Optimizing cutting plans using duality theory: practical applications in welded pipe manufacturing

Практические результаты поиска рациональных технологических планов раскроя на основе использования теории двойственности на примере производства электросварных труб

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Abstract:
This paper presents a practical application of the authors' universal algorithm for optimal planning in combinatorially complex problems, using the strip cutting problem in the production of electric-welded pipes as a case study. The algorithm is based on a synthesis of duality theory and an iterative approach. It implements a mechanism for selecting or generating new promising alternatives based on dual estimates, enabling the identification of a near-globally optimal solution without exhaustive enumeration. This holds true under conditions of both complete and partial a priori determinacy of the solution set.

KEYWORDS: *PRODUCTION PLANNING, OPTIMIZATION, CUTTING STOCK PROBLEM, DUALITY THEORY, COMPUTATIONAL COMPLEXITY*

1. Introduction

Modern manufacturing plants constitute complex technological systems, the effective management of which requires the coordinated operation of numerous subsystems. One of the key components is the production planning subsystem, which is responsible for generating optimal production schedules. The complexity of this task lies in the intricate interplay between available resources and production objectives, aimed at minimizing costs while maximizing the output of products that meet required quality and cost specifications.

In the production of electric-welded pipes, a critical stage of technological preparation involves determining the most efficient method for cutting metal coils (strip) into blanks of the required sizes with minimal waste. The calculation of these cutting patterns presents a combinatorial and computationally complex challenge. For instance, with just 50 types of blanks and a maximum of 10 blanks per coil, the number of potential cutting patterns exceeds the number of atoms on Earth, while the number of possible combinations for selecting these patterns surpasses the number of atoms in the observable universe.

This paper proposes a novel approach to solving this cutting-stock problem [1,1112; 2,254; 3,388], based on the Kantorovich mathematical framework [4, 87; 5, 11]. The proposed methodology is enhanced by the application of duality theory to identify rational technological cutting plans.

2. Preconditions and means for resolving the problem

2.1. Theoretical Model

In general, the problem of calculating an optimal cutting plan can be presented as a two-step procedure: step 1 – identification of all feasible cutting patterns [6, p.156; 7, p.4], step 2 – determination of an optimal production plan for the utilization of these cutting technologies, which fulfills production targets without generating surplus output [7, p.4].

One mathematical approach for solving this linear programming problem, capable of handling the complexity arising from a very large number of constraints, is the method based on the theory of duality. The mathematical model proposed by the authors, which generates dual estimates ("shadow prices") for production planning problems, leverages the duality theorems of linear programming. This model establishes a relationship between the constraints of the primal problem and the optimal values of the dual variables. This relationship allows for a substantive interpretation of the dual coefficients as a measure of the value of workpieces in the strip cutting problem. Consequently, this approach enables a reduction in the problem's dimensionality by transitioning to a dual problem of a smaller scale.

The general mathematical formulation of the optimal cutting plan problem, utilizing duality theory, can be stated as follows:

- *Objective Function*

$$(1) \quad Z = \sum_{j=1}^{s} b_j y_j - T y_{j+1} \rightarrow max$$

where: b_j – is the required production target for each type of finished product; y_j – is the dual variable (shadow price) associated with the inner width of the raw material strip; T – is the total available time capacity of the slitting line; s – is the number of raw material strip types.

- *Constraints*

$$(2) \quad \sum_{j=1}^{s} a_{ij} y_j - \frac{1}{p_i} y_{j+1} \leq 1$$

$$(3) \quad y_j >= 0 \; (j = 1 \dots s+1)$$

where: a_{ij} – is the quantity of the j-th type of blank ($j = 1 \dots s$) obtained from processing 1 tonne of steel (strip) using the i-th cutting technology (in pieces); y_j – is the dual (shadow) value of the blank width for the j-th type ($j = 1 \dots s+1$); p_i – is the production rate of the i-th manufacturing option (in tonnes per hour); s – is the number of blank types.

This study proposes an approach that encompasses two key solution scenarios, determined by the completeness of information regarding feasible cutting patterns:

- *Scenario 1 – presupposes the availability of the entire set of feasible technological cutting patterns for each product type.*
- *Scenario 2 – deals with a situation where the complete set of feasible technological cutting patterns for each product type is either unknown or cannot be formally defined. Only a certain initial subset of known cutting patterns is available.*

