CHOOSING A MODEL TO DETERMINE DEFORMATIONS

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Abstract: *If the material from which the equipment is made is known, constant and periodic loads are known and the displacement of equipment points are measured by the dynamic model, it is possible to determine the forces that cause these displacements and deformations themselves. If the deformations are within acceptable limits, then the equipment operation continues, otherwise the equipment must be stopped to eliminate deformations; at best, deformation can be eliminated without stopping the equipment.*

KEYWORDS: *STATIC, KINEMATIC AND DYNAMIC MODELS, DEFORMATIONS*

1. Introduction

If $\boldsymbol{S}(x)$ deformations depend on any of the parameters $\boldsymbol{x}$, then in static models they are zero, that is, $\boldsymbol{S}(\mathbf{x}) = 0$ for any time interval. The static model for deformation detection is the verification of definite points of an object for congruence (on an unchanged position) by geodesic measurements. In such models deformations that come from the outside and the established equilibrium of the object of observation under the influence of forces applied to it are excluded.

In kinematic models of deformation are described by the formulas of the uniform motion of the object without taking into account the forces acting on it, whose parameters are not known at the beginning, deformations determine the shape of only relatively well-known parameters.

In dynamical models, connections are established between the motions and the reasons for which they are called, and they provide an opportunity to specify unambiguous mathematical relations between geometric changes of an object, which are determined by periodic geodetic measurements and external forces acting on it, the deformations are determined in accordance with the form, so and in accordance with the parameters.

2. Presentation of the main material

Deformations $\boldsymbol{S}(x)$ are continuous, geometric changes in time that are distributed on the object, which are a function of the action of parameters x on an object.

We subdivide the object under consideration into separate points, we obtain its physical (design) coordinates $x_{p\ t}$ at the time t, which are determined

$$\boldsymbol{x}_{p.t} = \boldsymbol{x}_{a.t} + \boldsymbol{S}(x_a)_t,\ t = 0, 1,\dots, n, \qquad (1)$$

The set of coordinates of the points (1) can be determined by any geodesic method, after the alignment of the corresponding geodetic network, we use the approximate coordinates of the points $\boldsymbol{x}_t$ to the corresponding geodesic coordinates $\boldsymbol{x}_{g.t}$

$$\boldsymbol{x}_{.g.t} = \boldsymbol{x}_{a.t} + \boldsymbol{A}\,\boldsymbol{x}_{a.t},\ t = 0, 1,\dots, n \qquad (2)$$

where $\boldsymbol{A}$ is the operator of the transformation of measured values of coordinates $\boldsymbol{x}_{a\ t}$.

The motion of the points of the object is characterized by a trajectory, length, speed, acceleration and displacement. The movement of the point can be uniform and uneven, with acceleration [6].

Trajectory - a spatial line, which is described by a point in its movement. Trajectory of motion in space is rectilinear or curvilinear.

The length of the path (scalar) is the length of the segment of the trajectory that has passed the point for the time interval t.

Ideally, geodetic methods determine the coordinates of the points of an object to be equal to the design coordinates, so the identity can be written

$$x_{p.t} \equiv x_{.g.t},\ \ t = 0, 1, \dots, n, \qquad (3)$$

on the basis of which we get a nonlinear functional model

$$\Psi\,(t, x) = x_{p.t} + \boldsymbol{S}(x_a)_t - x_{a.t} - \boldsymbol{A}\,x_{\mathbf{a}.t} = 0,\ t = 0,1,\dots,n \ (4)$$

This system of equations corresponds to the Gauss-Helmert model, or the system of equations formulated on the basis of the corresponding evaluation results, which are conditional equations with unknown coordinates of the points of the object

$$\boldsymbol{Bv} + \boldsymbol{Ax} + \boldsymbol{w} = \mathbf{0}, \qquad (5)$$

where $\boldsymbol{v}$ is the vector of the resistance of the material of the object; $\boldsymbol{B}$ is the vector transformation vector of the material resistance; $\boldsymbol{w}$ is a free member.

