism for cutting sheets is considered. The mechanism fulfills the cutting task, but it is very complex and with large dimensions. In [4] and [5] a mechanism which contains two piston mechanisms is considered. The cutting tool has a straight-line motion. Using the published results we suggest a model of crank mechanism with a cutting tool. For the known driving torque, the cutting force is determined. The influence of system parameters on the cutting force of the mechanism is analyzed.

## 2. Model of the mechanism

In Fig.1 the suggested cutting mechanism is shown. It contains the driving bar AC fixed in A and in C connected with a slider. The slider moves along the bar BD. The bar is fixed at the end B and at the other end D the cutting tool is mounted. The length of AC is  $l_2$  and its weight is  $G_3$ . The weight of the slider is  $G_2$  and of the cutting tool is  $G$ . The length of the bar BD is  $l$  and its weight is  $G_1$ . It is assumed that the  $G_1$  and  $G_3$  act in the middle of corresponding bars. The driving torque  $M$  is constant and it acts on the driving bar AC. When the cutting tool is in contact with the sheet the cutting force  $F$  acts. Our aim is to compute the variation of the cutting force of the cutting tool for the constant driving torque  $M$ . It is assumed that the cutting process is slow enough and the system is considered as quasi-static.

### 2.1 Geometry of the mechanism

The angle position of the driving bar AC is  $\theta$ , while the position of the bar BD is  $\varphi$ . Analyzing the geometry of the mechanism the relation between these two angles is

$$c \tan \varphi = c \tan \theta + \frac{a}{l_2 \sin \theta} \quad (1)$$

where  $a$  is the constant distance between two fixed points A and B. Using the expression for the position of the slider C

$$BC = \sqrt{a^2 + l_2^2 + 2al_2 \cos \theta} \quad (2)$$

the relation (1) gives

$$\sin \varphi = \frac{l_2}{BC} \sin \theta, \quad \cos \varphi = \frac{a + l_2 \cos \theta}{BC} \quad (3)$$

The relations (2) and (3) will be of interest for further consideration.

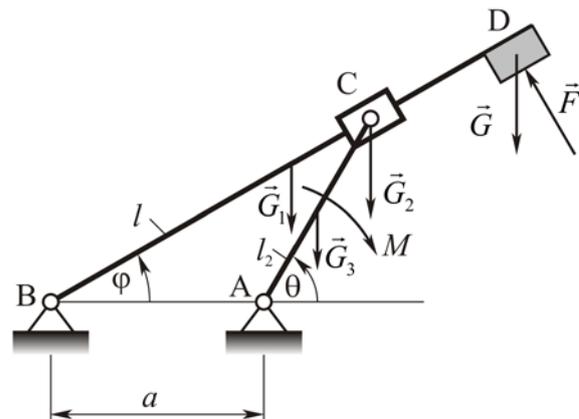


Fig. 1 Model of the sheet cutting mechanism and the load distribution.

For analysis of the geometry variation let us introduce the dimensionless value  $L_2 = l_2/a$  and

$$BC = a \sqrt{1 + L_2^2 + 2L_2 \cos \theta} \quad (4)$$

Then, the angle variation is

$$c \tan \varphi = c \tan \theta + \frac{1}{L_2 \sin \theta} \quad (5)$$

It is evident that for the same value of the driving angle  $\theta$  the angle  $\varphi$  is higher for higher values of  $L_2$ .

### 2.2 Kinetostatic analysis of the mechanism

In Fig.1 the load distribution of the mechanism is shown. Using the previous assumption, the inertial forces are neglected and only the input torque, output cutting force and weights of elements are taken into consideration. After decomposition (see Fig.2) the inner force  $F_c$  and reactions in A and B are shown.

For the driving bar with slider the inner force  $F_c$  is

$$F_c = \frac{1}{l_2 \cos(\theta - \varphi)} \left( M + G_2 l_2 \cos \theta + G_3 \frac{l_2}{2} \cos \theta \right) \tag{6}$$

The components of the reaction in A are

$$F_{Ax} = F_c \sin \varphi, \quad F_{Ay} = G_2 + G_3 - F_c \cos \varphi \tag{7}$$

Using the static relations for the bar BD we have

$$F = \frac{1}{l} \left( F_c BC + G l \cos \varphi + G_1 \frac{l}{2} \cos \varphi \right) \tag{8}$$

Substituting (4) and (6) into (8) it is

$$F = \frac{\sqrt{1 + L^2 + 2L_2 \cos \theta}}{L \cos(\theta - \varphi)} \left( \frac{M}{aL_2} + (G_2 + \frac{G_3}{2}) \cos \theta \right) + (G + \frac{G_1}{2}) \cos \varphi \tag{9}$$

where  $L=l/a$ . The force  $F$  depends not only on the input torque but also on the weight of the elements of the mechanism and geometric properties. If the distance between the fixed points A and B is longer, the cutting force is smaller. Besides, it is obvious that for the longer BD bar the force is smaller. It means that the construction has to be compact without long bars and large distance between the fixed points.

