Coupled two-carrier planetary gearboxes for two-speed drives

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Abstract: *The following paper reviews all the possible cases of coupled two-carrier planetary gears with four external shafts. An emphasis is made on the work of these gears with one degree of freedom, one input and one output shaft and brakes on the other two shafts. When switching over the gears, the speed ratio of the gear is changed, thus allowing the use in two-speed mechanical transmissions of technological lifting and other machines. Some relations are deduced for determining the speed ratios and the efficiency of all structural schemes.*
Recommendations for the selection of the most appropriate structural scheme according to the current necessities can be made. A 3D model of the S13V3 two-speed, two-carrier gearbox was created to demonstrate the process of determining the viability of a particular gearbox layout.
KEYWORDS: COUPLED PLANETARY GEARS, STRUCTURAL ANALYSIS, EFFICIENCY, 3D MODEL.

1. Introduction

In some cases of machinery there comes the need of using two-speed transmissions with a definite ratio between the two speeds. The ability to switch over while in motion and loaded is a certain advantage and in some cases an inevitable necessity. One proper solution is the use of a coupled two-carrier planetary gear with four external shafts and two brakes (Fig. 1).

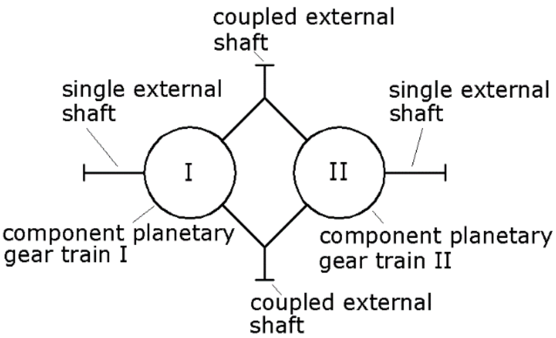
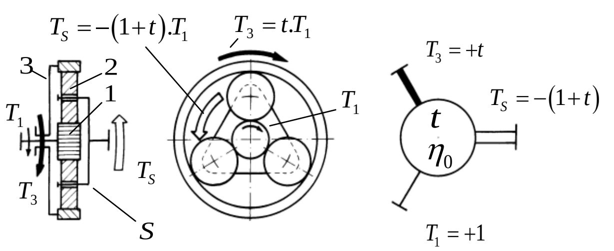


Fig. 1. *Schematic layout of a two-carrier planetary gear with four external shafts, two coupled and two single external shafts*

In [1-7] there is a review on the various possible ways of brake mounting and power flowing in the above-mentioned gear when it consists of two simple gears of the most common type (Fig. 2) [5-8] (consisting of a sun-gear 1, a ring-gear 3 and one-rimmed satellites 2, situated on the carrier *S*).
Practice shows that in the structural analysis of coupled planetary gears it is appropriate to use the structural symbol of Wolf, indicating the simple gear with a circle and the three external shafts with three different in width lines, corresponding to the torque magnitude of the corresponding shaft [1-7, 9, 10, 11].



Prerequisite: $\eta_0 = \eta_{13(S)} = \eta_{31(S)} = 1$

Torque ratio: $t = \dfrac{T_3}{T_1} = \dfrac{T_{D\max}}{T_{D\min}} = \left|\dfrac{z_3}{z_1}\right| > +1$

Torques: $T_1 : T_3 : T_S = T_{D\min} : T_{D\max} : T_\Sigma =$

$= T_1 : t.T_1 : -(1+t).T_1 = +1 : +t : -(1+t)$

$T_1 \equiv T_{D\min} < T_3 \equiv T_{D\max} < |T_S| \equiv |T_\Sigma|$

Fig. 2. *The most often used single-carrier planetary gear train and its torques*

The main force and kinematic characteristic of the simple planetary gear is the torque ratio *t*, through which the speed ratios in the gear can be expressed for all 6 cases of work with one degree of freedom [1-7, 9, 10, 11]:

$$t = \frac{T_3}{T_1} = \frac{T_{D\max}}{T_{D\min}}, \tag{1}$$

where: $T_1 \equiv T_{D\min}$ is the torque of the sun gear (the lowest torque), and $T_3 \equiv T_{D\max}$ is the torque of the ring gear (the higher differential torque).
The aim of the following paper is to analyze the possibilities given by all structural schemes of two-carrier planetary gears with four external shafts, consisting of simple planetary gears of the most common type (Fig. 2).