2.2. Scenario 1. Modeling Algorithm Framework

Fig.1 Schematic of the Simulation Algorithm for Scenario 1

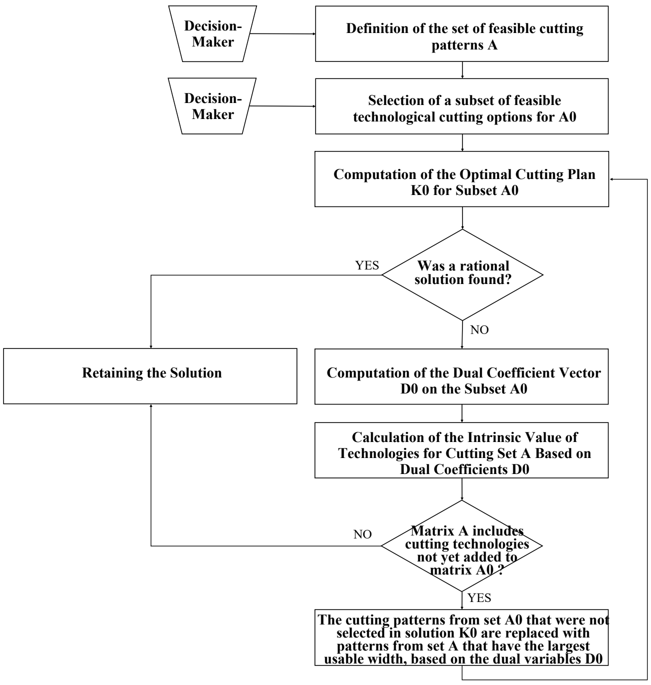


Table 1. Description of the modeling algorithm procedure for Scenario 1.

Stage	*Description*
1. Definition of the set of feasible cutting patterns A	*Consider a matrix A of dimensions <n,m>, where m>>n. The m columns of matrix A represent the set of feasible cutting patterns, while the n rows represent the set of required products (blanks) to be obtained from cutting the original strip. Consequently, the number of possible cutting patterns significantly exceeds the number of products that need to be manufactured.*
2. Selection of a subset of feasible technological cutting options for A_0	*From all columns (m) of matrix A, a specific subset of columns (m_0) is selected. This process yields a matrix A_0 of dimensionality <n,m_0>. The number of selected columns (m_0) must exceed the number of rows (n).*
3. Computation of the Optimal Cutting Plan K_0 for Subset A_0	*For matrix A_0, the primal linear programming problem is solved using an appropriate algorithm (e.g., the simplex method). The solution to this primal problem yields an optimal solution vector K_0 of length m_0. Each column of matrix A_0 represents a specific cutting pattern. The index of an element within vector K_0 corresponds to the specific cutting pattern used (i.e., the column index of matrix A_0). The non-zero elements of vector K_0 indicate the volume of raw material (strip) required to fulfill the production plan for the workpieces, according to each respective cutting pattern.*
4. Computation of the Dual Coefficient Vector D_0 on the Subset A_0	*The dual linear programming problem is formulated and solved for matrix A_0, producing a vector of dual coefficients, D_0, of length (v). These coefficients reflect the intrinsic "shadow price" of the blank.*
5. Calculation of the Intrinsic Value of Technologies for Cutting Set A Based on Dual Coefficients D_0	*For the entire set of feasible cutting patterns in matrix A, a reassessment of the total strip width is performed for each pattern, based on the values of the obtained dual variables D_0 (representing the internal blank width). This reassessed value is termed the "utility width based on dual values." This is calculated by substituting the actual blank width with its corresponding dual variable value. Subsequently, the set of feasible cutting patterns in matrix A is ranked in descending order of the calculated utility width based on dual values.*
6. The cutting patterns from set A_0 that were not selected in solution K_0 are replaced with patterns from set A that have the largest usable width, based on the dual variables D_0	*Zero-cutting patterns from the initial solution K_0 (i.e., patterns assigned a zero value in the solution vector for K_0), obtained by solving the primal linear programming problem for matrix A_0, are replaced with patterns exhibiting the highest usable width, as determined by the dual variables. During this replacement procedure, new patterns identified through the evaluation of usable width using dual variable values—which were not previously included in matrix A_0—must be incorporated into it. We will refer to such cutting patterns as "promising patterns."*

Steps 3 through 6 are repeated iteratively as long as either of the following conditions holds: 1) matrix A contains «promising cutting patterns» that have not yet been incorporated into matrix A_0, or 2) a viable cutting pattern satisfying the problem constraints is generated in Step 3.

2.3. Scenario 1. Empirical Results

Results of a practical calculation of the optimal cutting plan for the production of electric-welded pipes, given a known set of feasible technological cutting options. The general problem formulation is provided in Table 2.

