SYNTHESIS OF KINEMATICAL CONJUGATE SPATIAL GEARING

Assoc. Prof. Abadjieva E. PhD.[1,2], Prof. Sc. D. Abadjiev V. PhD.[2]
Graduate School of Engineering Science, Akita University, Japan[1]
Institute of Mechanics, Bulgarian Academy of Sciences, Bulgaria [2]

abadjieva@gipc.akita-u.ac.jp[1], abadjiev@imbm.bas.bg[2]

Abstract: *This study contains main views and scientific principles of its authors, used as a basis for mathematical modeling, oriented to the synthesis of kinematically conjugated spatial (hyperboloid) gear drives.*

Keywords: SPATIAL GEARING, HYPERBOLOID GEAR DRIVES, KINEMATIC SYNTHESIS, MATHEMATICAL MODELLING

1. Introduction

The main stage of the mechanical systems' design, an object of the *Applied Mechanics* and *Theory of Mechanisms*, is known under the name *Synthesis of Mechanisms.* In the most common case, the synthesis of mechanisms involves the following two general tasks [1, 2]:

- Choice of a structure of the designed mechanism by realizing a structural synthesis.
- Design of the chosen kinematic scheme of the mechanism. This task is known as a *kinematic synthesis of the mechanism*. Through its solution it is possible to determine the constant geometric parameters of the chosen structure of the mechanism, such to satisfy its preliminary defined kinematic characteristics, like the function of the law of motions transformation.

In case of the synthesis of spatial (hyperboloid) gears, the task of structural synthesis is not solved, since the spatial transformation of rotations is realized, as a rule, by three-link mechanisms with high kinematic joints.

Obviously, when it is talked about synthesis of gear mechanisms, it should be understood that *the task of kinematic synthesis has to be solved.* As for all mechanical systems and spatial gear mechanisms as well, the results of solution of the mentioned task have to provide to the designer an optimal amount of information in order to ensure the realization of the next stages of gear sets' design: (1) development of the design project of the gear drive, which determines a specific structural form of the mechanism, ensuring its strength, reliability, durability, optimal efficiency and etc.; (2) technological preparation of processes related to the production, providing the planned technical and economic indicators of the synthesized mechanics.

The work realizes studies on conjugated hyperboloid gear sets with linear tooth contact oriented to their kinematic synthesis.

2. Mathematical Modelling for the Synthesis

The main purpose of each transmission mechanism is to realize with a necessary exactness a preliminary defined law of motions transformation. Among the group of gear mechanisms, spatial three-links gear drives, transforming rotations between crossed axes, are essential for both industry and transport. The reason for this is that they are characterized by the presence of large number of free parameters and to search for suitable combinations among them creates the opportunity, in the synthesis of the considered type mechanisms, the desired exploitation characteristics to be obtained, such as: high reliability and durability; low vibrational activity and noiseless; high accuracy in realization of the motions transformation law, high hydrodynamic loading capacity, etc. As a rule, the positive technological and exploitation features of the spatial gear mechanisms are result from higher requirements to obtain their specific kinematic characteristics. All this determines a kinematic character of the approach to the synthesis, and respectively *kinematic character of the created mathematical models*.

The successful implementation of spatial gear drives with new kinematics and strength characteristics into industry is retained by the absence of defined adequate approaches to the mathematical modeling, oriented towards their synthesis and design. In their studies over the years, the authors of this work have offered two approaches for creation of a mathematical model for synthesis [2]: *upon a pitch contact point and upon a region of mesh*.

