EXPERIMENTALLY VERIFIED MATHEMATICAL MODEL OF THE POLYMER PLASTICIZATION PROCESS IN THE INJECTION MOLDING

M.Sc. Iwko J. PhD.[1], M.Sc. Wróblewski R. PhD. [1]
Faculty of Mechanical Engineering – Wroclaw University of Science and Technology, Poland [1]

jacek.iwko@pwr.edu.pl

Abstract: *The mathematical model of the polymer plasticization in the reciprocating screw injection molding machine is presented. According to the mathematical model, a computer program was developed. Based on the computer program, simulation studies of the injection molding process were conducted. Next, the experimental studies, evaluating the theoretical model from the accuracy and usefulness point of view, were carried out. Important output quantities, such as the temperature and pressure profiles, the power demand by the screw, the torque on the screw and the screw rotation time were measured. The studies were performed on a specially made research office. The simulation results were compared with the experimental data measured for the most popular polymers and different operating parameters of the injection machine. The experimental studies have indicated the need to introduce some corrections to the mathematical model. Several modifications have been made to the model, related to the methods of stress determining in the polymer layer. Finally, the output characteristics of the plasticization process in the injection molding are now correctly determined by the model with an average error less than 10%.*

Keywords: INJECTION MOLDING, PLASTICIZATION, POWER DEMAND, ENERGY CONSUMPTION, SEC

1. Introduction

The purpose of this article is to present the comprehensive model of the polymer plasticization process in the injection molding. The first version of this model and the simplified model verification was presented in [1,2]. Next, the experimental research which evaluates the theoretical model from the accuracy and usefulness point of view, was conducted. Important output quantities such as the temperature and pressure profiles of the polymer, the power demand of the plasticizing system, the torque of the screw and the recovery time, were measured. These tests were performed on a specially designed research office. The experimental studies indicated the need for introduction of some corrections to the mathematical model. Consequently, several modifications were made in the model. The changes were related to the methods of stress determining in the polymer layer in the screw-barrel system. Furthermore, another method for determining the temperature of the molten polymer in a slit between the top of the screw flight and the barrel was indicated. The above two groups of changes resulted in a significant improvement in determining the power demanded to the screw and the torque on the screw in the transition and melting zones. Because of these modifications, the output characteristics of the plasticization process in the injection molding are now correctly determined, with an average error less than 10%. The mathematical model and a summary of the results of its experimental verification is presented below.

2. Mathematical model

The full operation algorithm of the model is presented in Fig. 1. The model uses four groups of input data: the geometric parameters of the three-zone screw and the barrel, the adjustable operating parameters of the injection molding machine, the material data and the numerical data (rate and accuracy of calculations).

The plasticizing unit consists of a barrel with a feed hopper, a heating section and a three-zones-screw of diameter *D*, width W (constant on the whole screw length), lead *S* and flight width *e*. The channel depth *H* in feed and metering zones are constant and equal to H_f and H_m, respectively. The channel depth in a compression (transition) zone changes linearly from H_f to H_m. Additional geometric parameters of the model are flight clearance δ_S (thickness of the slit between the top of screw flight and the inner surface of the barrel) and helix angle φ. They are presented in Fig. 2.

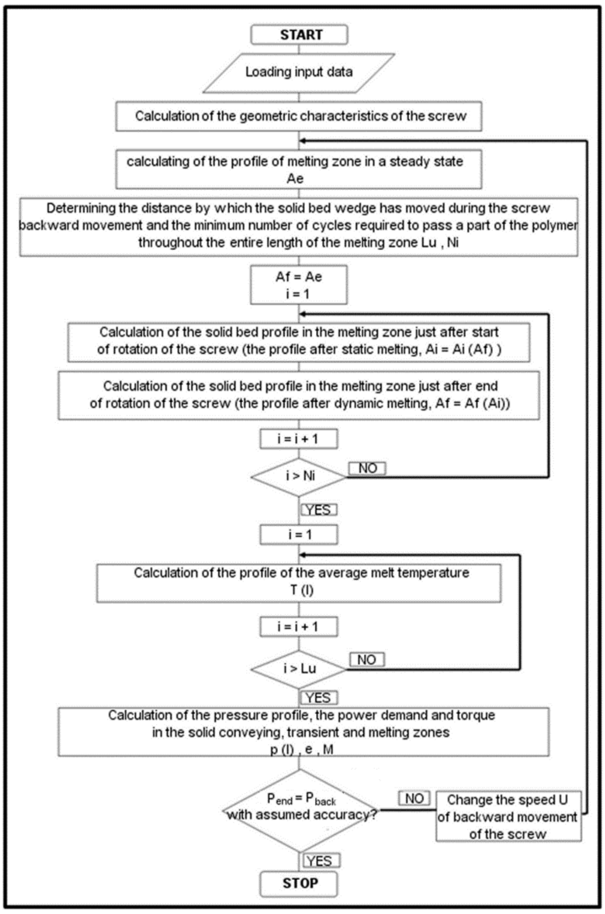

Fig. 1 *The algorithm of the mathematical model of polymer plasticization in injection molding*