2. Structural Analysis

Table 1 shows all the possible ways of connecting the coupling gears [1]. The arrows indicate the isomorphism, thanks to which there are only 12 structural schemes for review (11, 12, 13, 14, 15, 16, 33, 34, 35, 36, 55 and 56).
The conducted analysis shows that the most clarity is achieved when reviewing 6 basic variants (according to the situation of the brakes), each of which with two sub-variants according to which brake is activated (Fig. 3).
When situating the two brakes on the two single shafts (V6 – Fig. 3) only one of the coupling gears will be working and the speed ratio of the coupled gear will depend only on the torque ratio of the working gear.
When situating the two brakes on the two coupled shafts (V1 – Fig. 3) the two coupling gears are connected in series and the speed ratio of the coupled gear will be equal to the product of the corresponding speed ratios of the coupling gears. It will depend on the torque ratios t_I, t_{II}, the structural scheme and on which of the brakes is closed.

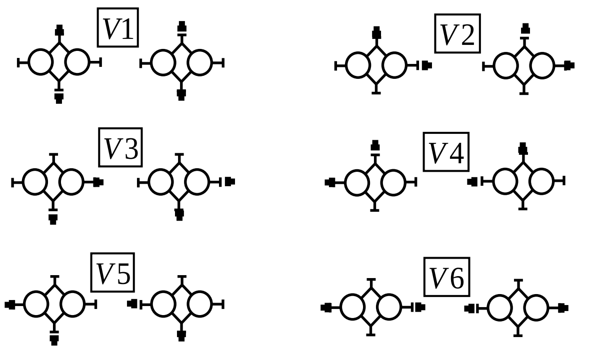


***Fig. 3.** Variants of work of the gears in review*

When situating the two brakes on one single and one coupled shaft there are four possible variants in one structural scheme. Having in mind that in the reviewed structural schemes there are different possible ways of power flowing (branching or circulation) with a fixed single shaft, the variants become eighth. When the brake of the coupled shaft is closed the speed ratio will depend only on the torque ratio of one of the coupling gears.

When the brake of the coupled shaft is open the speed ratio will depend on the torque ratios t_I, t_{II} of the coupling gears and the structural scheme in question. Essentially, the gear will be working as a two-carrier one with three external shafts, a case reviewed in detail in [4].

The cases from V1 – Fig. 3 can also be referred to the coupled two-carrier gears with three external shafts. However, they must work with a fixed coupled shaft.

***Table 1.** Structural schemes of coupled two-carrier planetary gears with four external shafts*

	...1 t_{II}	...2 t_{II}	...3 t_{II}	...4 t_{II}	...5 t_{II}	...6 t_{II}
1... t_I	11	12 21	13 31	14 41	15 51	16 61
2... t_I	21 12	22	23 32	24 42	25 52	26 62
3... t_I	31 13	32 23	33	34 43	35 53	36 63
4... t_I	41 14	42 24	43 34	44	45 54	46 64
5... t_I	51 15	52 25	53 35	54 45	55	56 65
6... t_I	61 16	62 26	63 36	64 46	65 56	66

3. Kinematic Analysis

The coupled two-carrier planetary gears with four external shafts are appropriate for two-speed drives.

The two-speed drive with one main speed and one micro-speed is common for mechanisms of lifting machines with higher positioning accuracy or operating with dangerous loads. It is also observed in technological and other machines.

In this case, given a certain angular velocity ω_A, three angular velocities of the output shaft when closing different brakes should be obtained. For example, a main speed ω_B and a micro-speed $\omega_{B\mu}$. Of particular interest is the possibility of reversing the motion - working displacement at low velocity (and high torque) and rapid return to initial position.

The torque ratios of the two coupling gears t_I and t_{II} can vary within the construction limited interval [4],

