

THE INFLUENCE OF THE TOOTH PROFILE SHAPE ON THE STRESS-STRAIN STATE IN THE GEAR

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Abstract: The shape of the tooth profile affects the stress-strain state in the gear. In the case of a fatigue failure the stress state is a decisive criterion for the lifetime of the gear. The shape of the tooth flank affects the magnitude of the contact pressure in the contact of the meshing teeth. The consequence of which are surface cracks and pitting. The shape of the tooth root influences the magnitude of the root stress, which, when limit is exceeded, leads to root cracks and teeth breakage. Many different types of gearing are known, but in the practice most widely used is the involute one. Other types of gearing become interesting especially when polymer materials are being used. If the gears are injection molded the type of gearing does not affect the cost of the tool. In the case of metal gears, standardized tools for involute gearing make the use of other special types of gearing economically unjustified. Our research is focused on the S-gears, which got their name from the S-shaped path of contact. The paper presents the research of how the defining parameters of S-gears impact the stress-strain state in the gear. This was done using a numerical model which simulates gear meshing. The stress state of two different types of S-gears was compared with the stress state in an involute gear of the same dimensions (same module, number of teeth and width). It was found that with a proper choice of gearing type we can improve the load bearing capability of the gear pair. With use of our numerical model we have also analyzed the impact of the tip relief on the stress state. The numerical model was validated for the case of meshing steel involute gears, where we can compare the results of the model with the results according the ISO 6336 calculation. A good match between the results of the model and results according the standard was obtained. After validation the same numerical model was used for the calculation of the stress-strain state in S-gears. This was calculated then for metal and polymer (POM/PA) gear pairs.

Keywords: S-GEARS, POLYMER GEARS, NUMERICAL SIMULATION, STRESS-STRAIN STATE

1. Introduction

Gears are often used machine elements for the transfer of mechanical power. With the increasing supply of polymeric materials and their ever-increasing mechanical properties, the use of polymer gears increases. The reason for the growing use of polymer gears are some of the advantages when comparing them to metal gears. The most important ones are the lower mass, cheaper mass production, easier production, operation without lubrication and better dampening of the vibrations. Partially crystalline thermoplasts are mainly used for the production of polymer gears. Various reinforcing fibers can be added to these materials as well as materials that reduce wear and improve sliding properties, e.g. PTFE, MoS₂ [1-4].

There is a valid international standard ISO 6336 [5] for the conversion of steel involute gears. Also, the German standard DIN 3990 [6] is often used. According to both of these standards, the conversion is divided into the calculation of the root and flank strength, as well as the calculation of thermal scuffing. There is no valid international standard yet for the conversion of polymer gears. Most commonly used is the recommendation VDI 2736 [7]. In the recommendation VDI 2736, the calculation of tooth root and flank strength is summarized and simplified according to DIN 3990. The recommendation offers a very limited set, of the necessary data for the calculation, as these are defined only for some basic engineering polymer materials. Existing calculation models apply only to involute gears. Our work will focus on the research of S-gears, which in certain conditions have better properties than the involute ones. This means a certain competitive advantage. Thus, the problem arises how to reliably dimension the polymer S-gears.

Hlebanja [8] proposed the shape of a tooth flank that defines the S-shaped path of contact and has some advantages over the involute flank profile. The main advantages are the convex/concave contact at the beginning and end of the meshing and consequently the smaller flank stresses, smaller sliding speeds, more favourable rolling/sliding ratio, and consequently less losses. The S-gears roots are wider than the involute ones at the same normal module, leading to higher root strength, and it is possible to produce gears with a much smaller number of teeth - at least 4 teeth. Hlebanja et al. [8-10] confirmed the advantages of S-gears in several studies, comparing them to the involute ones. They tested and compared the lifetime of steel involute and S-gears and analyzed the damage mechanisms. Kulovec and Duhovnik [11] analyzed the influence of

individual parameters on the S-gear tooth profile shape. Duhovnik et al. [12] tested the lifespan of injection molded polymer S-gears and compared the results of the tests with polymer involute gears. They found that the S-gears are sensitive to the quality of the manufacturing, for which, in the case of injection molding, a well-controlled technological process is required. In one of our previous works we experimentally compared the lifetime of the machine cut S and involute gears. S-gears have shown a longer lifetime [13]. The aim of our work presented in this article was to use the numerical methods to study the influence of the tooth shape on the stress-strain state in the gear.