In order to create a mathematical model of the plasticization process during the injection molding, several assumptions were made. Firstly, the existence of three dynamical zones in the plasticizing system was assumed:

1. a feed port and a solid conveying zone
2. a transient (delay) zone
3. a melting and melt conveying zone

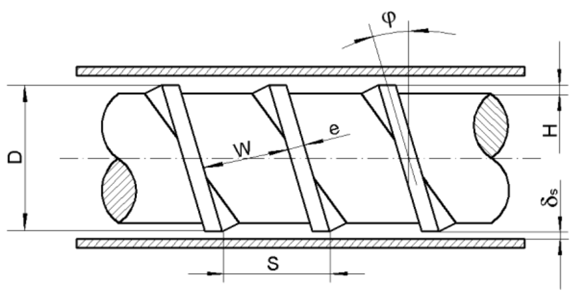

Fig. 2 *Geometry of the screw-barrel system of the injection screw machine*

Furthermore, the flat (rectangular) screw channel model was assumed.

The starting point for the model is the model of steady-state extrusion that is similar to the classical extrusion model of Tadmor and Klein [3]. However, in contrast to the steady conditions characteristic for extrusion, the lengths and positions of dynamical zones change in time within the injection cycle. To describe these time changes it was adopted, that two coupled states (appearing at two characteristic moments of time) are valid during the cycle:

1. at the end of screw rotation (the beginning of static melting)
2. at the beginning of screw rotation (the beginning of dynamic melting)

Moreover, it was assumed that the melt behavior can be described by the power law of the form:

$$\tau = k_0\, e^{-a(T-T_o)} \left(\frac{1}{2} II_d \right)^{n-1} d \tag{1}$$

where τ - the extrastress tensor, d – the rate-of-strain tensor, II_d – the second invariant of d-tensor, T_0 – the reference temperature (usually assumed as the polymer melting temperature), k_0, n, a - rheological parameters: k_0 – the consistency coefficient, n – the power-law exponent, a – the temperature coefficient.

solid conveying zone

It was assumed that the dynamic equilibrium in the solid conveying zone is established fast enough. Hence, its operating characteristics can be adequately described by means of relations, that are valid for the steady-state conditions [3]. However, the axial velocity component U of rotating and withdrawing screw should be taken into account.

Assuming the flow continuity, the mass flow can be calculated both from the solid bed velocity and from the screw withdraw velocity as

$$\dot{G} = H\, W\, V_{sz}\, \rho_s \tag{2}$$

$$\dot{G} = \frac{1}{4}\, \pi\, D^2\, U\, \rho_m \tag{3}$$

where H – the channel height, W – the average channel width, V_{sz} – the solid bed velocity along the screw channel (in z-direction), ρ_s – the density of solid polymer, D – the outer screw diameter, U - the axial screw velocity, ρ_m – the average density of polymer melt; The V_{sz} velocity is determined as

$$V_{sz} = V_b \frac{\sin\theta}{\sin(\varphi+\theta-\gamma)\, \cos\gamma} \tag{4}$$

where V_b – the barrel velocity, φ – the helix angle, $\gamma = arctan\left(\frac{U}{V_b}\right)$, θ – the solid conveying angle.

If the mass flow $\dot{G}$ is known, the values of V_{sz} and U can be calculated and this makes possible to calculate the solid conveying angle θ from Eq. (4). If this angle is known, the pressure profile in the solid conveying zone can be determined using the force and torque balance [4]. A general equation describing the pressure changes over the zone length has the form:

$$p_2 = p_1\, exp(k\, \Delta) \tag{5}$$

where k – the parameter determined from the balance of forces and moments of force acting on the material layer of elementary thickness dz [4], Δ – the length of one computational step in z-direction.

The initial pressure p_0 in the feed port region can be calculated according to the simple formula proposed in [5]:

$$p_0 = \rho_0\, g\, D \tag{6}$$

where ρ_0 – the bulk density, g – the gravitational acceleration.

For the given pressure profile it is possible to determine the power demand e_S in the solid conveying zone as the sum of power dissipated at the barrel, screw root, screw flights and the power used to increase the pressure in the solid bed:

$$e_s = p(l) f_b\, W\, \Delta\, V_j + p(l) f_s\, W\, \Delta\, V_{sz} + 2p(l)\, f_s\, H\, \Delta\, V_{sz} + H\, W\, V_{sz} \frac{dp}{dl} \tag{7}$$

where p(l) is the polymer pressure in l-location; l is the location on the screw channel length (in z-direction) measured by a number of computation steps from the beginning of the screw; f_b , f_s – the barrel and screw friction factors of solid polymer, respectively, V_j – the velocity of solid bed transport relative to the barrel, determined as

$$V_j = V_b \frac{\sin(\varphi-\gamma)}{\sin(\varphi+\theta-\gamma)\, \cos\gamma} \tag{8}$$

The torque M_S in the solid conveying zone was determined in the classical way as the product of the dry friction force and the lever arm:

$$M_s = p(l)\, f_b\, W\, \Delta\, \frac{D}{2} \tag{9}$$

From the presented relations it follows that the main difference in description of solid conveying zone action during extrusion and injection is the existence of the non-zero retraction velocity of the screw.