$$2 \le t_{I\min} \le t_I \le t_{I\max} \le 12 ,$$

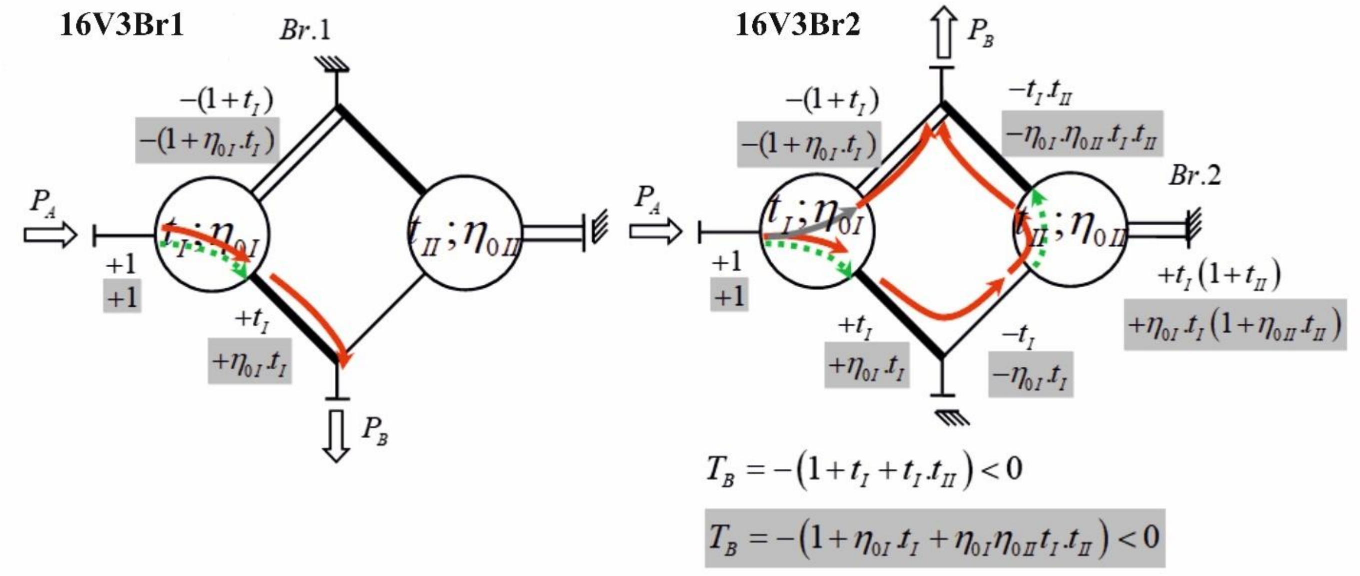
$$2 \le t_{II\min} \le t_{II} \le t_{II\max} \le 12 .$$

Therefore, if the necessary speed ratios of the two speeds $i_{AB} = \dfrac{\omega_A}{\omega_B}$ and $i_{AB\mu} = \dfrac{\omega_A}{\omega_{B\mu}}$ can be accomplished by a simple planetary gear, it is good to use structural schemes with brakes on the single shafts (V6 – Fig. 3).

With high-speed ratios i_{AB} and $i_{AB\mu}$ it is possible that the above-mentioned gear is a second stage, which gives the desired ratio between the two speed ratios. For example, in scheme 45 in Table 1 this ratio could be $2 \le \dfrac{\omega_B}{\omega_{B\mu}} \le 13$.

For all the different structural schemes in Table 1 relations are deduced for determining the speed ratios for the 12 cases of work with one degree of freedom.

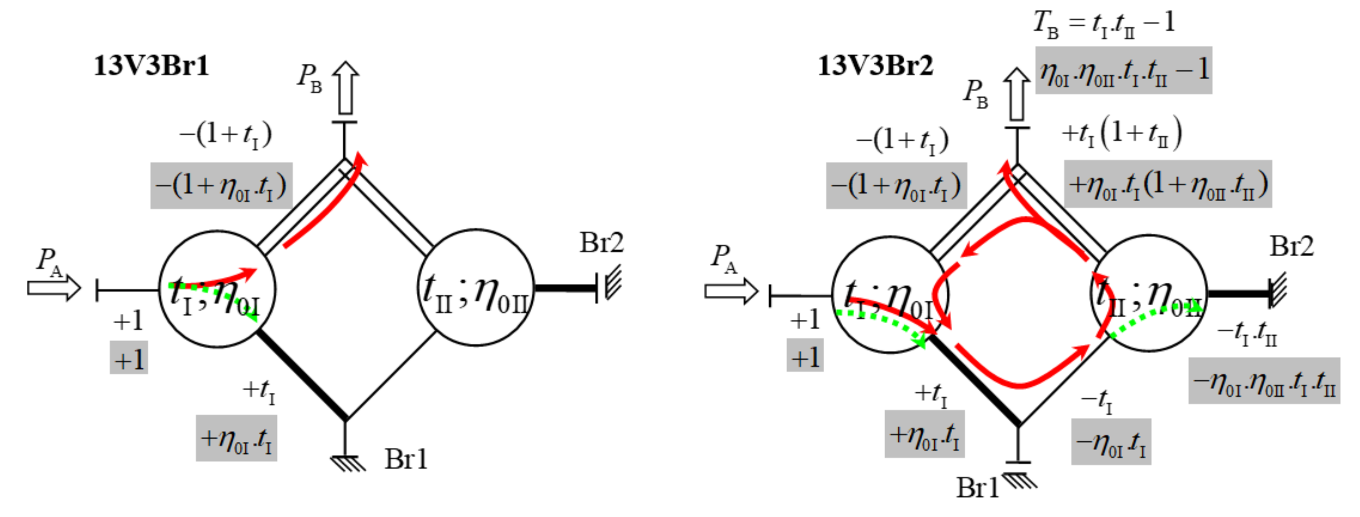
Fig. 4 exemplifies the way of determining the speed ratios for variant V2 in scheme 16 when activating the various brakes (V21 and V22). In this case there is an internal power division, while Fig. 5 shows an example with internal power circulation.