Table 2. The general problem formulation

Parameter	*Description*
Raw material	*A coil of steel (strip) is processed on a slitting line. The target width of the slit strip is specified as L = 750 mm.*
Production Plan	*The strip stock is to be cut into blanks. A total of six distinct blank types are required, each with the following specified widths: l_1 = 15, l_2 = 79, l_3 = 189, l_4 = 123, l_5 = 104, l_6 = 221.* *The required production plan for each blank type is defined by the following quantities: b_1 = 30, b_2 = 65, b_3 = 65, b_4 = 65, b_5 = 60, b_6 = 90.*
Constraints	*The minimum and maximum allowable trim loss values for the strip are defined as follows: e_min = 15, e_max = 36.* *The number of strip slitting knives is given as: N_g = 8.* *The scheduled operating time fund for the slitting line, in hours, is set at: T = 50.*
Goal	*The objective is to identify the optimal manufacturing process for producing the precise number of required blanks without surplus.*

In accordance with the specifications of Scenario 1, a defined set of available production technology alternatives is known. A total of 94 production alternatives are available, with illustrative examples provided in Fig.2.

Fig.2. Examples of a defined set of available production technology alternatives

Production Route	Blank_1	Blank_2	Blank_3	Blank_4	Blank_5	Blank_6	Edge Trim Width	Usable Strip Width	Blank Count per Strip	Production Scenario Output
Recept_0	0,000	0,527	0,000	0,000	0,138	0,295	0,040	1,00000	3	14
Recept_1	0,040	0,211	0,000	0,164	0,554	0,000	0,031	1,00000	4	24
Recept_2	0,020	0,527	0,000	0,000	0,138	0,295	0,020	1,00000	4	17
Recept_3	0,060	0,211	0,252	0,000	0,138	0,295	0,044	1,00000	5	20
Recept_4	0,040	0,211	0,000	0,000	0,415	0,295	0,039	1,00000	4	18
Recept_5	0,040	0,000	0,504	0,000	0,138	0,295	0,023	1,00000	4	14
Recept_6	0,020	0,000	0,504	0,000	0,138	0,295	0,043	1,00000	4	13
Recept_7	0,040	0,000	0,000	0,492	0,138	0,295	0,035	1,00000	4	15
Recept_8	0,080	0,211	0,252	0,000	0,138	0,295	0,024	1,00000	5	17
Recept_9	0,000	0,000	0,000	0,000	0,971	0,000	0,029	1,00000	1	17
Recept_10	0,080	0,000	0,000	0,000	0,000	0,884	0,036	1,00000	2	12
Recept_11	0,000	0,211	0,756	0,000	0,000	0,000	0,033	1,00000	2	17
Recept_12	0,000	0,316	0,000	0,656	0,000	0,000	0,028	1,00000	2	12
Recept_13	0,000	0,632	0,000	0,328	0,000	0,000	0,040	1,00000	2	16
Recept_14	0,000	0,421	0,000	0,000	0,555	0,000	0,024	1,00000	2	22
Recept_15	0,000	0,000	0,000	0,820	0,139	0,000	0,041	1,00000	2	18
…										
Recept_90	0,000	0,105	0,252	0,328	0,278	0,000	0,037	1,00000	4	19
Recept_91	0,100	0,000	0,000	0,000	0,278	0,589	0,033	1,00000	3	16
Recept_92	0,000	0,105	0,000	0,164	0,694	0,000	0,037	1,00000	3	14
Recept_93	0,000	0,105	0,000	0,000	0,278	0,589	0,028	1,00000	3	19

The solution to the optimal technological cutting plan problem yielded an optimum for all 94 variants. The total volume of feedstock required is 389.9 tons, with a total production time of 23.7 hours. The blank production plan has been fulfilled with no overproduction. The technological cutting plan by blanks is presented in Fig.3.

Fig.3. The technological cutting plan by blanks

Blanks	Recept_24	Recept_34	Recept_55	Recept_64	Recept_72	Recept_91	Total	Plan	Fulfillment
Blank 1	0,040	0,120	0,080	0,020	0,080	0,100	30,0	30,0	100%
Blank 2	0,632	0,000	0,211	0,632	0,000	0,000	65,0	65,0	100%
Blank 3	0,000	0,252	0,504	0,000	0,252	0,000	65,0	65,0	100%
Blank 4	0,000	0,000	0,164	0,164	0,492	0,000	65,0	65,0	100%
Blank 5	0,000	0,000	0,000	0,139	0,139	0,278	60,0	60,0	100%
Blank 6	0,295	0,589	0,000	0,000	0,000	0,589	90,0	90,0	100%

Let us examine the results of solving the problem of finding an optimal production plan using the approach proposed in Scenario 1. To this end in accordance with Step 2, a preliminary subset of options was selected, comprising 18 variants. The selected 18 options are presented in Fig.4.