transient zone

The transient zone in the model starts at a point, where melt layer appears at the solid bed surface. It was adopted, that this is the place of the screw channel which corresponds with the beginning of the barrel heating zone at a given moment of time. The end of the transient zone in the screw channel corresponds with the point, where the melt film thickness reaches a critical value δ_w [3]. In contrast to the solid conveying zone, the total length of the transient zone is variable and it depends on the process conditions. The length of this zone is very short and it usually reaches half to two coils. However it is important to consider this zone, because it allows continuity of the pressure profile as well as the correct calculation of the total power demand and torque on the screw. According to [3] it was assumed that the melt film thickness changes linearly from 0 to δ_w over the zone length. These changes depend on the rate of dynamic melting, that can be calculated from [3], assuming additionally the axial screw velocity U [1].

The calculations of the pressure changes in the transient zone base on the assumption that the pressure gradient in this zone can be determined as a weighted average of the pressure gradient at the end of the solid conveying zone and the pressure gradient at the beginning of the melting zone:

$$\left(\frac{\partial p}{\partial z}\right)_t = \left(\frac{\partial p}{\partial z}\right)_s (1-x) + \left(\frac{\partial p}{\partial z}\right)_m x \tag{10}$$

where the subscripts t, s and m mean: the transient zone, the solid conveying zone and the melting zone, respectively, x – the weight factor, changing from 0 to 1 on the length of the transient zone.

This semiempirical approach that provides a smooth pressure profile at the zone boundaries was introduced, because there is no exact method of pressure calculation in the case, if the flow is determined by both dry and viscous friction.

The power demand e_T in the transient zone was defined as the sum of the power dissipated in the thin melt film at the barrel surface, on the root surface and the flight surfaces, as well as the power desired to increase the pressure. It can be calculated for one computation step from the equation

$$e_T = \left[p(l) f_b\, (1-x) + \tau_j\, x\right] W\, \Delta\, V_j + p(l) f_s\, W\, \Delta\, V_{sz} + 2\, p(l) f_s\, (H-\delta_m)\, \Delta\, V_{sz} + H\, W\, V_{sz} \frac{dp}{dl} + \tau_e\, e\, \Delta \sqrt{V_b^2 + U^2} \tag{11}$$

where τ_j – the shear stress in the polymer melt layer in the screw channel, δ_m – the melt layer thickness in the screw channel in the transient zone, changing linearly from 0 to critical value δ_w : $\delta_m = \delta_w x$ (x – the weight factor), τ_e – the shear stress in the polymer melt layer in the slit between the top of the screw flight and the barrel surface, e – the width of the screw flight; τ_j and τ_e quantities are defined according to the power law as follows:

$$\tau_j = k_0 \exp\left(-a\,(T_b - T_m)\right)\left(\frac{V_j}{\delta_m}\right)^n \qquad (12)$$

$$\tau_e = k_0 \exp\left(-a\,(\bar{T}_{slit} - T_m)\right)\left(\frac{\sqrt{(V_b^2+U^2)}}{\delta_s}\right)^n \qquad (13)$$

where k_0, a, n – the parameters of the power law equation, δ_s – the slit thickness, T_b – the average barrel temperature in the heated zone, T_m – the melting (flow) temperature of polymer, $\bar{T}_{slit}$ – the average polymer temperature in the slit.

The first term in Eq. (11) assumes the occurrence of the polymer-barrel friction as a weighted average of dry (p (1-x)) and viscous (τ x) friction. The last term of the Eq. (11) refers to the power dissipated in the thin polymer layer existing between the top of the screw flight and the inner barrel surface as a result of the leakage flow. The determination of the polymer temperature in the slit bases on an energy equation, which assumes neglecting of the convection and the conductivity along and across the slit for the generalized Newtonian liquid and (assuming adiabatic flow) takes the simplified form:

$$\rho_m c_m \frac{\partial \bar{T}}{\partial t} = \bar{\eta}\left|\vec{\dot{\gamma}}\right|^2 \qquad (14)$$

wherein, according to the power law equation, the average viscosity can be defined by the formula

$$\bar{\eta} = k_0 \cdot e^{-a(\bar{T}-T_m)}\left|\vec{\dot{\gamma}}\right|^{n-1} \qquad (15)$$

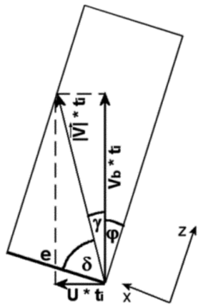

Fig. 3 *The vector analysis of the polymer particle displacement in the slit*

Analyzing the displacement of the polymer particle in the slit, the shear time t_i in the slit is equal to the transition time from one to the other edge of the slit with the width e and the helix angle φ in the direction and the velocity determined by the vector $\vec{V}$. The shear time can be easily calculated on the basis of the geometrical relationships from Fig. 3 as:

$$t_i = \frac{e}{|\vec{V}|\ \sin(\varphi+\gamma)} \qquad (16)$$

where $|\vec{V}| = \sqrt{V_b^2 + U^2}$ (17)

The equation (14) with the viscosity expressed by the formula (15), integrated with the initial condition $\bar{T}(t=0) = T_b$ and taking into account the formula (18) allows to obtain the expression which describes the polymer temperature in the slit:

$$\bar{T}_{slit} = T_b + \frac{1}{a} ln\left[\frac{k_0\, a\, e^{-a(T_b - T_m)}}{\rho_m\ c_m\ sin\,(\varphi+\gamma)}\left|\frac{\vec{V}}{\delta_s}\right|^n \frac{e}{\delta_s} + 1\right] \qquad (19)$$

where c_m – the average specific heat of the polymer melt.