$$i_{Br.1}\equiv i_{V21}=-\frac{T_B}{T_A}=-t_I \qquad i_{Br.2}\equiv i_{V32}=-\frac{T_B}{T_A}=1+t_I+t_I.t_{II}$$

$$\eta_{Br.1}\equiv\eta_{V21}=\frac{(T_B)_{losses}}{(T_B)_{without\ losses}}=\eta_{0I} \qquad \eta_{Br.2}\equiv\eta_{V22}=\frac{(T_B)_{losses}}{(T_B)_{without\ losses}}=\frac{1+\eta_{0I}.t_I+\eta_{0I}.\eta_{0II}.t_I.t_{II}}{1+t_I+t_I.t_{II}}$$

Fig. 4. *Determining the speed ratios and efficiency for variant V3 in structural scheme 16 in Table 1*
- transmitted power; - rolling power



$$i_{Br1}=-\frac{T_B}{T_A}=+1+t_I \qquad i_{Br2}=-\frac{T_B}{T_A}=-(t_I.t_{II}-1)$$

$$\eta_{Br1}=\frac{\left(\frac{T_B}{T_A}\right)_{losses}}{\left(\frac{T_B}{T_A}\right)_{whitout\ losses}}=\frac{1+\eta_{0I}.t_I}{1+t_I} \qquad \eta_{Br2}=\frac{\left(\frac{T_B}{T_A}\right)_{losses}}{\left(\frac{T_B}{T_A}\right)_{whitout\ losses}}=\frac{\eta_{0I}.\eta_{0II}.t_I.t_{II}-1}{t_I.t_{II}-1}$$

Fig. 5. *Determining the speed ratios and efficiency for variant V3 in structural scheme 13 in Table 1*
- transmitted power; - rolling power

4. Efficiency

In this case it is most appropriate to determine the efficiency η from the input T_A and output T_B torques following the expression [1, 3, 4, 6, 7]

$$\eta = \frac{\left(\frac{T_B}{T_A}\right)_{losses}}{\left(\frac{T_B}{T_A}\right)_{without\ losses}},$$

(2)

where the torques with losses are determined with reading the direction of rolling power flow in the coupling gears. Relations are deduced for all the structural schemes in Table 1 for determining the efficiencies for the 12 cases of work with one degree of freedom as a function of the torque ratios t_I and t_{II} and the basic (when working with a fixed carrier) efficiencies η_{0I} and η_{0II} of the coupling gears. The latter could be assumed as constants [4], but they had better be determined with reading the impact of the number of teeth (i.e., of t_I and t_{II}) on the mesh losses by the methods described in [5].

Fig. 4 and Fig. 5 exemplify the way of determining the efficiency of the given variants. The torques of the shafts (the values on a dark background) are determined with reading the losses and direction of the rolling power in the coupling simple gears. In the case in review $(T_A)_{without\ losses} = (T_A)_{losses} = +1$, which simplifies formula (2).

It has been established that in all cases the expression in the numerator of (2) is obtained when in the denominator the torque ratios of the coupling gears (t_I or t_{II}) are multiplied by the corresponding basic efficiencies (η_{0I} or η_{0II}) when in the simple gear the rolling power is transmitted from the sun 1 to the ring 3. When the rolling power is transmitted from the ring to the sun the torque ratio of this simple gear is divided by the basic efficiency. In the examples in Fig. 4 and Fig. 5 only the first case is shown.