The resistance vector is described by a stochastic model

$$\boldsymbol{v} = \Psi(t,x) + \boldsymbol{Bv} = x_{p.t} + \boldsymbol{S}(x_a)_t - x_{a.t} - \boldsymbol{A}\,x_{\mathbf{a}.t} -$$
$$-(x_p - x_p^{o}) + (x_a - x_a^{o}), \qquad (6)$$

where x_p^o, x_a^o - actual design and measured coordinates

$$\boldsymbol{v}_{\xi} = \boldsymbol{B}_a\,\boldsymbol{v}_a + \boldsymbol{x}_x - \boldsymbol{A}\,\boldsymbol{x}_a + \boldsymbol{w}, \qquad (7)$$

Regardless of this, in terms of the existing parameters on the object, one can proceed from such a basic dependence

$$x_a = x_a + \boldsymbol{v}_a = x_a^{o} + x_a. \qquad (8)$$

From (7) and (8) we get

$$\boldsymbol{v}_{y.\zeta} = A\,\boldsymbol{x}_{.a} - \boldsymbol{l}_a \qquad (9)$$

In order for the model to be reliable, taking into account (6), it is necessary that the overall variance is minimal

Thus, the system of equations (9) will be specified by other parameters, that is, unknown parameters of data enter once directly and once indirectly through xa in the decision vector. Between the found special solution xx and some solution x.a there is a dependence

$$\boldsymbol{x}_x - \boldsymbol{x}_a = \boldsymbol{A}\,(\boldsymbol{x}_x - \boldsymbol{x}_{.a}). \qquad (10)$$

Apply to the decision vector the minimum Euclidean norm

$$\boldsymbol{x}_x^{T}\ \boldsymbol{x}_x \rightarrow min, \qquad (11)$$

we obtain the following conditional equations

$$\boldsymbol{A}\,\boldsymbol{x}_x = 0. \qquad (12)$$

On the other hand, from the system of equations (9) with unknown parameters of data, taking into account (12), we observe

$$\boldsymbol{x}_a = -\,(\boldsymbol{A}^T \boldsymbol{A})^{-1} \boldsymbol{A}^T \{\ \boldsymbol{v}_a - \boldsymbol{B}_a\,\boldsymbol{x}_a -$$
$$-[\boldsymbol{w} - \boldsymbol{B}_a(\boldsymbol{x}_a - \boldsymbol{x}_a^{o})]\}. \qquad (13)$$

We substitute expression (13) in the original equation (9), we get

$$\boldsymbol{F} = \boldsymbol{I} - \ \boldsymbol{A}_{\zeta}^{T}(\boldsymbol{A}_{\zeta}^{T}\,\boldsymbol{A}_{\zeta})^{-1}\,\boldsymbol{A}_{\zeta}^{T}, \qquad (14)$$

where $\boldsymbol{F}$ is a symmetric idempotent matrix; $\boldsymbol{I}$ is a unit matrix.

Given (14) and (12), the expression (9) will look like

$$\boldsymbol{v}_a = \boldsymbol{x}_x + \boldsymbol{F}\,\boldsymbol{B}_a\,\boldsymbol{x}_a - \{-\boldsymbol{F}[\boldsymbol{w} -$$
$$-\boldsymbol{B}_a(\boldsymbol{x}_a - \boldsymbol{x}_a^{o})]\}, \qquad (15)$$

using the expression (9) for the vector va we obtain such a shortened model

$$\boldsymbol{v}_a = \boldsymbol{A}\,\boldsymbol{x}_a - \boldsymbol{w}\boldsymbol{l}_a, \qquad (16)$$

the matrix with the coefficients of the geodesic equations can be represented in a spectral form

$$\boldsymbol{Q}_{x.a} = \boldsymbol{H}\boldsymbol{G}^{-1}\,\boldsymbol{H}^{T}, \qquad (17)$$

while in the matrix H there are own vectors of constant eigenvalues, while the matrix G^{-1} contains its own vectors, which relate to free (geodesic) data parameters. The matrix $\boldsymbol{F}$ given by equation (14) can be described as follows

$$\boldsymbol{F} = \boldsymbol{I} - \boldsymbol{G}\,\boldsymbol{G}^{T} \quad (18)$$

from (17) and (18), we obtain

$$\boldsymbol{F}\,\boldsymbol{Q}_{x.a} = \boldsymbol{Q}_{x.a}\,, \quad (19)$$

Assuming that lo = l we obtain a stochastic model

$$\boldsymbol{v}_a = \boldsymbol{A}\,\boldsymbol{x}_a - \boldsymbol{l}_a, \quad (20)$$

in accordance with the size of the matrix of coefficients A, we establish that the system of equations (20) does not diverge without the introduction of the final vector va, from which it can be concluded that the corresponding variance and redundancy disappears.

Thus, expression (20) can be considered as a model-independent equation hypothesis, which is a solution of the problem described by the expressions (1), (2), (3).