Fig.4. The subset of 18 options

Production Route	Blank_1	Blank_2	Blank_3	Blank_4	Blank_5	Blank_6	Edge Trim Width	Usable Strip Width	Blank Count per Strip	Production Scenario Output
Recept_8	0,080	0,211	0,252	0,000	0,138	0,295	0,024	1,00000	5	17
Recept_10	0,080	0,000	0,000	0,000	0,000	0,884	0,036	1,00000	2	12
Recept_17	0,100	0,105	0,756	0,000	0,000	0,000	0,039	1,00000	3	20
Recept_25	0,060	0,316	0,000	0,000	0,000	0,589	0,035	1,00000	3	16
Recept_26	0,060	0,000	0,756	0,164	0,000	0,000	0,020	1,00000	3	21
Recept_27	0,060	0,000	0,252	0,656	0,000	0,000	0,032	1,00000	3	17
Recept_29	0,060	0,000	0,504	0,000	0,416	0,000	0,020	1,00000	3	20
Recept_30	0,080	0,000	0,756	0,000	0,139	0,000	0,025	1,00000	3	16
Recept_34	0,120	0,000	0,252	0,000	0,000	0,589	0,039	1,00000	3	21
Recept_55	0,080	0,211	0,504	0,164	0,000	0,000	0,041	1,00000	4	13
Recept_65	0,080	0,105	0,000	0,492	0,000	0,295	0,028	1,00000	4	14
Recept_72	0,080	0,000	0,252	0,492	0,139	0,000	0,037	1,00000	4	22
Recept_73	0,080	0,000	0,252	0,328	0,000	0,295	0,045	1,00000	4	17
Recept_74	0,080	0,000	0,000	0,328	0,277	0,295	0,020	1,00000	4	23
Recept_75	0,080	0,000	0,000	0,164	0,139	0,589	0,028	1,00000	4	17
Recept_88	0,100	0,000	0,252	0,328	0,000	0,295	0,025	1,00000	4	13
Recept_89	0,080	0,000	0,000	0,164	0,416	0,295	0,045	1,00000	4	17
Recept_91	0,100	0,000	0,000	0,000	0,278	0,589	0,033	1,00000	3	16

The optimization of the cutting-stock problem yielded a feasible solution. The total raw material requirement is 420.3 tons, with a total production time of 28.0 hours. The billet production target has been fulfilled, albeit with some overproduction. The detailed technological cutting plan for the finished products is presented in Fig.5.

Fig.5. The technological cutting plan by blanks

Blanks	Recept_25	Recept_55	Recept_65	Recept_89	Total	Plan	Fulfillment
Blank 1	0,060	0,080	0,080	0,080	31,5	30,0	105%
Blank 2	0,316	0,211	0,105	0,000	65,0	65,0	100%
Blank 3	0,000	0,504	0,000	0,000	65,0	65,0	100%
Blank 4	0,000	0,164	0,492	0,164	65,0	65,0	100%
Blank 5	0,000	0,000	0,000	0,416	60,0	60,0	100%
Blank 6	0,589	0,000	0,295	0,295	117,0	90,0	130%

The outcomes of the iterative algorithm, as outlined in Fig. 1, are presented in Fig.6. The results indicate that a viable solution, on par with the optimum in terms of material usage and differing only marginally in production time (a mere 9% difference), was identified as early as step 6.

Fig.6. Steps of iterative algorithm

The optimal solution for 94 cutting stock problems		Variant_24	Variant_34	Variant_55	Variant_64	Variant_72	Variant_91	**Total**	**Deviation from the optimum**
	Weight, tonnes	1,8	12,9	82,5	73,6	80,1	139	**389,9**	optimum
	Throughput of the option, tons/hour	18	21	13	17	22	16		
	Total production time (h)	0,10	0,61	6,35	4,33	3,64	8,69	**23,7**	optimum
The solution obtained for the selected 18 cutting patterns		Variant_25	Variant_55	Variant_65	Variant_89			**Total**	**Deviation from the optimum**
	Weight, tonnes	105,9	129,1	41,0	144,3			**420,3**	7,8%
	Throughput of the option, tons/hour	16	13	14	17				
	Total production time (h)	6,62	9,93	2,93	8,49			**28,0**	17,9%
Solution Obtained via Dual Evaluations: Step 1		Variant_24	Variant_25	Variant_55	Variant_65	Variant_89		**Total**	**Deviation from the optimum**
	Weight, tonnes	22,8	51,3	145,2	35,7	144,3		**399,3**	2,4%
	Throughput of the option, tons/hour	18	16	13	14	17			
	Total production time (h)	1,27	3,21	11,17	2,55	8,49		**26,7**	12,5%
Solution Obtained via Dual Evaluations: Step 2		Variant_24	Variant_25	Variant_55	Variant_65	Variant_89		**Total**	**Deviation from the optimum**
	Weight, tonnes	22,8	51,3	145,2	35,7	144,3		**399,3**	2,4%
	Throughput of the option, tons/hour	18	16	13	14	17			
	Total production time (h)	1,27	3,21	11,17	2,55	8,49		**26,7**	12,5%
Solution Obtained via Dual Evaluations: Step 3		Variant_24	Variant_25	Variant_55	Variant_65	Variant_89		**Total**	**Deviation from the optimum**
	Weight, tonnes	22,8	51,3	145,2	35,7	144,3		**399,3**	2,4%
	Throughput of the option, tons/hour	18	16	13	14	17			
	Total production time (h)	1,27	3,21	11,17	2,55	8,49		**26,7**	12,5%
Solution Obtained via Dual Evaluations: Step 4		Variant_24	Variant_25	Variant_55	Variant_65	Variant_89		**Total**	**Deviation from the optimum**
	Weight, tonnes	22,8	51,3	145,2	35,7	144,3		**399,3**	2,4%
	Throughput of the option, tons/hour	18	16	13	14	17			
	Total production time (h)	1,27	3,21	11,17	2,55	8,49		**26,7**	12,5%
Solution Obtained via Dual Evaluations: Step 5		Variant_24	Variant_25	Variant_55	Variant_65	Variant_89		**Total**	**Deviation from the optimum**
	Weight, tonnes	22,8	51,3	145,2	35,7	144,3		**399,3**	2,4%
	Throughput of the option, tons/hour	18	16	13	14	17			
	Total production time (h)	1,27	3,21	11,17	2,55	8,49		**26,7**	12,5%
Solution Obtained via Dual Evaluations: Step 6		Variant_24	Variant_55	Variant_65	Variant_88	Variant_89	Variant_91	**Total**	**Deviation from the optimum**
	Weight, tonnes	60,3	112,3	30,5	33,4	125,8	27,6	**389,9**	0,0%
	Throughput of the option, tons/hour	18	13	14	13	17	16		
	Total production time (h)	3,35	8,64	2,18	2,57	7,40	1,73	**25,9**	9,0%