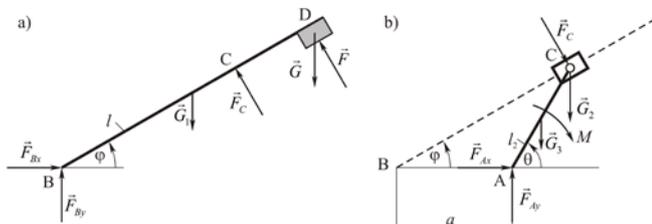


Fig.2 Decomposition of the mechanism: a) Bar BD with cutting tool, b) Driving bar AC with slider.

Analysing Fig.2b components of the reaction force in B are calculated

$$\begin{aligned} F_{Bx} &= (F - F_c) \sin \varphi, \\ F_{By} &= (G + G_1) - (F - F_c) \cos \varphi. \end{aligned} \tag{10}$$

Based on (10) the link in B is designed.

### 3. Result analysis and discussion

Our aim is to obtain a strong cutting force which would be approximately constant for certain angle interval and suitable to cut the sheet approximately straight along the thickness of the sheet. For the numerical data  $M=100$  kNm,  $G=G_2=10$  N,  $G_1=G_3=0$ ,  $L=1$ ,  $a=1$  m and variable value of  $L_2$  the cutting force variation and the position of the cutting tool in x and y direction is calculated and presented in Table 1.

Table 1: Cutting force  $F$  as the function of the driving angle  $\theta$  for various values of  $L_2$ .

$L_2=1/3$

$\Theta$ (o)	$L \cos \varphi$	$L \sin \varphi$	F
-90	2.8460	-0.9487	342.82
-60	2.9122	-0.7206	185.93
-45	2.9460	-0.5620	165.51
-30	2.9752	-0.3850	154.76
0	3	0	147.78
30	2.9752	0.3850	154.76
45	2.9460	0.5620	165.51

60	2.9122	0.7206	185.93
90	2.8460	0.9487	342.82

$L_2=1/2$

$\Theta$ (o)	$L \cos \varphi$	$L \sin \varphi$	F
-90	2.6833	-1.3416	96.667
-60	2.8347	-0.9820	88.705
-45	2.9026	-0.7782	79.517
-30	2.9554	-0.5156	67.946
0	3	0	115
30	2.9554	0.5156	117.59
45	2.9026	0.7582	121.58
60	2.8347	0.9820	129.03
90	2.6833	1.3416	175.61

$L_2=1$

$\Theta$ (o)	$L \cos \varphi$	$L \sin \varphi$	F
-90	2.1213	-2.1213	73.738
-60	2.5980	-1.5000	78.660
-45	2.7716	-1.1480	80.620
-30	2.8978	-0.7765	82.099
0	3	0	83.33
30	2.8978	0.7765	82.10
45	2.7716	1.1480	80.62
60	2.5980	1.5000	78.66
90	2.1213	2.1213	73.74

$L_2=2$

$\Theta$ (o)	$L \cos \varphi$	$L \sin \varphi$	F
-90	1.3410	-2.6838	46.139
-60	2.2678	-1.9640	58.893
-45	2.2588	-1.5163	63.640
-30	2.8172	-1.0312	67.137
0	3	0	70
30	2.8172	1.0312	67.137
45	2.2588	1.5163	63.640
60	2.2678	1.9640	58.593
90	1.3410	2.6838	46.139

Analyzing data given in Table 1 it is seen that the cutting force is higher for  $L_2 < 1$  than for  $L_2 > 1$  (for  $L_2 = 1$  the length of the driving bar AC and the distance between fixed points A and B is equal). The

smaller the value of  $L_2$  in comparison to 1, the force is stronger. Otherwise, the higher the value of  $L_2$  in comparison to 1 the force is smaller. For  $L_2 < 1$  stronger force acts for larger angle than for  $\theta = 0$ . For  $L_2 > 1$  maximal force is for  $\theta = 0$  and the force decreases for higher values of angle  $\theta$ .

For comparison of the mechanisms with different geometric properties the following criteria are introduced:

1. The force variation ratio

$$F_{\%} = 100 \frac{F_m - F_0}{F_0} \quad (11)$$

where the difference between the force for two angles  $F_m$  and  $F_0$  are divided with smaller value  $F_0$ .

2. The variation of the cutting tool position in x and y direction

$$X = x_m - x_0 \quad Y = y_m - y_0 \quad (12)$$

where

$$x = L \cos \theta, \quad y = L \sin \theta \quad (13)$$

and  $x_m, y_m, y_0$  and  $x_0$  are corresponding coordinates for two angles.