In this model, based on the fact that for each case a special vector of coordinates xx.t is introduced for the description of the adopted initial situation, however, if the deformation function S ($\boldsymbol{x}_a$) indicates the absence of deformation, these vectors xx.t must coincide for all periods of time, and thus the hypothesis can not be rejected

$$\boldsymbol{H}_0 : \boldsymbol{x}_{x.t} - \boldsymbol{x}_{x.t} = 0, \quad t = 1,\dots, n. \quad (21)$$

Accordingly, the model of equation (20), taking into account that $\boldsymbol{l}o = \boldsymbol{l}$ will be $v_a = \boldsymbol{A}\,\boldsymbol{x}_a$-$\boldsymbol{l}_a$, assuming that the periodic correlation is equal to zero, then.

$$\boldsymbol{Q}_{a.i.j} = 0, \quad i, j = 1,\dots, n, \; i \neq j. \quad (22)$$

Introducing into the functional model, following the vector of solutions from (20), we obtain

$$\boldsymbol{x}_a = \boldsymbol{N}_a\,\boldsymbol{n}_a, \quad (23)$$

to test the hypothesis (21). The test size *F* - distribution is proposed

$$F_{ry0,r\xi\alpha} = \frac{R}{bs_0^2} \approx F(r_{y,0}, r_{\xi,\alpha}, \lambda_F) \quad (24)$$

When it comes to H_o, the noncentricity parameter λ_F (systematic biasing) disappears and has a probabilistic relationship

$$P\left\{F_{ry0,r\xi\alpha} \geq F_{ry,0,r\xi,\alpha;1-\alpha}\left|H_0\right.\right\} = \alpha \quad (25)$$

where α is the probability of a zero hypothesis. If the test value F_{ry0}, $r_{\xi\alpha}$ is greater than the corresponding *F*-distribution of F_{ry0}, $r_{\xi\alpha}$; 1-α, then H_0 should be corrected in accordance with (21). Due to the hypothesis of reducing the sum of squares of deviations, as well as the redundancy of measurements *r* is manifested

$$R = v_a^T Q_a,\; v_a = 0,\; b = r(Q_a) - r(Q_{x\,a}) \quad (26)$$

Taking into account the variation introduced in (24) and the redundancy $r_{\xi\alpha}$ must be removed from the previous equations. If you need to change H_0 then you need to examine two variations of the change, and analyze identity and deformation.

After analyzing the identity, we localize our own offsets of the points, which should not be completely excluded, while they may occur regardless of the deformations of the object.

Static strain model. Static model is a body equilibrium model. Such a model describes such objects that do not change the geometric form for consideration of the period of time. In this respect, it is better to refer to the correlation model if applying these processes solely for the analysis of identity. Need to answer the question whether there are selected points of equipment personal movements (bias), that is, to what extent the same points match. The static model implies the inadmissibility of any deformations $\boldsymbol{S}$ (*x*). However, the exclusion coefficients of the matrix B and coefficients of the matrix of the corresponding influence parameters $\boldsymbol{Q}_x$,

$$B_{a,e} = O, Q_{x,a} = O, \forall e = 1,\dots,n \quad (27)$$

In the future it is necessary to take into account again that the pseudo-inverse symmetric matrix is identical to itself

$$Q_{x,\alpha}^{+} = Q_{x,\alpha}, \forall Q_{x,\alpha} = Q_{x,\alpha}^{T} = Q_{x,\alpha}^{2}\,. \quad (28)$$

At the same time, the parameters of (23) belonging to the system of normal equations

$$N_{y,0}^{s} x_{y,0}^{s} - n_{y,0}^{s} = 0\,. \quad (29)$$

The usual static model of deformation is the same when taking into account (27) the specific cases of the general solution described, the advantage of this process is in a relatively simple structure, and accordingly deformations excluded are excluded. The identified natural disadvantage and in fact justifies that no deformation can be taken into account. In the geodetic definition of many destinations this process is optimal, that is, the geodetic network is stable, but this model is not acceptable for determining deformations.

Kinematic model of deformation. Kinematics is the theory of body motion without taking into account the force that affects the movement of bodies. Such a model describes objects that we know that they are deformed. It is known, however, that deformations are definite functions of the place (object), time, and other specific parameters that influence deformation processes. Proceeding from this, the deformation will not be determined for an object by only one continuous function. It is known that a definite relationship between cause and effect of deformation is not unambiguous.