The nesting plan generated in Step 6, detailed by blanks, is provided in Fig.7.

Fig.7. The technological cutting plan by blanks generated in Step6

	Blanks	Recept_24	Recept_55	Recept_65	Recept_88	Recept_89	Recept_91	**Total**	**Plan**	**Fulfillment**
Solution Obtained via Dual Evaluations: Step 6	Blank 1	0,040	0,080	0,080	0,100	0,080	0,100	**30,0**	30,0	**100,0%**
	Blank 2	0,632	0,211	0,105	0,000	0,000	0,000	**65,0**	65,0	**100,0%**
	Blank 3	0,000	0,504	0,000	0,252	0,000	0,000	**65,0**	65,0	**100,0%**
	Blank 4	0,000	0,164	0,492	0,328	0,164	0,000	**65,0**	65,0	**100,0%**
	Blank 5	0,000	0,000	0,000	0,000	0,416	0,278	**60,0**	60,0	**100,0%**
	Blank 6	0,295	0,000	0,295	0,295	0,295	0,589	**90,0**	90,0	**100,0%**

The implementation of the iterative algorithm yielded the following outcomes:

• A novel cutting plan was identified, incorporating cutting patterns that were not previously part of the optimal solution derived from the set of 94 patterns.

• A solution was obtained that is devoid of overproduction.

2.4. Scenario 2. Modeling Algorithm Framework

Fig.8 Schematic of the Simulation Algorithm for Scenario 2

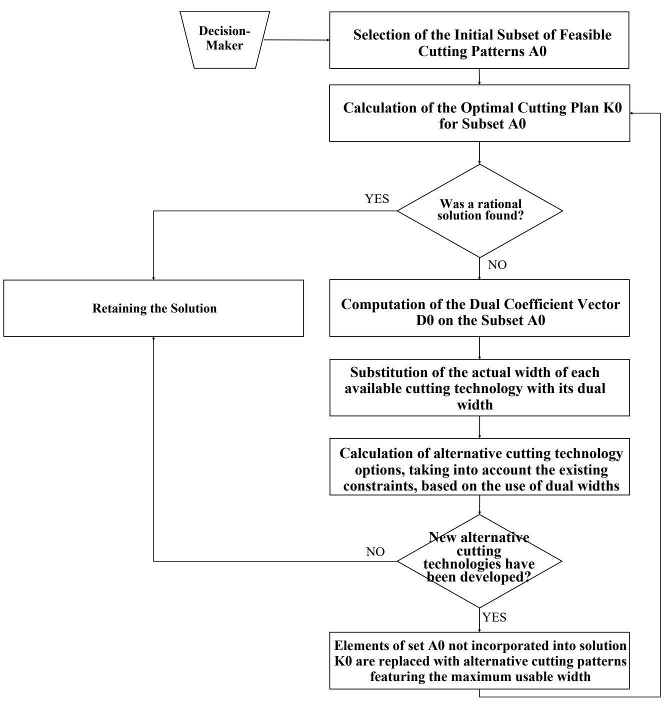


Table 3. Description of the modeling algorithm procedure for Scenario 2.