In Table 2 the values of X, Y and  $F_{\%}$  for various values of  $L_2$  are presented. The boundary values are  $\theta = 0$  and  $\theta = 60^\circ$ .

**Table 2:** Values of  $F_{\%}$ , X and Y for various values of  $L_2$ .

$L_2$	$F_{\%}$	X	Y
1/3	25.815	0.0878	0.721
1/2	12.200	0.1653	0.982
1	5.5174	0.4020	1.500
2	18.860	0.7322	1.964

Analyzing the results in Table 2 it is seen that the cutting force variation for the same angle interval is higher for smaller values of  $L_2$  than for  $L_2 = 1$ . The same conclusion is obtained for the bar whose length  $L_2$  is higher than 1.

Based on the X values in the Table 2 we conclude that the motion of the cutting tool is approximately in straight line and  $X \ll 1$  only for the case when  $L^* = L_2 \ll 1$ . The higher the value of  $L_2$  but satisfying the relation  $L^* < L_2 < 1$  the cutting line becomes an arc. For these mechanisms the straight line cutting is satisfied only for small cutting angle.

For  $L_2 = 1/3$  the force variation is high: the higher the angle  $\theta$ , the higher the value of force. The parameter X is small in spite of the fact that the angle interval is 60 degrees.

For  $L_2 = 1/2$  the force variation is smaller and the force is smaller than for  $L_2 = 1/3$ . X and T parameters are higher than for the previous case.

For  $L = 1$  the parameter of force variation is the smallest in comparison to other systems. However, it has to be mentioned that the maximal force exist for  $\theta = 0$  and is smaller for higher values of  $\theta$ . X and Y parameters are increasing with  $L_2$ .

For  $L_2 = 2$  the force parameter is increasing with increasing of  $L_2$ . The force is smaller than for  $L_2 = 1$ . Besides, the force changes significantly with  $\theta$ . The force has higher value for  $\theta = 0$  than for  $\theta = 60^\circ$ . The geometric parameters X and Y are increasing with  $L_2$ .

The mechanism is convenient to give the cutting force which is approximately constant only for small cutting angle.

Using the relations (7) and (10) reactions in A and B are computed as

$$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2}, \quad F_B = \sqrt{F_{Bx}^2 + F_{By}^2} \quad (14)$$

According to (6) and (14) reactions  $F_A, F_B$  and  $F_C$  as functions of the input angle  $\theta$  are presented in Table 3.

**Table 3:** Reactions  $F_A, F_B$  and  $F_C$  as the function of the driving angle  $\theta$  for various  $L_2 = 1/2$ .

$\theta$ (o)	$F_A$	$F_B$	$F_C$
-90	438.29	280.58	447.21
-60	261.76	151.64	271.19
-45	230.32	128.10	239.98
-30	212.35	114.47	222.20
0	200	105	210
30	212.35	114.47	222.20
45	230.32	128.10	239.98
60	261.76	151.64	271.19
90	438.29	280.58	447.21

It can be seen that the variation of the reactive forces has the same tendency as the change of the cutting force F. The reactive forces are even two times higher than the force F.

#### 4. Conclusion

In this paper a sheet cutting mechanism is analyzed. For the known input torque the cutting force is computed. It is concluded that the relation between the length of the driving bar and the distance between the fixed points A and B of the bars have significant influence on the cutting properties of the mechanism. Using this ratio the mechanism are divided into three groups: 1) the ratio is much smaller than 1, 2) ratio is smaller than 1 and 3) the ratio is higher than 1.

1. The mechanism with driving bar which is much shorter than the distance between the fixed bar points is convenient for cutting sheets of symmetric composites or multilayer sheets where the boundary layers need stronger cutting force than those in the middle of the sheet. The motion of the tool is approximately straightforward during cutting. The thickness of the cutting sheet may match the driving angle  $\theta = 60^\circ$ .

2. Mechanism with the driving bar shorter than the distance between the fixed points of bar gives straight line cutting only for small cutting angles. Then the cutting force is much smaller than in the previous case but approximately constant. The thickness of the sheet has to be small. As the difference between the force at the boundary and in the middle are approximately equal, we recommend the mechanism to be applied for cutting sheets of a homogenous materials.

3. Mechanism, where the driving shaft is long and longer than the distance between the fixed points of the bars, is suitable to be applied for cutting only thin sheets which need not a strong cutting force. The mechanism gives the straight line cutting for small interval of cutting angle. Otherwise, this mechanism is recommended for cutting those symmetrical structures where the boundary layers are soft, but in the middle the strong cutting force is necessary. Namely, increasing the driving angle from zero the force decreases from the maximal to the minimal value. For wide angle cutting the cutting line is an arc.

For all of previously mentioned cutting mechanisms it is common that the cutting force is smaller if the length of the cutting bar is longer and the weight of elements of the mechanism, including the cutting tool, is larger.

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