2.1. Mathematical Modeling for the Synthesis upon a Pitch Contact Point

It is based on the assumption that all the necessary quality characteristics, defining concrete exploitation and technological requirements to the active tooth surfaces are guaranteed for only one contact point P (respectively and for its close vicinity) of the active tooth surfaces Σ_1 and Σ_2 (see Fig. 1) [2]. This model is applicable to the synthesis of spatial gear mechanisms with both point and linear contact. According to it, the common contact point P of the conjugated tooth surfaces Σ_1 and Σ_2 is a common contact point of the circles H_i^c $(i=1,\ 2)$, called *pitch circles* $(H_1^c : H_2^c)$ i.e., *point* P *is the pitch contact point*. The tangent plane T_m, which includes tangents to H_i^c $(i=1,\ 2)$ at point P is a pitch plane, and $m-m$ is the pitch normal to the T_m at point P. In Fig. 1 a case of mutual placement of pitch circles in the fixed space, corresponding to the traditional constructions of hyperboloid gear mechanisms with externally meshed tooth surfaces is illustrated. The diameters d_i $(i=1,\ 2)$ of H_i^c $(i=1,\ 2)$, together with the parameters $a_i,\ \theta_i,\ \delta_i$ $(i=1,\ 2)$, δ and a_w determines the type of the construction and its dimensions, shape, and mutual position of the movable links in the fixed space (in the coordinate systems $S_i(O_i, x_i, y_i, z_i)$, $(i=1,\ 2)$. At the same time, these dimensions are related to the definition of the longitudinal and cross orientation of the active tooth surfaces Σ_i $(i=1,\ 2)$ at the pitch contact point.

In other words, the pair of circles $(H_1^c : H_2^c)$ has a direct relation to determining the pitch of the teeth and, respectively, the tooth module of the designed gear system. The parameters d_i, δ_i $(i=1,\ 2)$ are the defining dimensions of the reference coaxial rotation surfaces, i. e. the overall dimensions of the blanks of the gears depend on them.

The parameters, mentioned above, are also used in determining the dimensions of the mounting of the synthesized gear mechanism.

From the above, it follows that the mathematical model for synthesis upon a pitch contact point ensures an algorithmic solution of two basic problems:

- Synthesis of the pitch configurations [2-4];

- Synthesis of the active tooth surfaces [3, 4].

The necessary (preliminary defined) geometric characteristics of the synthesized gear mechanism in a close vicinity of the pitch contact point are found by solving these two problems together.

In conclusion, it should be pointed out, that the described here approach to synthesis of spatial gears is based on the following kinematical condition: *The relative velocity vector* $\overline{V}_{12} = \overline{V}_1 - \overline{V}_2$ ($\overline{V}_i$ *is a circumferential velocity of the link* i *at point* P) *at the pitch contact point* P *has to lie both in the pitch plane* T_m *and in the common tangent plane* T_n *of the tooth surfaces* Σ_1 *and* Σ_2 *contacting at* P, *and this vector has to be oriented along the common tangent to the longitudinal lines of the active tooth surfaces* Σ_i $(i = 1,\ 2)$.

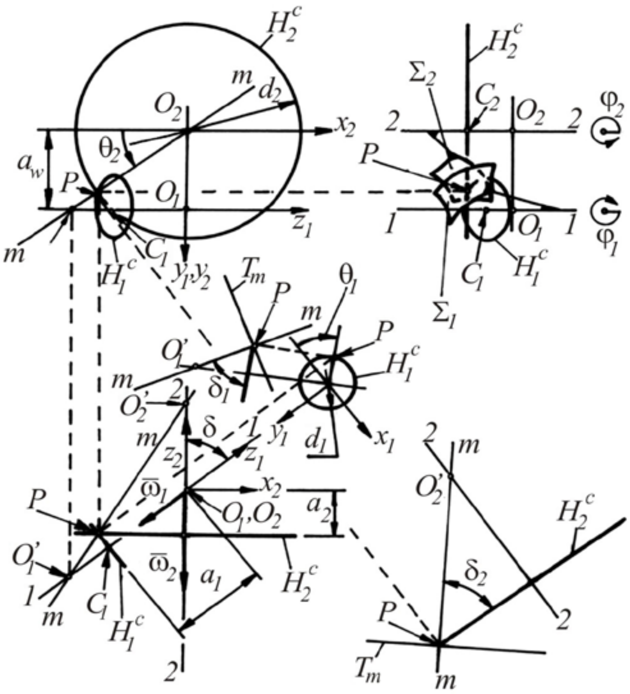

Fig. 1 *Geometric and kinematic interpretation of the mathematical model for synthesis based on a pitch contact point:* H_i^c $(i = 1,\ 2)$ *- pitch circles;* T_m *- pitch plane;* $m - m$ *- pitch normal to* T_m *at point* P*;* Σ_i $(i = 1,\ 2)$ *- tooth surfaces, contacting at the point* P