2. Methodology

Gear geometry

The S-gear profile shape depends on two parameters, namely the size factor a_p and the exponent n [8]. The stress state was calculated for two different S-gear geometries and compared to the involute one. The parameters of the analyzed geometries are given in Table 1, and the analyzed teeth geometries are shown in Fig. 1.

Table 1: Specification of the gear parameters. *Note: For the S-gears this is the initial pressure angle at the kinematic point C. In the case of S-gears the pressure angle is variable along the path of contact.

Parameter	involute	S-gear 1	S-gear 2
module [mm]	1	1	1
number of teeth	20	20	20
width [mm]	6	6	6
pressure angle [°]	20	18*	23*
exponent - n	/	2.05	1.3
size factor - a_p	/	1.5	1.8
reference diameter [mm]	20	20	20
tip diameter [mm]	22	22	22
root diameter [mm]	17.30	17.70	17.60
center distance [mm]	20	20	20
contact ratio	1.557	1.383	1.352

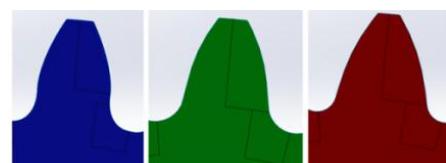


Fig. 1 The geometry of analyzed gears: involute gear (blue), S-gear 1 (green), S-gear 2 (red)

Numerical model

Due to the nature of the problem, the numerical model was set in 2D, taking into account the plane stress state. This reduces the use of computer resources and the calculation times are much shorter. The time it takes to solve the problem on a better desktop computer is approximately 20 minutes, which also makes it easy to use the model in practice. The advantage of the modeling in 2D is also the possibility to obtain a denser mesh in the contact area of the teeth, Fig. 2, without making solving times too long. Therefore, we expect a more accurate calculation of the flank stresses. We modeled the entire gear body and only 5 teeth, since the load is cyclically the same for each tooth. Stresses are examined on the central tooth, which passes through all the characteristic points of meshing.

In order to validate the model, in the first step we simulated the meshing of steel gears without considering the friction between the tooth flanks. This is an example that can be compared with the calculation of the stress according to ISO 6336. After the standard calculation, the maximum values of the root and flank stresses that occur during meshing are calculated. With our numerical model, we can calculate the stresses at all points of contact. According to ISO 6336, the actual root stress in the gear is calculated by the equation:

$$\sigma_F = \sigma_{F0} \cdot K_A \cdot K_V \cdot K_{F\beta} \cdot K_{F\alpha}$$

The nominal tooth root stress σ_{F0} is calculated by the equation:

$$\sigma_{F0} = Y_{Fa} \cdot Y_{Sa} \cdot Y_\epsilon \cdot Y_\beta \cdot Y_B \cdot \frac{F_t}{b \cdot m}$$

where: Y_{Fa} is the form factor, Y_{Sa} is the stress correction factor, Y_ϵ is the contact ratio factor, Y_β is the helix angle factor, Y_B is the rim thickness factor, F_t is the nominal tangential force, b is the gear width and m is the normal module.