Stage	Description
Initial state	*Consider a set of strip cutting patterns, which is not known a priori, represented by a matrix A of dimension <n,m>. The columns (m) of matrix A represent the set of all feasible cutting patterns, also not known in advance, while the rows (n) correspond to the set of required products (blanks) to be obtained from cutting the original strip.*
1. Selection of the initial subset of feasible cutting patterns A_0	*Consider a matrix A_0 of dimensions $<n,m_0>$, where $m_0>>n$. The m_0 columns of matrix A_0 represent the set of known cutting patterns, while its n rows correspond to the set of required products (workpieces) to be obtained from cutting the raw strip material.* *Consequently, the number of feasible cutting patterns significantly exceeds the number of products (workpieces) that need to be manufactured.*
2. Calculation of the optimal cutting plan K_0 for subset A_0	*For the initial matrix A_0, the primal linear programming problem is solved using an appropriate algorithm (e.g., the simplex method).* *The solution to the primal problem yields an optimal solution vector K_0 of length m_0. Each column of matrix A_0 represents a specific cutting pattern. The index of an element in the solution vector K_0 corresponds to the specific cutting pattern (i.e., the column index in matrix A_0) used. The non-zero elements of K_0 indicate the amount of raw material (master coil) required to fulfill the production plan for the final products (blanks) according to each respective cutting pattern*
3. Computation of the dual coefficient vector D_0 on the subset A_0	*A dual linear programming problem is formulated and solved for matrix A_0. The solution to the dual problem yields a vector of dual coefficients, D_0, of length (v). In the context of the cutting stock problem, these dual coefficients D_0 possess significant economic and mathematical interpretations. They represent the marginal value of each type of final product (workpiece) in relation to the constraints of the primal problem.*
4. Substitution of the actual width of each available cutting technology with its dual width	*The actual width of the workpieces is replaced by its dual representation.*
5. Calculation of alternative cutting technology options, taking into account the existing constraints, based on the use of dual widths	*This stage involves the identification of new feasible cutting patterns, subject to existing constraints, through the application of dual coefficients. These patterns are hereafter referred to as "alternative" patterns. During this step, the objective function of the problem is modified by incorporating these dual coefficients. The objective function aims to maximize the "value" of a cutting pattern, defined as its utility based on dual widths.*
6. Elements of set A_0 not incorporated into solution K_0 are replaced with alternative cutting patterns featuring the maximum usable width	*The obtained set of alternative cutting patterns is sorted in descending order of the resulting scrap amount. All zero elements in the vector K_0 (obtained in Step 2) are then replaced with the most efficient alternative cutting patterns (obtained in Step 5).*

"Steps 2 through 6 are repeated until an acceptable solution is obtained at Step (2), or until no further alternative cutting patterns remain."

2.5. Scenario 2. Empirical Results

The general problem formulation is provided in Table 4.

Table 4. The general problem formulation

Parameter	Description
Raw material	*A coil of steel (strip) is processed on a slitting line. The target width of the slit strip is specified as L = 750 mm.*
Production Plan	*The strip stock is to be cut into blanks. A total of six distinct blank types are required, each with the following specified widths: l_1 = 15, l_2 = 79, l_3 = 189, l_4 = 123, l_5 = 104, l_6 = 221.* *The required production plan for each blank type is defined by the following quantities: b_1 = 30, b_2 = 65, b_3 = 65, b_4 = 65, b_5 = 60, b_6 = 100.*
Constraints	*The minimum and maximum allowable trim loss values for the strip are defined as follows: e_min = 15, e_max = 36.* *The number of strip slitting knives is given as: N_g = 8.* *The scheduled operating time fund for the slitting line, in hours, is set at: T = 50.*
Goal	*The objective is to identify the optimal manufacturing process for producing the precise number of required blanks without surplus.*

As in Scenario 1, 94 feasible cutting patterns were identified for the task defined in Table 10. Subsequently, the problem of determining the optimal cutting plan was solved. An optimal solution was identified. The total volume of raw material required is 394.1 tons, with a total production time of 23.3 hours. The production target for the blanks was met with no overproduction. The detailed technological cutting plan by blanks is presented in Fig.9.

Fig.9. The technological cutting plan by blanks

Blanks	Recept_9	Recept_12	Recept_20	Recept_74	Recept_78	Recept_91	Total	Plan	Fulfillment
Blank 1	0,200	0,040	0,060	0,100	0,080	0,080	30,0	30,0	100%
Blank 2	0,632	0,105	0,000	0,000	0,000	0,211	65,0	65,0	100%
Blank 3	0,000	0,000	0,504	0,252	0,000	0,252	65,0	65,0	100%
Blank 4	0,328	0,000	0,000	0,328	0,328	0,000	65,0	65,0	100%
Blank 5	0,000	0,832	0,416	0,000	0,277	0,139	60,0	60,0	100%
Blank 6	0,000	0,000	0,000	0,295	0,295	0,295	100,0	100,0	100%

Let us assume that for solving the problem, we are only aware of a subset of 10 cutting patterns, as presented in Fig.10.