5. Geometrical modelling of the S13V3 planetary gearbox

To validate the feasibility of a gearbox layout, a kinematic or geometrical model should be used. Taking S13V3 as an example, we can summarize the following from the Wolf structural symbols:

1. The sun gear of the first gearset is the input element.
2. The planet carriers of the gearsets are coupled together to a common output shaft.
3. The ring gear of the first gearset is coupled to the sun gear of the second gearset.
4. Brake Br2 acts on the ring gear of the second gearset.
5. Brake Br1 acts on the ring gear of the first gearset, and consequently also on the sun wheel of the second gearset as it is coupled to the ring gear of the first gearset.

As soon as these basic concepts have been determined, a kinematic model of the gearbox can be created (Fig. 6), and appropriate attention may be given to specific parts of the design (e.g., the need for hollow shafts or specific brake placement).

With the kinematic model settled, a geometrical model may be created (Fig. 7).

This model will have appropriately sized gears fit for the purpose for which the gearbox is being designed, as well as a preliminary layout of bearings. Finally, an actual layout will be created, and brake actuation will be dealt with at this stage. Hydraulically actuated clutch-type brakes are used in modern planetary gearboxes.

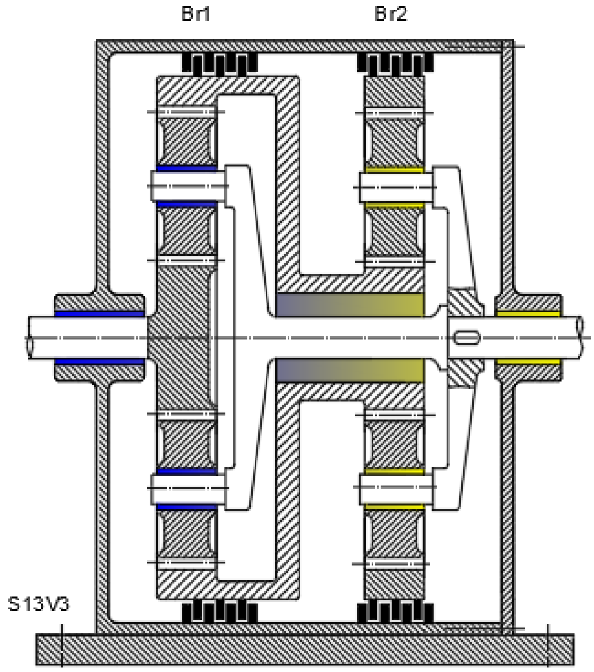


Fig. 6. *Kinematic model of S13V3 planetary gearbox*

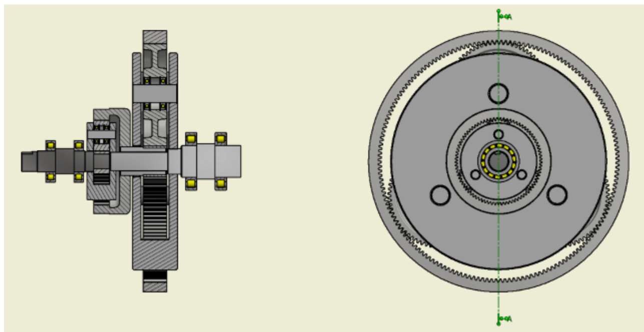


Fig. 7. *Geometrical model of S13V3 planetary gearbox*

However, band brakes and electric or electromechanical actuation may be used for ease of servicing or operation. In this case, electrically actuated band brakes are used to change the transmission ratio of the gearbox (Fig. 8).