The actual tooth flank stress σ_{H0} is calculated by the equation:

$$\sigma_H = \sigma_{H0} \cdot \sqrt{K_A \cdot K_V \cdot K_{H\beta} \cdot K_{H\alpha}}$$

The nominal tooth flank stress σ_{H0} is calculated by the equation:

$$\sigma_{H0} = Z_H \cdot Z_E \cdot Z_\epsilon \cdot Z_\beta \cdot \sqrt{\frac{F_t}{b \cdot d_1} \cdot \frac{u+1}{u}}$$

where: Z_H is the zone factor, Z_E is the elasticity factor, Z_ϵ is the contact ratio factor and Z_β is the spiral angle factor. The application factor K_A takes into account the overloads during the operation, the dynamic factor K_V takes into account the internal dynamic forces and factors $K_{F\beta}$, $K_{F\alpha}$, $K_{H\beta}$, $K_{H\alpha}$ take into account the manufacturing accuracy on the load distribution between the teeth. These effects are not considered in our numerical model, so the calculated stresses are compared with the nominal stresses σ_{F0} and σ_{H0} . According to the recommendation VDI 2736, the same equations are used to calculate the nominal root and flank stresses. The difference in the calculation of the actual stresses is that the coefficients K_A , K_V , $K_{F\beta}$, $K_{F\alpha}$, $K_{H\beta}$, $K_{H\alpha}$ are not defined for polymer gears, but rather it is recommended to take a factor $K_F \approx K_A$, the value of which is $1 - 1.25$.

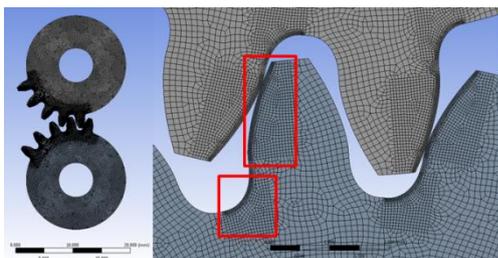


Fig. 2 Geometry of the numerical model and presentation of the mesh density

For all gear pairs analyzed, a load of 0.4 Nm was simulated. When a good match between the results of our numerical model and the ISO 6336 calculation was achieved, we upgraded the simulation by modeling friction in the contact between the meshing flanks. This is not taken into account by the standard ISO 6336. Steel gears are mostly lubricated with oil, so the influence of friction on the stress is also negligible. The frictional coefficient $\mu = 0.1$ was prescribed when modeling the frictional contact between the steel gears. Polymer gears often operate without additional lubrication, in which case the friction affects the stress in the gear. In order to calculate the stress state in the polymer gear, the same numerical model was used, which was previously validated in the case of steel gears. The material combination POM/PA6 was analyzed, where the drive gear was made from POM, and driven from PA6. This material combination was tested in our previous works [12, 13]. Due to the short duration of the load, the polymer material was modeled as linear elastic. The material parameters considered were: $E_{POM} = 2800$ MPa, $E_{PA6} = 3500$ MPa, $\nu_{POM} = 0.35$, $\nu_{PA6} = 0.4$. The friction coefficient $\mu = 0.28$ was prescribed for this material combination.

3. Results

Steel gear pair, frictionless contact

When simulating the meshing of the involute gear pair, the stress pattern (Fig. 3) corresponds to the theoretical stress, which is found in the literature [14]. The stress at the tooth root and on the flank appears at the initial meshing point A. When the gears are in mesh, the contact point moves towards the tip of the tooth. In point B, the single-tooth contact zone begins, where the previous pair of teeth leaves the meshing. In the area of single-tooth contact, the stresses increase rapidly, as the total load is transmitted only over one pair of teeth. The more the contact point moves towards the tip of the tooth, the longer is the lever and consequently the greater the root stress. This reaches the maximum value in the point D, which is the outer point of the single-tooth contact area of the drive gear. This appears shortly before the next pair of teeth gets into the mesh. The reversed image of the stress pattern occurs in the driven gear, where the maximum value of the root stress appears in point B. On Fig. 3 all the characteristic meshing points are indicated by the dotted lines for individual gear geometries. The stress pattern for both S-gear shapes is similar to the involute one, except that both S-gear geometries have a wider area of single-tooth contact as a result of a smaller contact ratio. The minimum root stress is calculated in the gear S2, which has the widest root, slightly higher is the root stress in the involute gear and the highest in the gear S1, where the calculated stress is higher in spite of the broader root. The higher stress in the gear S1 occurs due to the wider area of single-tooth contact, where the distance from the point D to the root is larger than in the involute one. This means a greater bending moment of the tooth and, consequently, higher stress at the tooth root. The gear S1 also has a smaller rounding radius in the root, which is also the reason for higher root stress.

