AN ANALYTICAL METHOD FOR DYNAMIC ANALYSIS OF CATERPILLAR VEHICLE'S STRAIGHT MOTION

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Abstract: In this article some of main characteristics of caterpillar vehicle’s straight motion are depict. An analytical method for working out formulas for maximum speed, acceleration and time and route for acceleration is suggested.

Keywords: DRIVING FORCE, TIME AND ROUTE FOR ACCELERATION

1. Introduction

In theory of caterpillar vehicles [Гуськов 1984], [Никитин 1971] and [Забавников 1968], the problem for straight motion dynamics is examined like following task solving:

1.1 Determination of forces, which effect vehicle;
1.2 To work out a deferential equation for straight motion without taking notice of caterpillar slipping as driving force, inert force and resistant forces from rolling and moment on the slope take part in the balance of forces influencing on vehicle mass center. The driving force is shown trough moment of rotation of engine's crankshaft, transmitted to the driving wheel and reduced with losses in transmission and caterpillar;
1.3 A numerical identification of speed - force characteristic from several points, as driving force is shown depended from vehicle straight speed trough Laiderman’s equation. The energy losses are estimated only for average motion speed for every gear.

Using shown characteristics for analysis of dynamic features of a concrete caterpillar vehicle depending of its constructive parameters goes along with a large amount of estimate operations and low accurateness (because of making average speed in the formula for losses from carpentiliar).

2. Aim

Working out system of equations for analysis of main dynamic characteristics of straight motion.

3. Dynamic characteristics

To work out analytical functions, illustrated characteristics of straight motion, is necessary to develop examined in [Гуськов 1984], [Никитин 1971], [Забавников 1968] deferential equation of straight motion

\[ m \ddot{\delta}_a \bar{x} = F_n - (F_i + F_r), \]

where m is mass of vehicle;

\[ F_n = \frac{M_k}{r_k} \eta_r \eta_i \bar{x}_i \bar{x}_0 \]

- driving force; \( M_k \) - moment of rotation from driving wheels; \( \eta_r \) - transmission's efficiency; \( \eta_i \) - efficiency of engine's crankshaft; \( \eta_a \) - caterpillar's efficiency; \( \bar{x}_1, \bar{x}_2 \) and \( \bar{x}_3 \) - coefficients of concrete vehicle; \( G \) - weight of vehicle; \( i_k \) - transmission ratios of gear box and gearing; \( r_k \) - driving wheel's radius; \( F_i = -G \sin \alpha \) - resistant force from longitudinal slope \( \alpha \); \( F_r \) - resistant force from ground deformation; \( f \) - coefficient of resistance for straight motion on the ground; \( \delta_a = 1 + \frac{m_g}{m} + \frac{1}{r_k^2 m} \sum J_j i_j^2 \) - adjusted coefficient of rotating masses, cinematically connected with the engine; \( m \) - caterpillar mass; \( J_j \) - axis inert moment of j rotating mass; \( \eta \) - summary transmission ratio.

Moment of rotation of engine's crankshaft is shown depended of straight speed using Leidernan's formula

\[ M_{au} = P_{\max} \left( c_1 \frac{1}{\omega_p^2} + c_2 \frac{\omega_p}{\omega_p} - c_3 \frac{\omega_p^2}{\omega_p^2} \right), \]

where \( P_{\max} \) is a power of engine;

\[ \omega_p = \frac{V_i}{r_k} \]

- crankshaft's angle speed;
\( \omega_p^2 \) - crankshaft's angle speed for maximum power; \( c_1, c_2 \) and \( c_3 \) are Leidernan coefficients. Driving force is shown after substitution (3), (4) and (5) in (2):

\[ F_n = c_1 a_i k_1 - G k_2 + c_2 b k_i V^2 - c_3 \eta_a \eta_i k_1 \eta c V^2, \]

where are constant parameters for concrete vehicle. After these mathematical transformations, the deferential equation (1), displayed in Koshi form, is

\[ a = \frac{dV}{dt} = \frac{g}{G \delta_a} \left[ -c_i k_i \eta c V^2 + G k_2 + f \cos \alpha + \sin \alpha \right]. \]

3.1. Maximum speed

The acceleration is

\[ a = \frac{dV}{dt} = 0 \]

That’s why (7) is equal to zero. The achieved quadratic equation solving defines possible speeds for steady motion.
\[ V_{12} = \frac{c_2 b k_i i_k^2 \pm \sqrt{D}}{2(c_1 k_i c_i^3 + Gk_i)} , \text{ where} \]
\[ D = (c_2 b k_i i_k^2)^2 - 4(c_1 k_i c_i^3 + Gk_i) \left( c_1 a_i i_k - G(k_2 + f \cos \alpha + \sin \alpha) \right). \]

The bigger of them is in stable zone of speed-force characteristic and it is maximum speed for motion in concrete road conditions.