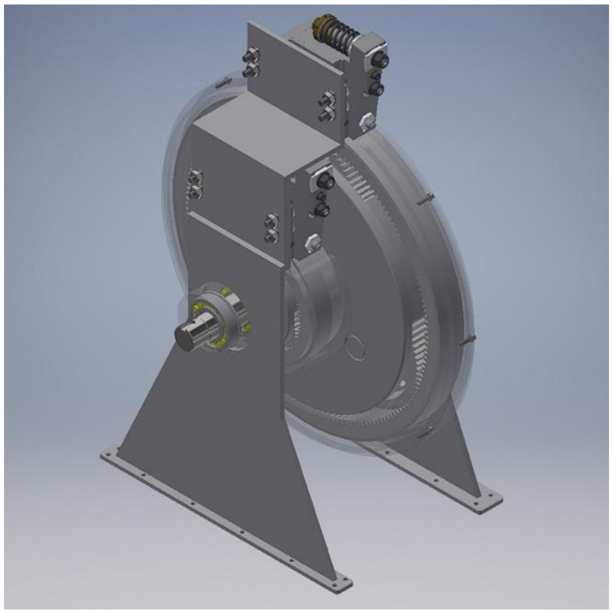


Fig. 8. *Complete model of S13V3 planetary gearbox*

6. Analysis and Discussion of the Results

The conducted structural analysis and the deduced formulae give the following conclusions (generalizations):
According to the structural scheme and the way of situating the brakes (the variant) different ratios of the output angular velocities are obtained with one and the same input velocity (i.e., of the two speed ratios).
In some cases when switching over the brakes the rotational direction of the output shaft is retained (12V1, 12V3, 13V1, 13V5, etc.), whereas in other cases it is reversed (12V2, 13V2, 13V3, 13V4, 14V2, etc.) In both cases it is possible that the gear continues its work as a reducer (respectively as a multiplier) or it might change its way of working – from reducer to multiplier (respectively from multiplier to reducer) [8, 12-15].

6.1 Retaining the rotational direction

In some of the cases of retaining the rotational direction the two speed ratios are close in value (15V3), sometimes close to 1 (15V4), even when the gear works as a reducer with one brake and as a multiplier with the other one (16V5, 36V5).
In other cases of retaining the rotational direction there is a significant difference between the two speed ratios (13V1, 13V6, 34V6, 36V2 and particularly 16V3) when the gear works as a reducer. In schemes 12V6, 13V5, 14V6, 15V5, 35V5 it works as a reducer with one brake and as a multiplier with the other one. Thus, a higher difference between the two speed ratios could be obtained (especially in 35V5). In scheme 56V6 when $i_{Br.1} = t_I$, $i_{Br.2} = \frac{1}{t_{II}}$, which allows the obtaining of $i_{Br.1} = \frac{1}{i_{Br.2}}$ when necessary.

6.2 Reversing the rotational direction

A great variety of options exists in the variants where the rotational direction is reversed. In 36V1 both speed ratios are close to 1, in 12V2 $i_{Br.1} = -t_I$ and $i_{Br.2} = 1 + \frac{1+t_I}{t_{II}}$, in 13V2 $i_{Br.1} = -t_I$ and $i_{Br.2} = \frac{t_I.t_{II} - 1}{1+t_{II}}$, whereas in 14V2 a great difference is obtained between the two speed ratios - $i_{Br.1} = -t_I$ and $i_{Br.2} = 1 + t_{II} + t_I.t_{II}$. The case is similar in 16V1. These schemes are appropriate for a technological machine with one displacement with high resistance (and low speed) and reversion to initial position with a higher speed (for increasing the productivity).
In other variants the gear works as a reducer with one brake and as a multiplier with the other one (15V6, 34V5, 35V1, 35V6) as in some cases there is a significant difference between the speed ratios.
In the structural schemes of the main diagonal in Table 1 for variants V2 and V3, according to the values of the torque ratios t_I and t_{II} of the coupling gears the rotational direction could be either retained or reversed. In the borderline case, when $t_I = t_{II}$, no motion is transmitted. The input shaft rotates, while the output one is motionless. In these cases, $i = \frac{\omega_A}{\omega_B} = \infty$ is obtained analytically, but the efficiency equals zero.
The formulae deduced for the speed ratios as a function of the torque ratios of the coupling gears serve as a basis for the establishment of a kinematic analysis program for the structural schemes in Table 1.