The torque M_T in the transient zone was determined as

$$M_T = [p(l)\, f_b\, (1-x) + \tau\, x]\, W\, \Delta\, \frac{D}{2} \qquad (20)$$

where τ is the shear stress defined as

$$\tau = k_0 \exp\left(-a\,(T_b - T_m)\right)\left(\frac{V_b}{\delta_m}\right)^n \qquad (21)$$

Melting zone

The melting process during the injection molding is more complicated in comparison with the extrusion, mainly due to the existence of the static melting phase (for the stationary screw). Moreover, the phase of the dynamic melting must additionally take into account the axial screw motion. Both phases (static and dynamic melting) are coupled. The final conditions for one of them are the initial conditions for the second one.

Defining of the function of the solid bed width distribution in time and space (along the screw channel) is the basis for the plasticization model with the three-zone-screw. It can be described with the general relation

$$\frac{X}{W} = f(l,t) \qquad (22)$$

The spatial coordinate l (along the screw channel) in the equation (22) is expressed in the number of computation steps.

It was assumed that three areas of different behavior of the polymer material, shown in Fig. 4, can be distinguished in the screw channel cross section in the melting zone [3,6]. There is an area occupied by a homogeneous wedge of solid polymer, a melted polymer layer at the barrel surface and a melt pool.

Due to the variable channel height along the screw it is convenient to express the solid bed profile as the function

$$A(l,t) = H(l)\frac{X(l,t)}{W} \qquad (23)$$

where $A(l,t)$ - the ratio of the cross-sectional area of the channel occupied by the solid bed to the total cross-sectional area of the channel (see Fig. 4), $H(l)$ – the relative height of screw channel, determined as $H(l)$ = H/H_f , changing from 1 (in the geometrical feed zone) to H_m/H_f (geometrical metering zone) on the screw channel length.

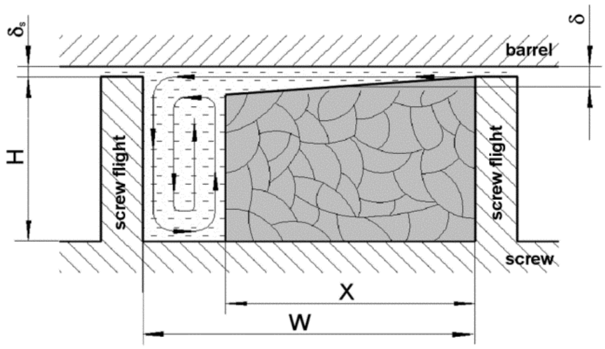

Fig. 4 *The screw channel cross-section in the melting zone*
(δ – average thickness of the polymer melt layer)

Knowing the $A(l)$ function in the two basic moments of time, is necessary to describe of the melting process in the injection molding:

1. just after the start of the screw rotation (beginning of dynamic melting): $A(l,0) = A_i(l)$
2. just after the finish of the screw rotation (beginning of static melting): $A(l,t_r) = A_f(l)$, where t_r is the recovery time defined as

$$t_r = \frac{N_s\, S}{U} \qquad (24)$$

where N_S is the screw stroke (expressed in number of coils).

The static melting begins after the stopping the rotational screw motion. The solid polymer is molten in a certain time interval, which is approximately equal to the cooling time, and then the screw is shifted forward on the distance of the screw stroke. The polymer is molten now in the time approximately equal to the hold time. According to the known theories of static melting [6,7] it was assumed that the time dependent melt film thickness δ_t molten in contact with the hot barrel surface is given by the equation:

$$\delta_t = \delta_0 + k\sqrt{t} \qquad (25)$$

where δ_0 is the initial thickness of the melt, k is the root of the algebraic, non-linear, complex equation [6].

Assuming that the state after dynamic melting A_f is the initial state for the phase of static melting, the solid bed profile A_i after static melting can be determined as follows:

$$A_i(l) = A_f(l)\frac{H-\delta_t(l)}{H-\delta_0(l)} \quad (26)$$

Dynamic melting starts at the moment of the beginning of the screw rotation. The calculation of the solid bed profile after the screw rotation period was done using the theory of dynamic extrusion [8]. The basic equation, that describes the differential mass balance in the solid bed under unsteady conditions can be presented in the following way:

$$\frac{\partial A}{\partial \tau} + \frac{\partial A}{\partial l} = -\frac{\Phi}{V_{sz}\,\rho_s\,\sqrt{H}}\,\sqrt{\frac{A}{H(l)}} \quad (27)$$

where Φ - the auxiliary variable associated with the rate of dynamic melting [3], τ - the dimensionless time variable defined as

$$\tau = \frac{t\,V_{sz}}{\Delta} \quad (28)$$

The expression (27) is a non-linear partial differential equation of the first order, which describes the evolution of the relative cross-section area of the solid bed in time and space during the screw rotation. The equation (27) has been solved analytically for the purposes of the model with the assumption of the 3-zone screw [1]. This solution has been adapted in the model.