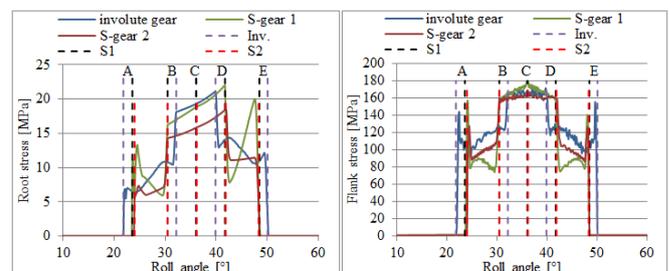


Fig. 3 Root stress (left) and flank stress (right) in the driver gear

If we look at the flank stresses, we see that stress peaks occur at the beginning and at the end of the meshing. These peaks occur due to the narrow contact area, where the tooth tip is pressed against the flank of the other tooth. The flank stresses are similar to theoretical ones, with the difference in the stress peaks at the beginning and

end of the meshing, resulting from the transfer of force through the narrow contact area. We can see that the flank stresses in the two S-gear geometries in the A-B and D-E range are smaller. In the single-tooth contact area B-D the flank stresses in all three analyzed geometries are approximately the same.

Polymer gear pair, frictionless contact

The stress pattern is different for the polymer gears comparing them to the steel gears, Fig. 4. Due to the small elastic module, the same load will result in a significantly greater deformation of the teeth. This results in an increase of the contact ratio, which results in the narrower single-tooth contact zone. The maximum root stress in the drive gear appears in the point D and in the driven gear in point B. The calculated root stresses are practically unchanged between the polymer and steel gear pairs, as the gear pair in both simulations was loaded with the same moment load.

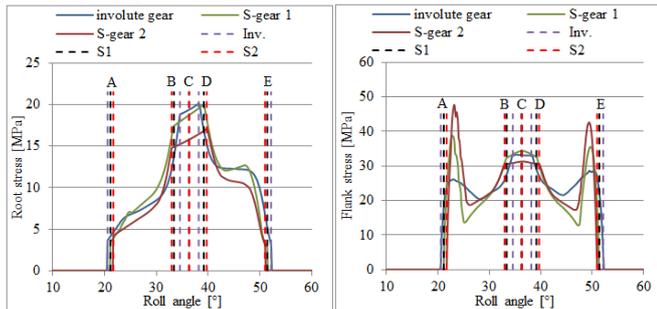


Fig. 4 Root stress (left) and flank stress (right) in the driver gear

Large differences can be observed when looking at the flank stresses, where these calculated for polymer gear pairs are much smaller. The elastic modulus of the material has an effect on the magnitude of the flank stress. In the standard calculation this effect is taken into account by the coefficient of elasticity Z_E . The flank stress peaks that occur at the start and end of the meshing are also present in the polymer gears.

Steel gear pair, frictional contact

In the following step, the numerical model was upgraded, taking into account the friction between the meshing flanks. The results obtained with this model are shown in Fig. 5. If the friction is taken into account, the stress pattern is similar, only in the kinematic point C there is a jump in the stress. This is due to the change in the direction of sliding, consequently the frictional force reverses the sign. Due to the wider tooth root in the S2 gear, the root stress was expected to be smaller. In the gear S1 with a similar profile to the involute gear, the root stress was even slightly larger. Even for the flank stresses, a smaller leap in the stress at the kinematic point C is observed.