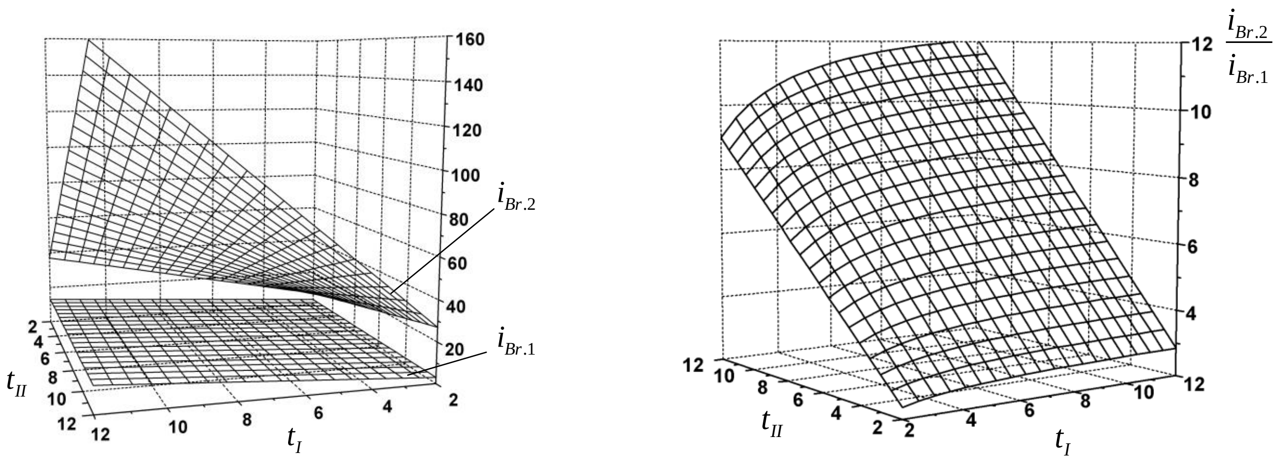


Fig. 9. *The speed ratios i_{Br1} and i_{Br2} (left) and their correlation (right) for variant V3 in structural scheme 16 (with activating different brakes) as a function of the torque ratios and of the coupling gears*

It can be used for picking out the structural schemes which can provide the desired ratio between the speed ratios in the case of a two-speed drive. The graphical representation of valid transmission ratios $i_{Br.1}$ and $i_{Br.2}$ as a function of the torque ratios of the coupling gears t_I and t_{II} (Fig. 9), facilitates the quick orientation of the designer about the capabilities of the various structural schemes and their variants.
The diagrams in (Fig. 9) may be used for computer-based generation of induced diagrams, an example of which may be seen in (Fig. 10).
This diagram provides an insight into the valid transmission ratio combinations of the gearbox, white the equations enable the basic torque ratios to be determined for the respective numbers of gear teeth.

Based on the formulae deduced for the efficiency as a function of the torque ratios and the basic efficiencies of the coupling gears, a program is established for determining the most appropriate structural scheme and its parameters from the standpoint of minimal losses.

6.3 Power circulation

Of all the schemes in review 15, 16, 35 and 36 are with internal power division (branching out) and the others are with internal power circulation (as in the case of gears with three external shafts [4]).
It has been established that in the schemes of the main diagonal of Table 1 (the symmetrical structural schemes) the direction of power circulation in one and the same variant is changed according to the magnitude of the torque ratios of the coupling gears. In the rest of

the structural schemes with power circulation the direction of the circulating power is not changed.

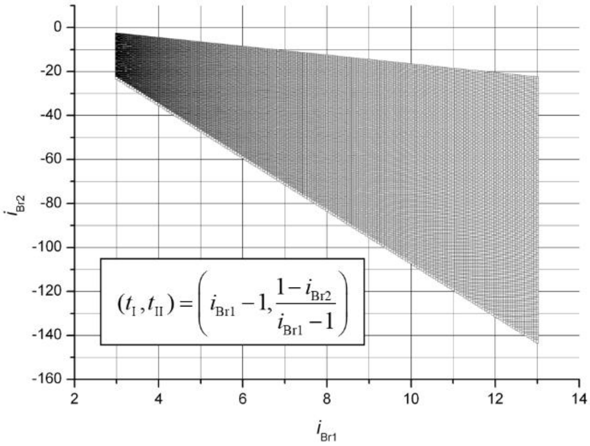


Fig. 10. *Diagram of valid transmission ratio combinations for S13V3 planetary gearbox*

7. Conclusion

All possible structural schemes of coupled two-carrier planetary gears with four external shafts and two brakes, working with one degree of freedom have been reviewed.

The analysis shows that it is sufficient to review just 12 structural schemes with the 6 possible variants for locating the brakes.

Relations have been deduced for determining the speed ratios and efficiencies for all the variants of the schemes in review.

An analysis has been conducted and recommendations have been given for when a certain scheme is appropriate to be used.

The analytical results are shown in Table 2, which allows the quick orientation in the kinematic capabilities of the structural schemes in review.