The quantity L_u defined by equation (29) describes the solid bed displacement due to the rotary-backward motion of the screw and it is expressed in the computation steps. The solid bed evolution during the screw rotation can be followed by changing L_u from 0 to the end value given by the equation (29).

$$L_u = \frac{t_r\,V_{sz}}{\Delta} = \frac{N_5\,S\,V_{sz}}{U\,\Delta} \quad (29)$$

Equilibrium values of A after the static and dynamic melting can be calculated using the iteration method. The steady-state profile A_e characteristic for the extrusion process [3] taking additionally into account the backward screw motion could be assumed as the first approximation of A [1]. Hence, the approximated A_i profile after static melting can be determined. It is the initial value for the new profile of the solid bed A_f calculated for dynamic melting. The iteration is repeated until A profiles (after static and dynamic melting) will be established. It could be shown that the time required for stabilization of A_f and A_i is not shorter than the passage time of the first polymer portion over the whole channel length. It corresponds to a certain value of N_i iteration cycles [1].

If the solid bed profiles are known, the pressure and temperature profiles in screw channel can be calculated. Knowing these profiles makes it possible to calculate other quantities: the power demand, the screw torque and the energy consumption that are important for the detailed characterization of the plasticization process. All the quantities were calculated for the A_f profile after the dynamic melting, which is characterized by the maximal filling of the screw channel with the solid polymer.

The temperature profile is the result of thermal processes during the whole screw rotation phase. The methods of polymer melt temperature calculation, which are valid for the steady-state conditions, could not be applied for the calculation of the temperature profile in the injection molding. In this case, the temperature profile was determined by an approximated method described in [5], which was adapted to the model requirements.

The averaging equation of energy for the polymer melt region is represented by the expression:

$$\rho_m\,c_m\,\frac{\partial T}{\partial t} = k_m\,\overline{\frac{\partial^2 T}{\partial y^2}} + \overline{\left(\tau_{xy}\,\frac{\partial v_x}{\partial y} + \tau_{zy}\,\frac{\partial v_z}{\partial y}\right)} - \rho_m\,c_m\,\overline{V_{mz}}\,\frac{\partial T}{\partial z} - \rho_m\,c_m\,\overline{V_{mx}}\,(T - T_b) \quad (30)$$

where k_m – the average thermal conductivity of the molten polymer, $\overline{V_{mz}}$ – the average flow rate of the polymer melt along the screw channel, defined as

$$\overline{V_{mz}} = \frac{\dot{q}_z}{H\,W} \quad (31)$$

$\dot{q}_z$ is the volumetric flow rate. The method of the exact determination of $\dot{q}_z$ is shown later.

$\overline{V_{mx}}$ – the average inflow rate of the polymer melt from the melting layer:

$$\overline{V_{mx}} = \frac{\Phi\sqrt{\frac{A(l)}{H(l)}}}{\rho_m\,H\left(1-\frac{A(l)}{H(l)}\right)} \quad (32)$$

The respective terms in the equation (30) represent the following quantities: the heat accumulation rate, the heat conduction rate, the rate of the heat generation by viscous friction, the heat convection rate and the rate of heat input from the melt film. The average values of the terms describing the energy conduction and dissipation can be calculated in a similar way as presented in [9] for the steady-state extrusion. Assuming the constant barrel temperature and the neutral screw, after several transformations the equation (30) takes the following dimensionless form:

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial l} = -B(l,\tau)\,\theta + C(l,\tau)\exp(-a\,\theta) \quad (33)$$

where θ, τ, l – the dimensionless values of T, t and z, respectively; B, C – the complex variables [1], a – the temperature coefficient; the dimensionless time variable τ is defined as

$$\tau = \frac{t\,\overline{V_{mz}}}{\Delta} \quad (34)$$

The equation (32) is a nonlinear, partial differential equation. It was solved numerically (by the similar method presented in [9]). This solution has been adapted in the model. This made it possible to calculate the temperature profile of the molten polymer in the melting zone.