### 3.2. Maximum acceleration

Driving force has maximum value for speed, determined \[ \frac{dF}{dV} = 0 \]
from equation. After substitution in (7), \[ a_{\text{max}} = \frac{g}{G\delta_c} \left[ k_1 c_1 a_i i_k - G(k_2 + f \cos \alpha + \sin \alpha) \right] \]
(8)

### 3.3 Time and route for acceleration

Analytic identification of time for acceleration is achieved from \[ a = \frac{dV}{dt} , \text{ therefore} \]
\[ t = \int_{V_a}^{V} \frac{1}{a} dV \]  \hspace{0.5cm} (9)

where \( V_a \) is initial speed, which vehicle accelerates from, and \( V \) is its final value for each of gears. After substitution (7) in (9) and solving of gotten integral, the following expression is achieved

\[ t = \frac{G\delta_c \ln \left( \frac{V_a - V_1}{V - V_1} \left( \frac{V - V_2}{V_1 - V_2} \right) \right)}{g(k_1 c_1 c_i^3 + Gk_i) \left( V_1 - V_2 \right)} \]  \hspace{0.5cm} (10)

\[ \begin{array}{|c|c|c|c|c|c|c|c|}
\hline T, s & 0 & 20 & 40 & 60 & 80 & 100 & 120 \\
\hline V, m/s & 0 & 2 & 4 & 6 & 8 & 10 & 12 \\
\hline \end{array} \]

Fig. 1. Graph of time for acceleration for caterpillar vehicle with following parameters:
\[ \begin{align*}
P_{\text{max}} &= 507 \text{ kW}; \quad \omega = 187 \text{ s}^{-1}; \quad \eta = 0.9; \quad \eta_1 = 0.3; \\
c_1 &= 0.822; \quad c_2 = 1.356; \quad c_3 = 1.178; \quad k_1 = 0.95; \\
k_2 &= 0.25; \quad k_3 = 0.000003; \quad i_1 = 3.85; \quad i_2 = 4.4; \\
i_3 &= 3.48; \quad i_4 = 2.787; \quad i_5 = 2.027; \quad i_6 = 1.467; \quad i_7 = 1; \\
i &= 0 \%; \quad f = 0.04 \\
\end{align*} \]

where \( V_1 \) and \( V_2 \) are roots of equation (7). Expression (10) shows time for acceleration for a specific gear. For whole of stage for acceleration to maximum speed is necessary to solve (10) for each of gears. The final time for acceleration (фиг. 1) is achieved after recapitulation of all times for each gears

\[ t = \sum_{i=1}^{n} t_i \]  \hspace{0.5cm} (11)

The route, which vehicle pass for acceleration from \( V_a \) to \( V \), is determined with following integral solving:

\[ S = \int_{V_a}^{V} \frac{V}{a} \frac{dV}{a} . \]  \hspace{0.5cm} (12)

After substitution (7) in (13) and gotten integral solving, the expression for route, which vehicle passes for acceleration for one of gears

\[ S = \frac{G\delta_c \ln \left( \frac{V_a - V_1}{V - V_1} \left( \frac{V - V_2}{V_1 - V_2} \right) \right)}{G(k_1 c_1 i_k^3 + Gk_i) \left( V_1 - V_2 \right)} \]  \hspace{0.5cm} (13)

The route, which vehicle pass for acceleration from \( V_a \) to \( V \), is determined with following integral solving:

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The minimum values of integrals (9) and (13) getting depends on limits, which they are integrated. The optimum values for initial and final speed, for achieving time and route for acceleration for concrete gear are determined from graph on fig. 3.

\[ S = \sum_{i=1}^{n} S_i \]  \hspace{0.5cm} (15)

\[ \begin{array}{|c|c|c|c|c|c|c|c|}
\hline S, m & 0 & 200 & 400 & 600 & 800 & 1000 & 1200 \\
\hline V, m/s & 0 & 2 & 4 & 6 & 8 & 10 & 12 \\
\hline \end{array} \]

Fig. 2. Graph of route for acceleration of caterpillar vehicle with parameters:
\[ \begin{align*}
P_{\text{max}} &= 507 \text{ kW}; \quad \omega = 187 \text{ s}^{-1}; \quad \eta = 0.9; \quad \eta_1 = 0.3; \\
c_1 &= 1.356; \quad c_2 = 1.178; \quad k_1 = 0.95; \quad k_2 = 0.025; \quad k_3 = 0.000003; \\
i_1 &= 3.85; \quad i_2 = 4.4; \quad i_3 = 3.48; \quad i_4 = 2.787; \quad i_5 = 2.027; \\
i_6 &= 1.467; \quad i_7 = 1; \quad i = 0 \%; \quad f = 0.04 \\
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c_1 &= 0.822; \quad c_2 = 1.356; \quad c_3 = 1.178; \quad k_1 = 0.95; \quad k_2 = 0.025; \quad k_3 = 0.000003; \\
i_1 &= 3.85; \quad i_2 = 4.4; \quad i_3 = 3.48; \quad i_4 = 2.787; \quad i_5 = 2.027; \\
i_6 &= 1.467; \quad i_7 = 1; \quad i = 0 \%; \quad f = 0.04 \\
\end{align*} \]
Fig. 3. Graph of reciprocal values of accelerations of caterpillar vehicle with parameters

$P_{\text{max}}=507 \text{ kW}; \quad w_p=187 \text{ s}^{-1}; \quad n_t=0.9; \quad r_k=0.3; \quad c_1=0.822; \quad c_2=1.356; \quad c_3=1.178; \quad k_1=0.95; \quad z=0.025; \quad k_3=0.000003; \quad i_1=3.85; \quad i_2=8; \quad i_3=4.4; \quad i_4=3.48; \quad i_5=2.787; \quad i_6=2.027; \quad i_7=1.467; \quad i=0 \% ; \quad f=0.04$