It has to be noted that the considered approach for the basic synthesis upon on a pitch contact point is described by the fact that the created mathematical model and the worked out on it algorithm are characterized with a ***universal structure*** for all types of hyperboloid gear sets. The universalization of the algorithm can be continued and at a certain stage it can be transformed into an optimization synthesis' algorithm by constructing criteria for control of the quality of meshing in a vicinity of the pitch contact point.

When the briefly described approach for the synthesis is applied, then a mathematical model for synthesis of Spiroid gear sets [5] is elaborated. On its bases is elaborated a computer program for their optimization synthesis and design (see Fig. 2) [2].

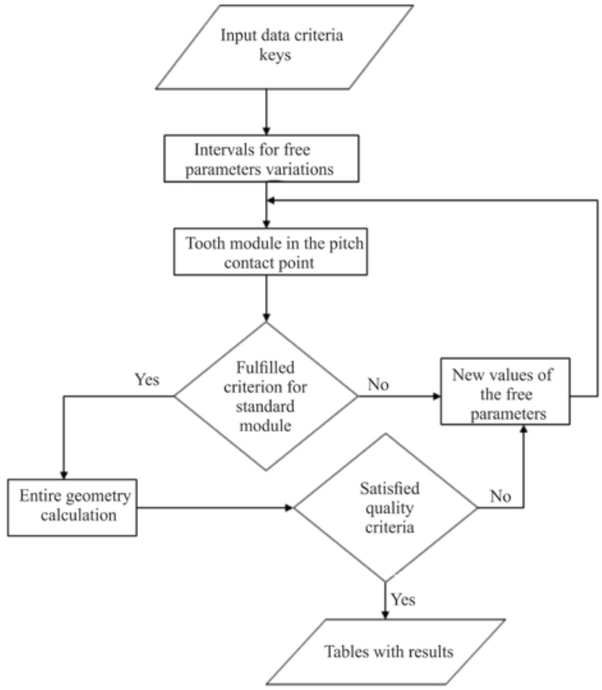

Fig. 2 *Program scheme for an optimization synthesis of a Spiroid gear pair*

2.2. Mathematical Modelling upon Region of Mesh

When hyperboloid gear sets with a linear contact between the tooth surfaces are synthesized, it is obvious that it is necessary to control the quality of meshing characteristics over the entire region of mesh or in a certain area for a specific reason. Such an approach to the task for the synthesis requires the construction of an adequate mathematical model. Its general kinematic scheme is shown in Fig. 2 [2].

Mathematical modelling for synthesis upon mesh region *is **not a structurally universal***. The reason for this is that the concrete geometric and kinematic characteristics of the mesh region depend on its placement in the fixed space, as well as on the geometric characteristics of the instrumental tooth surfaces Σ_J, that generate the active tooth surfaces Σ_i $(i = 1,\ 2)$.

This mathematical model is suitable for application when:

- It is not possible to define the pitch circles, respectively the pitch contact point;
- the condition for the special orientation of the longitudinal lines of the synthesized tooth surfaces is not mandatory or cannot be kept;
- the specific technological requirements has to be kept (for example, when the same tool is used in the manufacturing of gear sets, that realize different gear ratios, but based on the same size blanks for the gear sets, housing details and etc.).

In the basis of the mathematical modelling for synthesis of spatial gear drives upon mesh region is the kinematic model of surface of action /mesh region. Following this approach, we will note that the optimization process, in its essence, is a determination of both the optimal geometry and limitations on the mesh region, as part of the action surface (Fig. 3).