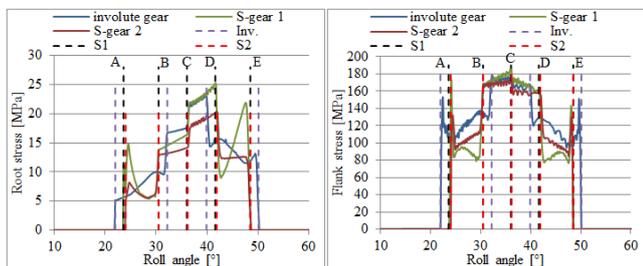


Fig. 5 Root stress (left) and flank stress (right) in the driver gear

Polymer gear pair, frictional contact

In the last step, the same numerical model was used to calculate the stresses in the polymer gears. The stresses, both the root and the flank ones, differ considerably from those of the steel gear pairs, Fig. 6. The reason is in the much smaller elastic modulus of the polymer gears, which leads to a greater deformation of the teeth. A rather large difference in the pattern of root stress also occurs in comparison with a frictionless model. The calculated root stress

here is a bit larger, the maximum occurs in point D in both the drive and the driven gear.

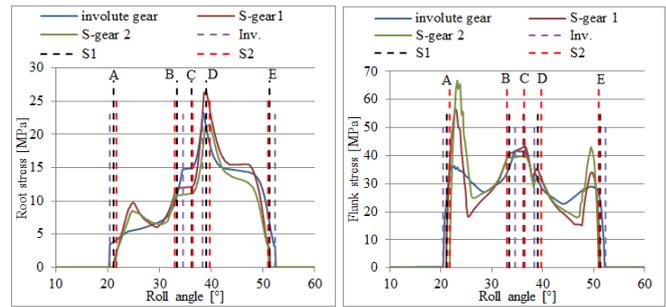


Fig. 6 Root stress (left) and flank stress (right) in the driver gear

In the case of flank stresses, the initial peaks are considerably higher for S-gears. This could most likely be avoided with a greater rounding of the tooth tip, which will be analyzed in the future. The teeth of the S-gears are also less deformable than involute, so no adaptation is given which leads to a greater force at the stroke.

4. Discussion

Stresses calculated with the numerical models were compared with those analytically determined according to the standard or recommendation. Comparison of the root and flank stresses in the steel gears is shown in Fig. 7. With the numerical model obtained maximum root stresses correlate well with the stresses calculated according to the standard. With the model that simulates a frictional contact between the meshing flanks, the calculated root stresses are approximately 8.5% larger, while in the frictionless model they are 2% smaller, comparing to the standard calculation. A big difference occurs when comparing the flank stresses. Here, it turns out that our model does not calculate the flank stress sufficiently precisely. In the case of steel gears, the contact area is very narrow, so there was a lack of elements in the contact area in spite of the very dense mesh. With further mesh refinement in the contact area, we managed to reduce the deviations to 7.5%.

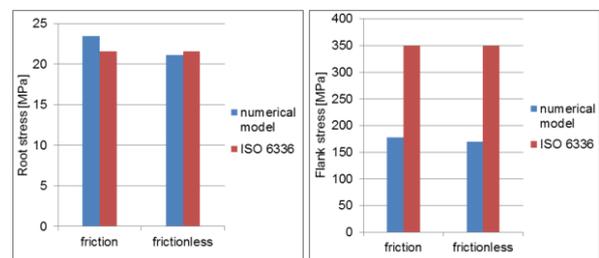


Fig. 7 Comparison of the stresses calculated with our numerical model and according to the standard ISO 6336

Comparison of the root and flank stresses in polymer gears is shown in Fig. 8. In the case of polymer gear pairs, both the root and the flank stresses have a good match with the calculation according to VDI 2736. Based on this, we claim that our numerical model is also good for calculating the stress state in S-gears. Results obtained with the numerical model, which models a frictional contact, deviated by 2% from the calculation according to VDI 2736 in the case of root stress and in the case of flank stress by 3.6%, Fig. 8.