In recent years, there have been many similar connections (functions), a large number of authors and ideas in choosing the most diverse mathematical functions, often point to such extended functions as general methods for determining deformations, although they represent only isolated cases of simulation of deformation processes [1, 2, 3, 5]. All kinematic models have the disadvantage that they represent an unambiguous mathematical connection between the physical cause of deformation and the geometric action of forces that cause deformation. The definition of kinematic models in this way is due to the fact that the deformation function $\boldsymbol{S}$ (*x*) is presented in a general form. The defining parameters of the influence of these bonds remain unknown again. The coefficients of the matrix B can be determined, and the coefficients of the matrix of the corresponding parameters of influence $\boldsymbol{Q}_x$ exactly as in static models are withdrawn in accordance with (27).

$$B_{a,e} = B_{a,e}, Q_{x,a} = O, \forall e = 1,\dots,n\,. \quad (30)$$

In taking (30) we obtain from (29) the system of normal equations

$$N_{y,0}^{k} x_{y,0}^{k} - n_{y,0}^{k} = 0\,, \quad (31)$$

The kinematic model is also a partial case when considering a general solution. The advantage of this model is that deformations of the object can be taken into account. The disadvantage lies in the fact that the impact parameters can be estimated as the difference between geodetic measurements at different times. They are direct in determining the coefficients of the matrix $\boldsymbol{B}$ and depend on the accuracy of the measurements. External information for determining the deformation values is not available and therefore this model is not acceptable.

Dynamic strain model. Using such a model, it is possible to describe objects that are known for their changes in size, shape and / or volume. In order to ask this model you need to know the material of the object and its properties, the external forces that act on the object [7]. Determination of the deformation of the object is detected by the force of alignment

$$\boldsymbol{Ka} + \boldsymbol{f} = 0, \quad (32)$$

the value of K is a global rigidity matrix, and the vector a represents the displacement of the point, the vector f represents the forces that cause these displacements. If known coefficients of the matrix of rigidity, which depend on the properties of the material of the object and the load vector, then it is possible to calculate the displacement vector of point a. Deformation S (xa) is determined by interpolation. If the expression (32) is determined by linear functions, then the estimate of expression (32) will be simple (linear). If the load of the object exceeds some limits then the material begins to react plastically or visco-plastic. The dynamic model assumes that the deformation function is entirely given, while the coefficients of the matrix $\boldsymbol{B}_a$, e, as well as the matrix of the coefficients $\boldsymbol{Q}_x$, a are known.

$$B_{a,e} = B_{a,e}, Q_{x,a} = Q_{x,a}, \forall e = 1,...,n . (33)$$

The corresponding system of normal equations that follows from (23) will be $N^d_{y,0} x^d_{y,0} - n^d_{y,0} = 0$, the same as (29) and (31).

Expression (32) is an integral part of the general model, which makes it possible to calculate the forces that cause a change in the shape of the volume and size of the object.

For example, let's take the measured values of the displacements of equipment points from work [4] and make 21 equations in expression (32), which are given in the table.

Table Measured displacement of equipment points and load

№ points, j	Load Measure. f, т	Load Calcul f_o, т	$\Delta f = f - f_o$
1	1,3	1,051	-0,249
2	1,2	1,066	-0,134
3	1,1	1,073	-0,027
4	1,1	0,997	-0,103
5	0,9	0,952	0,052
6	0,9	0,583	0,317
7	0,8	0,004	0,796
8	1,0	0,680	0,320
9	1,0	0,674	0,326
10	0,9	0,398	0,602
11	0,8	0,397	0,603
12	0,9	0,545	0,455
13	0,8	0,752	0,048
14	1,2	0,951	0,249
15	1,3	1,604	-0,304
16	1,4	1,707	-0,307
17	1,3	1,594	-0,294
18	1,1	1,366	-0,266
19	1,0	0,955	0,045
20	0,7	0,901	-0,201
21	0,7	0,628	0,072

Having solved these equations using the least squares method, we find the coefficients of the matrix of rigidity $\boldsymbol{K}$ with the mean square error of the approximation m = 0,294

$\boldsymbol{K}$ = □ - 0,08939 - 0,03983 + 0,02517□ ,

Substituting these values (table) and values a of the displacements (table) and in expression (32) we find the values f_o of fobch and the difference Δf that are close to zero (because the bias $a_{h.j}$, a_{xj} and $a_{y.j}$ are measured with some error), in the general solving these equations ΣΔf = 0 and the corresponding solution found satisfies (32).

3. Conclusion

By comparing the corresponding matrices of normal equations (static, kinematic, and dynamic models), we find that the definition of deformation is better achieved using a dynamic model, another advantage is the interpolation approach. Expression (32) is an integral part of the general model, which makes it possible to calculate the forces that cause changes in the shape, volume and size of the object. From the table and calculations of the coefficients of the matrix of stiffness K, it follows that from the equation (32), which is a dynamic model, if we know the coefficients of the matrix of stiffness and the load vector, it is possible to calculate the vectors of the displacement of the points of the object a and vice versa.

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