For the pressure calculation we have assumed that the polymer pressure in the screw channel is stabilized fast enough. Hence, for its calculations the same methods can be used as for the steady-state conditions. The pressure was calculated according to the own method based on the results of the analysis of the two-directional, non-isothermal flow of the Ellis fluid in the rectangular channel [10]. The flow of the power law fluid is described by the following set of equations:

$$\chi\int_0^1 (\eta - \eta_x)\,\Psi(\eta)\,d\eta = U_x \quad (35a)$$

$$\int_0^1 (\eta - \eta_z)\,\Psi(\eta)\,d\eta = U_z \quad (35b)$$

$$\chi\int_0^1 \eta\,(\eta - \eta_x)\,\Psi(\eta)\,d\eta = U_x \quad (35c)$$

$$\int_0^1 \eta\,(\eta - \eta_z)\,\Psi(\eta)\,d\eta = U_z - \frac{\dot{q}_z}{W\,H\,\nu_0} \quad (35d)$$

where η, η_x, η_z, χ are the dimensionless quantities. η_x, η_z are the integration constants of the equation of motion and the $\Psi(\eta)$ function is defined as

$$\Psi(\eta) = \left[(\eta - \eta_z)^2 + \chi^2\,(\eta - \eta_x)^2\right]^{\frac{1-n}{2n}} \quad (36)$$

$$\nu_0 = H\,\left|\frac{\partial p}{\partial z}\,F_p\,\frac{H}{k_0}\,exp[a(\bar{T} - T_m)]\right|^{\frac{1}{n}}\,sgn\left(\frac{\partial p}{\partial z}\right) \quad (37)$$

$$U_x = \frac{V_{bx}^*}{\nu_0} \quad (38)$$

$$U_z = \frac{V_{bz}^*}{\nu_0}\,F_d \quad (39)$$

F_d and F_p are the shape factors for the drag and pressure flow for the Newtonian fluid [3]. $\bar{T}$ is the average temperature of the polymer melt. The transversal and longitudinal velocity components of the barrel V_{bx}^* and V_{bz}^* with respect to the screw channel in the presence of the backward screw motion can be defined by expressions:

$$V_{bx}^* = V_b\,\frac{\sin(\varphi-\gamma)}{\cos\gamma} \quad (40)$$

$$V_{bz}^* = V_b\,\frac{\cos(\varphi-\gamma)}{\cos\gamma} \quad (41)$$

For known V_{bx}^*, V_{bz}^*, $\dot{q}_z$, W and H values, the local values of η_x, η_z, χ and v_0 can be calculated by solving the system of equations (35a-d). It was solved with the iteration method. As the first approximation the Newtonian values of η_x, η_z, χ and v_0 were used [10]. The integrals were calculated by the Gaussian quadratures method with five nodes. This made it possible to calculate the local pressure gradient

$$\frac{\partial p}{\partial z} = \frac{1}{\Delta}\,\frac{\partial p}{\partial l} \quad (42)$$

from the equation (37) and then to calculate the pressure profile, if the average polymer melt temperature $\bar{T}$ is determined.

The total power demand in melting zone e_M is the sum of the power dissipated in the melt region due to the longitudinal and transversal flow, the power dissipated in the slit between top of screw flight and the barrel inner surface and the power required for pressure changes:

$$e_M = \tau_z\, W\, \Delta\, V_{bz}^*\, F_d + \tau_x\, W\, \Delta\, V_{bx}^* + \tau_e\, e\, \Delta \sqrt{V_b^2 + U^2} + H\, W\, \overline{V_{mz}}\, \frac{dp}{dl} \quad (45)$$

where

$$\tau_z = k_0 \exp[-a(\bar{T} - T_m)] \left(\frac{V_{bz}^*\, F_d}{H}\right) \quad (46)$$

$$\tau_x = k_0 \exp[-a(\bar{T} - T_m)] \left(\frac{V_{bx}^*}{H}\right) \quad (47)$$

τ_e – the shear stress in the polymer melt layer in the slit, defined by eq. (13).

The torque M_M in the melting zone was determined as a sum:

$$M_M = \tau_{xz}\, W\, \Delta\, \frac{D}{2} + \tau_e\, e\, \Delta\, \frac{D}{2} \quad (48)$$

where τ_{xz} and τ_e are defined respectively as

$$\tau_{xz} = \sqrt{{\tau_x}^2 + {\tau_z}^2} \quad (49)$$

$$\tau_e = k_0 \exp\left(-a\left(\bar{T}_{slit} - T_m\right)\right) \left(\frac{V_b}{\delta_s}\right)^n \quad (50)$$

One of the most fundamental questions in the model is the determination of the screw retraction velocity U and the pressure profile, where the pressure value at the screw end is equal to the known back pressure (operating parameter). Both quantities are strictly coupled and their determination closes the computation cycle. Hereafter, that makes it possible to calculate the most important process characteristics such as the plasticization rate, the power requirement, the screw torque, the average melt temperature and the specific energy consumption. The choice of the proper backward velocity U for a given back pressure was done with the iteration method using a special control algorithm. It increases or decreases the U value depending on the calculated pressure on the screw end and the assumed back pressure, until both pressures become equal with a desired accuracy.

3. Experimental results and discussion

The test office for the measurements of the output parameters of the plasticization process during the injection molding consists of the suitably instrumented injection molding machine linked to the collecting and processing data module and the computer for imaging and saving of the collected data. The test office shown in Fig. 5 consists of:

1. the injection molding machine Battenfeld Plus 350/70,
2. four pressure/temperature sensors (analog CDTAI200-1/2-1500-1-1-1J (Bagsik Sp. z o.o.), range 0-150 MPa, 0-300 °C, OE: ±0.5% FS),
3. the torque - measuring device (analog sensor DMF2X-250 (MEGATRON Elektronik GmbH & co. KG), range 0-250Nm, OE: ±1% FS),
4. the inductive sensor for the screw rotational velocity measurements (induction detector E2A-S08KS02-WP-B1, Omron Corp.),
5. the screw linear displacement sensor (analog sensor LWH 0150 (Novotechnik U.S. Inc.), range 0-150 mm, LE: ±0.08%),
6. the control cabinet with the touch screen.