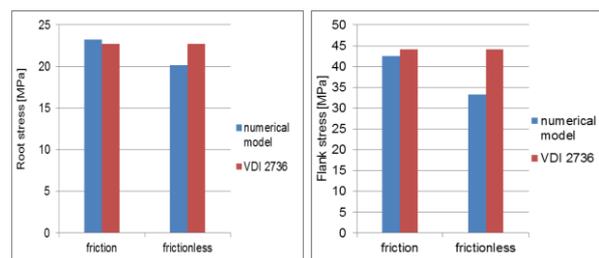


Fig. 8 Comparison of the stresses calculated with our numerical model and according to the recommendation VDI 2736

When calculating the flank stresses with our numerical model, the stress peaks that occurred at the beginning and end of the meshing were observed. These peaks are ignored by the standard calculation. In practice, these peaks can be avoided by correcting the tooth profile, adding tip relief. This is reflected by improved meshing, leading to an extension of the lifetime of the flank and a reduction in noise during the operation of the gears. For the analysis of the effect of the profile correction, three different examples were examined. These were: a gear pair without any flank modification, a gear pair with the tip rounding of $R = 0.05$ mm and a gear pair with tip relief ($Ca = 0.015$ mm to the diameter of the outer point of single-tooth contact) and tip rounding of $R = 0.05$ mm. When adding the tip rounding, the results are much better, Fig. 9. The stress peaks at the start and end of the meshing are in this case smaller than the flank stress at the kinematic point C. When adding a tip relief, it is necessary to pay attention to the diameter to which it will go. In our example, the relief ran to the diameter where the single-engagement point B is located on the driven gear and point D on the drive gear. Due to the large teeth deformation of the polymer gears, we expected this correction to be appropriate. It turned out that when there is a tip relief the flank stress peaks are completely avoided, Fig. 9.

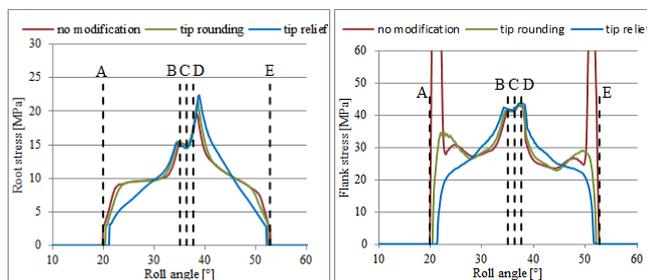


Fig. 9 The impact of the tip relief on the root stress (left) and flank stress (right)

5. Conclusion

From the calculated stresses we can see the difference between the involute and S-gears. The stresses in the S-gears are also affected by tooth geometry, as this can be quite different, depending on the choice of parameters n and a_p . By using a suitable S-gear shape, we get smaller stresses in the material when transferring the same load. Thus, with the S-gear pair, we can transmit heavier loads without exceeding the permissible stresses; the gear unit may be smaller. By changing the parameters n and a_p , we obtain a considerable number of different shapes of S-gears with different characteristics. In future research, we intend to analyze and set up a computational model for dimensioning of the S-gears.

Differences in the results between the frictional and frictionless models were observed. A jump in the tooth root stress was observed at the kinematic point C when modeling a frictional contact. In the case of polymer gears, no jump occurs, but the maximum stress of both the drive and the driven gear is calculated at the point D. The calculated root stresses are 25% higher when the friction is considered for polymer gears and 12% higher for steel gears. The calculated flank stresses in the kinematic point C are 27% higher in the case of frictional contact for polymer gears and 5% higher for steel gears.

In the case of polymer involute gears, the deviation between results obtained with our numerical models and those according the recommendation VDI 2736 is minimal. Thus, our assumption is that our numerical model can accurately calculate the nominal stresses also for polymer S-gears. In the recommendation, for the calculation of the actual stress, the nominal stress is multiplied by the factor of application K_A , whose recommended values are from 1 to 1.25. In the case of dimensioning of S-gears, the same value of the application factor should be used.

In the case of steel gears, the deviation between the numerical models and the calculation according to ISO 6336 is slightly larger. The calculation of the tooth root stress is still acceptable, but an excessive deviation occurs in the calculation of flank stress. We checked for the mesh density that would be needed to reduce the deviation between the flank stresses. When the mesh on the flank was refined for three times, the deviation of the flank stress was 7.5%.

With the use of a numerical model, we have proved that with a proper tip relief we can avoid the stress peaks at the beginning and end of the meshing. From practice we have experience that this has a beneficial effect on the lifetime and the noise of the gears.

6. References

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