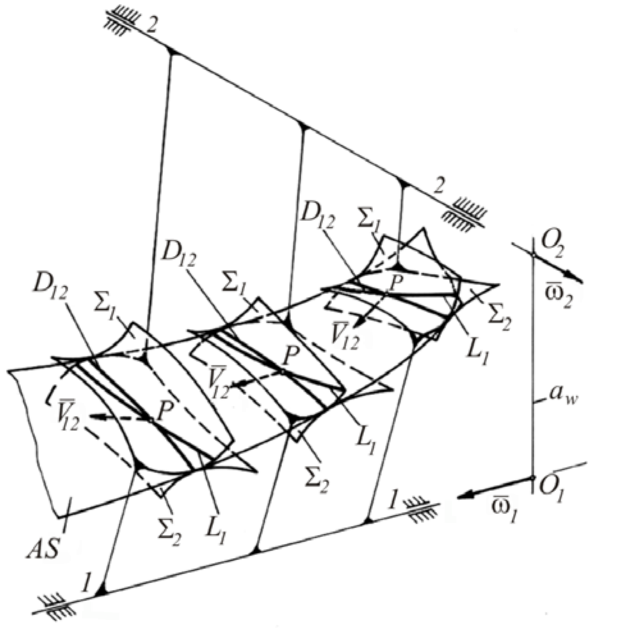

Fig. 3. *Geometric-kinematic interpretation of the model for synthesis upon mesh region: L_1 - longitudinal line of Σ_1; D_{12} - contact line between Σ_1 and Σ_2; AS - action surface*

As it has already been noted, when the spatial gear pair with a linear contact is synthesized, it is necessary to control their quality characteristics in the whole mesh region. The technology of the tooth surfaces generation of these class gear mechanisms is based on the second principle of T. Olivier. For this reason, the action surface of every gear drive is defined through the active tooth surfaces of one of the movable links of the three-link hyperboloid gear mechanism.

In order to illustrate this approach to the mathematical modelling, let us accept that Σ_1 - the active tooth surface of one of the movable links ($i = 1$ - pinion), is known. Let it be presented by its vector equation:

$$\overline{\rho}_{1,p} = \overline{\rho}_{1,p}(u, \vartheta), \tag{1}$$

where $\overline{\rho}_{1,p}$ is a radius-vector of Σ_1 in the coordinate system $S_p(O_p, x_p, y_p, z_p)$, fixed to the pinion; u, ϑ - parameters of Σ_1.

The equation (1) also describes the generating tooth surfaces Σ_2 of the second movable link ($i = 2$), instrumental surfaces $\Sigma_J \equiv \Sigma_1$. Let's accept that on Σ_1 are no folds and interruptions, i.e. of this surfaces the following condition is accomplished:

$$\overline{n}_{1,p} = \frac{\partial \overline{\rho}_{1,p}}{\partial u} \times \frac{\partial \overline{\rho}_{1,p}}{\partial \vartheta} \neq \overline{0}. \tag{2}$$

Here $\overline{n}_{1,p}$ is a normal vector in an arbitrary point of Σ_1.

In accordance with the kinematic approach to the synthesis, through the direct application of the basic theorem of meshing [2-4], the contact lines on the active tooth surface Σ_1 can be defined, i.e.

$$\overline{\rho}_{1,p} = \overline{\rho}_{1,p}(u, \vartheta), \ \ \overline{n}_{1,p}.\overline{V}_{12,p} = 0, \tag{3}$$

where $\overline{V}_{12,p}$ is a relative velocity vector in an arbitrary contact point.