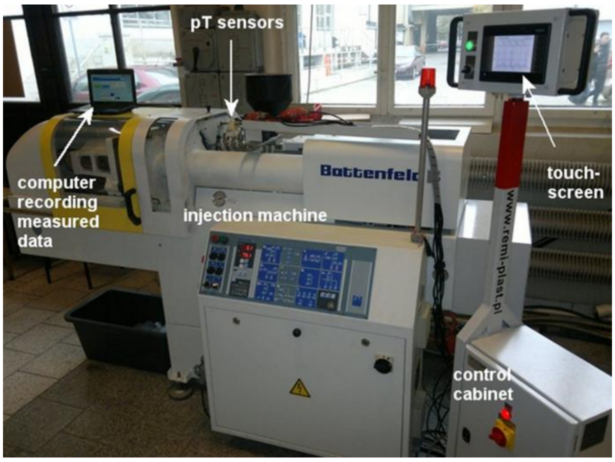

Fig. 5 *The test office for the plasticization process in the injection molding*

The results presented below are related to the studies involving selected examples from five typical thermoplastic polymers characterized in Table 1. The variable parameters of the injection molding machine during the plasticization process study were the following:

1. the back pressure (changed in range of 4-24 MPa)
2. the screw velocity (changed in range of 30-70% of the maximal screw velocity)
3. the dwell time (changed in range of 8-50 s) - approximately equal to the cooling time of the product in the mold
4. the average barrel temperature in the heated section

Tab. 1 *The types of tested polymers*

PE-LD	PE-HD	PP	POM	PS
Malen E FABS 23D022	Hostalen GC 7260	HP 515M	Schulaform 9A	Krasten 154

Table 2 shows the values of the adjustable parameters of the injection molding machine used in the experiments. Other process parameters were kept constant.

Tab. 2 *The adjustable parameters of the injection molding process*

	back pressure [MPa]				
	3	6	10	16	24
	screw rotational velocity [rpm]				
	154	200	240	286	333
	dwell time [s]				
	8	12	20	30	50
	average barrel temperature [°C]				
	T1	T2	T3	T4	T5
PE-LD	140	160	180	200	220
PE-HD	150	170	190	210	230
PP	190	210	230	250	270
POM	-	-	210	-	-
PS	-	180	200	220	240

The studies on the plasticization process in the injection molding were carried out as two independent series of experiments:

1. The study by changing only one of the parameters listed in Tab. 3, and keeping constant the values of the other parameters, which were always equal to the third value in Tab. 2. The back pressure, the screw velocity and the dwell time were the same for all polymers. The symbols of T1-T5 were introduced due to the different average barrel temperature for processing of five polymers mentioned above. All three heating zones of the barrel were kept at the same (constant) temperature during experiments.
2. The study by changing simultaneously two of the adjustable parameters listed in Tab. 2 and keeping constant the values of the other parameters, which were always equal to the third (middle) value from Tab. 2.

During the experiment, all the most important characteristics of the process were measured: the pressure and temperature profiles on the screw length, the torque on the screw, the power supplied to the heaters, the mass yield of the plasticization process and the recovery time.

Because there is a lot of results and they are very similar for different polymers in aspect of the shape of curves and the differences for the relevant theoretical and experimental characteristics, it was decided to present the output characteristics of one polymer for the study with simultaneously two of the adjustable parameters changing. In order to standardize the charts, the characteristics obtained from the model are shown as thick lines without markers, while the experimental profiles represent the markers indicating the measurement points.

In the analyzed example, the rotational velocity of the screw and the plasticizing pressure in the tests were changed at the same time. These parameters are presented in Table 3. Other working parameters of the injection machine were constant, as in the first part of the studies. The tests were performed for PE-LD, PP and PS. The exemplary results are shown for PP.

Tab. 2 *Changed parameters in the second part of the studies*

screw rotation velocity [rpm]	back pressure [MPa]
60	1.0
154	4.0
240	10
333	16
400	21

A comparison of the output characteristics for the injection molding of PP is presented in Figs. 6 and 7. We can see a very good agreement between the model curves and the experimental points. The model provides a very good pressure profile as well as the power demand, the torque and the process yield. Only the temperature profile is predicted by the model with the differences of approx. 5-7% for high screw rotational velocity. The same situation occurs for PE-LD and PS. In order to generalize these results, similar tests should be performed by changing other pairs of the injection working parameters. Research works in this area are continued.