Program products have been established for kinematic and force analysis of the structural schemes, facilitating the designer in the choice of the most appropriate structural scheme and its parameters.

A 3D model of the S13V3 was developed for $i_{Br1} = 5$ and $i_{Br2} = -30$.

Table 2. *Kinematic capabilities of the different variants of the structural schemes in review*

11	**12**	**13**	**14**	**15**	**16**
V1 – **ret.** V2 – **ret.** $t_I > t_{II}$ – **rev.** $t_I < t_{II}$ V3 – **rev.** $t_I > t_{II}$ – **ret.** $t_I < t_{II}$ V4 ≡ V2 V5 ≡ V3 V6 – **ret.**	V1 – **ret.** V2 – **rev.** V3 – **ret.** V4 ≡ V3 V5 ≡ V2 V6 – **ret.** r.-m.	V1 – **ret.** ! V2 – **rev.** V3 – **rev.** ! V4 – **rev.** V5 – **ret.** r.-m. V6 – **ret.** !	V1 – **ret.** ! V2 – **rev.** ! V3 – **ret.** V4 – **ret.** V5 – **rev.** V6 – **ret.** r.-m. $i_{V61} = 1 + t_{II}$ $i_{V62} \approx 1$	V1 – **rev.** V2 – **ret.** V3 – **ret.** V4 – **ret.** $i_{V41} \approx i_{V42} \approx 1$ V5 – **ret.** r.-m. V6 – **rev.** r.-m.	V1 – **rev.** ! V2 – **ret.** V3 – **ret.** !! V4 – **ret.** $i_{V41} \approx i_{V42} \approx 1$ V5 – **ret.** r.-m. $i_{V51} \approx i_{V52} \approx 1$ V6 – **rev.** $i_{V61} \approx 1$ $i_{V62} = t_{II}$
33	**34**	**35**	**36**	**55**	**56**
V1 – **ret.** V2 – **ret.** $t_I > t_{II}$ – **rev.** $t_I < t_{II}$ V3 – **rev.** $t_I > t_{II}$ – **ret.** $t_I < t_{II}$ V4 ≡ V2 V5 ≡ V3 V6 – **ret.**	V1 – **ret.** r.-m. V2 – **rev.** r.-m. **!** V3 – **ret.** $i_{V31} \approx 1$ $i_{V32} = t_{II} + 1$ V4 – **ret.** $i_{V41} \approx i_{V42} \approx 1$ V5 – **rev.** r.-m. **!** V6 – **ret.** r.-m. **!**	V1 – **rev.** r.-m.**!** V2 – **ret.** r.-m. **!** V3 – **ret.** $i_{V31} \approx i_{V32} \approx 1$ V4 – **ret.** $i_{V41} \approx i_{V42} \approx 1$ V5 – **ret.** r.-m. **!** V6 – **rev.** r.-m. **!**	V1 – **rev.** $i_{V11} \approx i_{V12} \approx 1$ V2 – **ret.** ! V3 – **ret.** $i_{V31} \approx 1$ $i_{V32} = 1 + \frac{t_I}{t_{II}}$ V4 – **ret.** V5 – **ret.** r.-m. V6 – **rev.** !	V1 – **ret.** $\frac{i_{V11}}{i_{V12}} = \frac{t_I}{t_{II}}$ V2 – **ret.** $t_I > t_{II}$ – **rev.** $t_I < t_{II}$ V3 – **rev.** $t_I > t_{II}$ – **ret.** $t_I < t_{II}$ V4 ≡ V2 V5 ≡ V3 V6 – **ret.**	V1 – **rev.** ! V2 – **ret.** V3 – **rev.** r.-m. V4 ≡ V3 V5 ≡ V2 V6 – **ret.** r.-m. Suitable for $i_{Br.2} = \frac{1}{i_{Br.1}}$

Note: The following abbreviations are used in the table:
ret. – the rotational direction is retained when working with different brakes;
rev. – the rotational direction is reversed when working with different brakes;
r.-m. – the gear works either as a reducer or as a multiplier according to which brake is activated;
! – the variant is capable of obtaining a significant difference between the two output angular velocities.

It was proven that the layout variant is kinematically feasible, and that there is a noticeable difference in the size of the simple component gearsets. The gearset will have a complex arrangement, as hollow shafts will be required for its operation, and electrically operated band brakes will be used to change the transmission ratio.

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