Fig.10. The subset of 10 options

Production Route	Blank_1	Blank_2	Blank_3	Blank_4	Blank_5	Blank_6	Edge Trim Width	Usable Strip Width	Blank Count per Strip	Production Scenario Output
Recept_34	0,000	0,211	0,252	0,492	0,000	0,000	0,045	1,00000	3	22
Recept_55	0,060	0,105	0,252	0,000	0,555	0,000	0,028	1,00000	4	12
Recept_65	0,020	0,316	0,000	0,328	0,000	0,295	0,041	1,00000	4	13
Recept_72	0,000	0,000	0,504	0,328	0,139	0,000	0,029	1,00000	3	23
Recept_73	0,080	0,000	0,252	0,492	0,139	0,000	0,037	1,00000	4	19
Recept_74	0,100	0,000	0,252	0,328	0,000	0,295	0,025	1,00000	4	22
Recept_75	0,080	0,000	0,252	0,328	0,000	0,295	0,045	1,00000	4	20
Recept_88	0,060	0,211	0,252	0,164	0,277	0,000	0,036	1,00000	5	18
Recept_89	0,040	0,105	0,252	0,000	0,277	0,295	0,031	1,00000	5	23
Recept_91	0,080	0,211	0,252	0,000	0,139	0,295	0,024	1,00000	5	13

The solution to the optimal cutting stock problem, considering the 10 variants presented in Table 12, was obtained. The total volume of raw material required is 419.7 tons, and the total production time required is 29.14 hours. The production plan for the blanks has been fulfilled, albeit with an overproduction of blank No. 3. The detailed technological cutting plan by blanks is presented in Fig.11.

Fig.11. The technological cutting plan by blanks

Blanks	Recept_55	Recept_65	Recept_73	Recept_74	Recept_91	Total	Plan	Fulfillment
Blank 1	0,060	0,020	0,080	0,100	0,080	30,0	30,0	100%
Blank 2	0,105	0,316	0,000	0,000	0,211	65,0	65,0	100%
Blank 3	0,252	0,000	0,252	0,252	0,252	87,7	65,0	135%
Blank 4	0,000	0,328	0,492	0,328	0,000	65,0	65,0	100%
Blank 5	0,555	0,000	0,139	0,000	0,139	60,0	60,0	100%
Blank 6	0,000	0,295	0,000	0,295	0,295	100,0	100,0	100%

Next a rational solution to the problem will be found using the algorithm described in Fig. 8. Fig 12 presents the calculation results for the volumes of raw material required to produce the target number of blanks, plotted against the algorithm iteration.

Fig.12 Raw material required to produce the target number of blanks

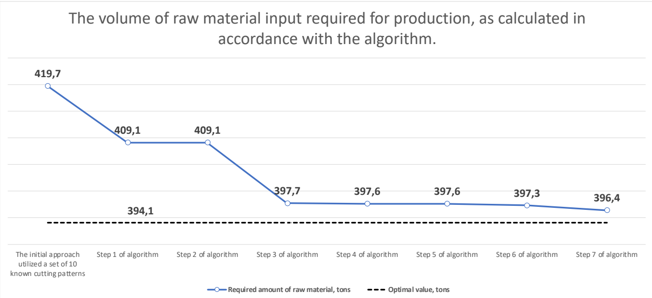
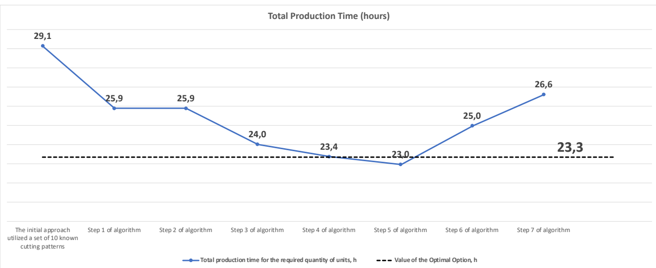


Fig. 13 shows the relationship between the solution variant and the time required to execute the production plan.

Fig.13 Time required to execute the production



The application of the proposed algorithm yielded solutions that are nearly optimal in terms of both material usage and production time. Five novel cutting plans were identified, which preclude the possibility of overproduction. Technological cutting layouts based on these previously unknown patterns are presented in Fig. 14.