When the equation systems (3) is written in the fixed coordinate system $S(O, x, y, z)$, then the mesh region is obtained as a locus of the contact lines between Σ_1 and Σ_2 in the fixed space:

$$\begin{aligned}
&x_p = x_p(u, \vartheta),\\
&y_p = y_p(u, \vartheta),\\
&z_p = z_p(u, \vartheta),\\
&n_{1,x_p} = n_{1,x_p}(u, \vartheta),\\
&n_{1,y_p} = n_{1,y_p}(u, \vartheta),\\
&n_{1,z_p} = n_{1,z_p}(u, \vartheta),\\
&[x \ \ y \ \ z \ \ t]^{\mathrm{T}} = M_{sp}[x_p \ \ y_p \ \ z_p \ \ t_p]^{\mathrm{T}},\\
&[n_{1,x} \ \ n_{1,y} \ \ n_{1,z}]^{\mathrm{T}} = L_{sp}[n_{1,x_p} \ \ n_{1,y_p} \ \ n_{1,z_p}]^{\mathrm{T}},\\
&n_{1,x}V_{12,x} + n_{1,y}V_{12,y} + n_{1,z}V_{12,z} \equiv f_s(u, \vartheta, \varphi_1) = 0.
\end{aligned} \tag{4}$$

Here M_{sp} and L_{sp} are correspondingly 4x4 and 3x3 transition matrices from S_p into S.

Analogically, the contact lines on Σ_1 can be written in the coordinate system $S_g(O_g, x_g, y_g, z_g)$ of the second movable link $i = 2$.

In general, the defined geometry, dimensions and placement of the mesh region in the fixed space, as a part of the action surface, are the optimal ones, if:

- singular points of second order (***undercutting points)*** are registered and eliminated;
- singular points of first order (***ordinary nodes of contact***) are registered and eliminated;
- the orientation and placement of the contact lines on the mesh region is chosen, in order to achieve the maximum loading capacity and coefficient of efficiency.

In many cases the synthesis of spatial gear drives with crossed axes is appropriate to be realized by combination of the two main approaches for synthesis [2]. The general strategy of such approach is based on:

- to construct such models, which allow that the basic geometric-kinematic characteristics at the pitch contact point to be determined.
- establishment of system of criteria, which will control concrete set of characteristics, that define different quality features for the whole mesh region or at least for some limited and preliminary defined part of it.

The approach to the synthesis upon a mesh region is offered by the authors of this study, when the mathematical model and computer program for synthesis of some less studied, not only in Bulgaria, hyperboloid gear drives (gear mechanisms of type Wildhaber) (see Fig.4) [2, 6,-10] are organized.

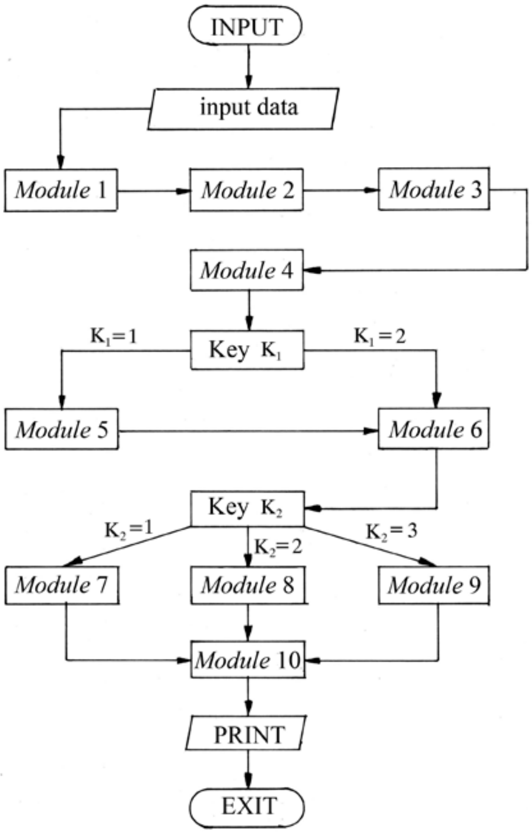

Fig. 4. *General structure of the research program for synthesis of gear mechanisms of type Wildhaber*

The structure of this program for the synthesis contains Modules and it is described as follows:

Module 1. In this first module, the offset a_w is directly chosen with a standard value or by the maximum torque $M_{2,\max}$ of the driven link;

Module 2. It serves to realize an adequate choice of the number of the helical teeth (threads) z_1 of the torus shaped worm of type Wildhaber gears;