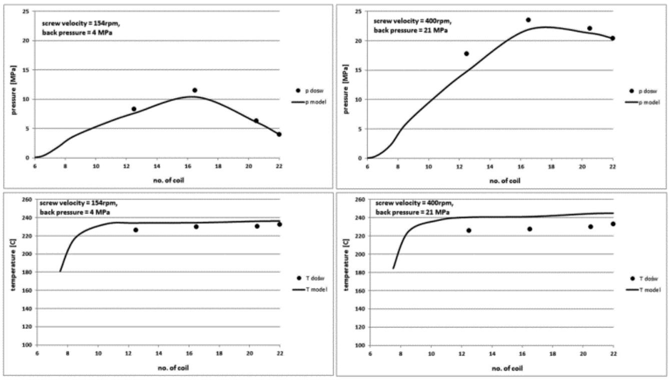

Fig. 6 *Comparison of the theoretical and experimental characteristics of the pressure (top) and temperature profiles (bottom) on the screw channel length in PP injection process for the screw rotational velocity of 154 rpm and back pressure of 4 MPa (left) and 400 rpm and 21 MPa (right)*

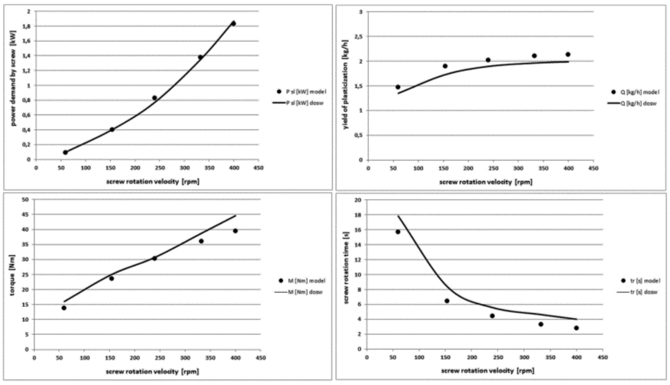

Fig. 7 *Comparison of the other theoretical and experimental characteristics (power demand, torque, yield of plasticization and screw rotation time) for the different screw rotation velocity and back pressure values in PP injection process*

4. Conclusions

The paper presents the comprehensive model of the plasticization process during the injection molding. The experimental verification of the model using the specially designed and built research office was described. The plasticization studies of five typical thermoplastic polymers during the injection molding with different values of the back pressure, the screw rotational velocity, the dwell time and the barrel temperature were carried out. The values of experimental characteristics with the results generated by the simulation model were compared. It was found that the model correctly determines the dynamics of the plasticization process under the changes of the most important input parameters. The model predicts well the power demand and the torque values as well as the process yield. The average differences in the theoretical and experimental values do not exceed 10%. Slightly larger average differences (about 20%) occur in the determination of the pressure and temperature characteristics. It is worth noting, however, that the knowledge of the pT profiles is scientific and cognitive in nature. On the other hand, the results such as the power demand, the torque, the process yield and the SEC are of a practical importance. Here the model shows the results more compatible with the experience.

The model uses three groups of input parameters: geometrical parameters of the plasticizing system (mainly the screw), working parameters of the injection molding machine and material parameters of the polymer being processed. In the experimental verification of the model, the second and third parameter groups are changed. All tests were conducted using one injection molding machine and one screw. It is worth performing similar studies for a larger injection molding machine as well as at least two different screws with the different length of feed, compression and metering zones to generalize or detail the results presented in this work.

References

[1] Steller R., Iwko J.: Polymer Plastication during Injection Molding Part 1 Mathematical model, *Int. Polym. Process.*, **2008**, *23*, 252-262. DOI: 10.3139/217.2041

[2] Iwko J., Steller R.: Polymer Plastication during Injection Molding Part 2 Simulations and Experiments, *Int. Polym. Process.*, **2008**, *23*, 263-269. DOI: 10.3139/217.2042

[3] Tadmor Z., Klein I.: *Engineering Principles of Plasticating Extrusion*, Wiley-Interscience, New York 1970.

[4] Darnell W.H., Mol E.A., Solids Conveying in Extruders, *SPE Journal*, 1956, *12*, 20-29.

[5] Basow N.I., Kazankow Yu.W.: *Injection Moulding of Polymers*, Chimija, Moskwa 1984. (in Russian)

[6] Donovan R.C.: The plasticating process in injection molding, *Polym. Eng. Sci.*, **1974**, *14*, 101-111. DOI: 10.1002/pen.760140205

[7] Lipshitz S.D., Lavie R., Tadmor Z.: A melting model for reciprocating screw injection-molding machines, *Polym. Eng. Sci.*, **1974**, *14*, 553-559. DOI: 10.1002/pen.760140803

[8] Tadmor Z., Lipshitz S.D., Lavie R.: Dynamic model of a plasticating extruder, *Polym. Eng. Sci.*, **1974**, *14*, 112-119. DOI: 10.1002/pen.760140206

[9] Tadmor Z., Gogos C.G.: *Principles of Polymer Processing*, John Willey & Sons, New York 1979.

[10] Steller R., Iwko J.: Generalized Flow of Ellis Fluid in the Screw Channel Part 1 Parallel Plates Model, *Int. Polym. Process.*, **2001**, *16*, 241-248. DOI: 10.3139/217.1649