Fig.14 The technological cutting plan by blanks

	Blanks	Recept 74	Recept 91	Recept 9D	Recept 35D	Recept 20D	Recept 5D	Total	Plan	Fulfillment
Execution of the production plan according to the variant obtained at Step 3 of the Algorithm.	Blank 1	0,100	0,080	0,040	0,080	0,040	0,000	**30,0**	30,0	**100,0%**
	Blank 2	0,000	0,211	0,421	0,000	0,000	0,000	**65,0**	65,0	**100,0%**
	Blank 3	0,252	0,252	0,000	0,000	0,000	0,000	**65,0**	65,0	**100,0%**
	Blank 4	0,328	0,000	0,492	0,164	0,328	0,820	**65,0**	65,0	**100,0%**
	Blank 5	0,000	0,139	0,000	0,416	0,000	0,139	**60,0**	60,0	**100,0%**
	Blank 6	0,295	0,295	0,000	0,295	0,589	0,000	**100,0**	100,0	**100,0%**

	Blanks	Recept 74	Recept 91	Recept 9D	Recept 35D	Recept 5D	Recept 4D	Total	Plan	Fulfillment
Execution of the production plan according to the variant obtained at Step 4 of the Algorithm.	Blank 1	0,100	0,080	0,040	0,080	0,000	0,000	**30,0**	30,0	**100,0%**
	Blank 2	0,000	0,211	0,421	0,000	0,000	0,632	**65,0**	65,0	**100,0%**
	Blank 3	0,252	0,252	0,000	0,000	0,000	0,000	**65,0**	65,0	**100,0%**
	Blank 4	0,328	0,000	0,492	0,164	0,820	0,328	**65,0**	65,0	**100,0%**
	Blank 5	0,000	0,139	0,000	0,416	0,139	0,000	**60,0**	60,0	**100,0%**
	Blank 6	0,295	0,295	0,000	0,295	0,000	0,000	**100,0**	100,0	**100,0%**

	Blanks	Recept 74	Recept 91	Recept 9D	Recept 35D	Recept 4D	Recept 25D	Total	Plan	Fulfillment
Execution of the production plan according to the variant obtained at Step 5 of the Algorithm.	Blank 1	0,100	0,080	0,040	0,080	0,000	0,000	**30,0**	30,0	**100,0%**
	Blank 2	0,000	0,211	0,421	0,000	0,632	0,105	**65,0**	65,0	**100,0%**
	Blank 3	0,252	0,252	0,000	0,000	0,000	0,000	**65,0**	65,0	**100,0%**
	Blank 4	0,328	0,000	0,492	0,164	0,328	0,164	**65,0**	65,0	**100,0%**
	Blank 5	0,000	0,139	0,000	0,416	0,000	0,693	**60,0**	60,0	**100,0%**
	Blank 6	0,295	0,295	0,000	0,295	0,000	0,000	**100,0**	100,0	**100,0%**

	Blanks	Recept 74	Recept 91	Recept 9D	Recept 35D	Recept 8D	Recept 21D	Total	Plan	Fulfillment
Execution of the production plan according to the variant obtained at Step 6 of the Algorithm.	Blank 1	0,100	0,080	0,040	0,080	0,040	0,100	**30,0**	30,0	**100,0%**
	Blank 2	0,000	0,211	0,421	0,000	0,105	0,000	**65,0**	65,0	**100,0%**
	Blank 3	0,252	0,252	0,000	0,000	0,000	0,000	**65,0**	65,0	**100,0%**
	Blank 4	0,328	0,000	0,492	0,164	0,820	0,000	**65,0**	65,0	**100,0%**
	Blank 5	0,000	0,139	0,000	0,416	0,000	0,277	**60,0**	60,0	**100,0%**
	Blank 6	0,295	0,295	0,000	0,295	0,000	0,589	**100,0**	100,0	**100,0%**

	Blanks	Recept 74	Recept 91	Recept 9D	Recept 8D	Recept 21D	Recept 17D	Total	Plan	Fulfillment
Execution of the production plan according to the variant obtained at Step 7 of the Algorithm.	Blank 1	0,100	0,080	0,040	0,040	0,100	0,060	**30,0**	30,0	**100,0%**
	Blank 2	0,000	0,211	0,421	0,105	0,000	0,000	**65,0**	65,0	**100,0%**
	Blank 3	0,252	0,252	0,000	0,000	0,000	0,000	**65,0**	65,0	**100,0%**
	Blank 4	0,328	0,000	0,492	0,820	0,000	0,492	**65,1**	65,0	**100,0%**
	Blank 5	0,000	0,139	0,000	0,000	0,277	0,416	**60,0**	60,0	**100,0%**
	Blank 6	0,295	0,295	0,000	0,000	0,589	0,000	**100,0**	100,0	**100,0%**

3. Conclusion

The algorithm proposed in this study addresses the problem of identifying rational cutting-stock plans by applying duality theory. This paper presents practical results from the computational testing of the algorithm.
It can be concluded that the proposed algorithm offers several advantages, including its ability to rapidly identify promising cutting patterns in high-dimensional linear programming environments. The developed algorithm facilitates finding a near-optimal production plan with respect to cost and allows for the rapid recalculation of production alternatives with negligible computational overhead.

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