Module 3. Here, the choice of the coefficient q of the reference diameter of the Wildhaber worm in the throat sector is realized;

Module 4. This module is oriented into determining the profile angle $\alpha^{(j)}$, $(j = 1,\ 2)$ of the teeth of the cylindrical gear with the straight plane teeth;

Modules 5 and 6. They analyze the existence of the singular points of first and second order respectively;

Module 7. In this part of the program, the algorithm is oriented towards the optimization of the synthesized gear pair of type Wildhaber on the basis of its hydrodynamic characteristic*;*

Module 8. It ensures the evaluation of the contact strength of the studied gear mechanisms. This assessment is realized in the process of the synthesis and analysis;

*Module 9***.** This module covers the optimization of the synthesized gear sets Wildhaber, in relation to their durability;

Module 10 serves to print the table, which consists the basic geometric parameters of the synthesized gear set of type Wildhaber.

3. Conclusion

The presented here study has evolved and improved over time. As a result of the created computer programs, dozens of gear mechanisms were synthesized [2, 6-10]: Spiroid, Helicon, Wildhaber and Planoid. Some of these spatial gear drives have been realized as industrial models, and many of gear sets of type Helicon have been manufactured and implemented into various engineering constructions. The theoretical and applied studies carried out are accompanied by an active innovation activity related to the creation of nine Bulgarian inventions registered in the Bulgarian Patent Office [2].

References

1. Levitsky, N. Theory of Mechanisms and Machines, Pulbishing house “Nauka”, Main Editorial “Fiziko- Matemaitcheskoi Literaturi”, Moscow, 1979, p. 567, (in Russian).

2. Abadjiev,V. “Theory and technical applications of hyperboloid mechanisms,” Sc. D. Dissertation, Institute of Mechanics, Bulgarian Academy of Sciences, Sofia, Bulgaria, 2007, (in Bulgarian).

3. Litivn, F. Geometry and Applied Theory, PTR Prentice Hall, A Paramount Communication Company, Englewood Eliffs, New Jersey 07632, 1994, p. 724.

4. Litivin, F and Al. Fuentes, Gear Geometry and Applied Theory, 2nd Ed., Cambridge University Press, Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, Sau Paulo, Delhi, Tokyo, Mexico City, 2004, p. 800.

5. Radzevich,S. Dudley's Handbook of Practical Gear Design and Manufacture, CRC Press, Taylor & Francis Group, 2012., p. 862.

6. Abadjiev V., D. Petrova, E. Abadjieva. An Optimization of Type Wildhaber Gear Sets Based on Loading Capacity. Kinematic Approach to Criteria Construction, In Proc.: 15th Int. Conf. on Manufacturing Systems ICMaS 26-27 October 2006, Proceedings, Editura Academia Romane, Bucharest, 2006, pp. 421-424.

7. Abadjiev V., D. Petrova, E. Abadjieva. An Optimization of Type Wildhaber Gear Sets Based on Loading Capacity. A Software Estimate of the Hydrodynamic Loading Capacity, In Proc.: 15th Int. Conf. on Manufacturing Systems ICMaS 26-27 October 2006, Proceedings, Editura Academia Romane, Bucharest, 2006, pp. 425-428.

8. Kolchin, N. Gears and Worm Gear Sets. Some Questions to the Geometry, Kinematics, Calculations and Manufacture. Publishing House "Mashinostroenie", Leningrad, 1968, p. 362, (in Russian).

9. Abadjieva E., V. Abadjiev. On the Synthesis of Hypoid Gears with Linear Contact. Part I – Geometric Synthesis Practices in the Pitch Contact Point. 10th Jubilee National Congress on Theoretical and Applied Mechanics, 13-16 September 2005, pp. 1-6.

10. Abadjieva E., V. Abadjiev. On the Synthesis of Hypoid Gears with Linear Contact. Part II – Geometric Synthesis of the Planoid Gearing. 10th Jubilee National Congress on Theoretical and Applied Mechanics, 13-16 September 2005, pp. 7-11.