DESIGN OF CONTROL SYSTEM WITH INCREASED POTENTIAL OF ROBUST STABILITY FOR NONLINEAR OBJECT USING LYAPUNOV FUNCTION

ПОСТРОЕНИЕ СИСТЕМЫ УПРАВЛЕНИЯ С ПОВЫШЕННЫМ ПОТЕНЦИАЛОМ РОБАСТНОЙ УСТОЙЧИВОСТИ НЕЛИНЕЙНЫМ ОБЪЕКТОМ С ИСПОЛЬЗОВАНИЕМ ФУНКЦИИ ЛЯПУНОВА

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Abstract: *The article considers approach to construction of control system with increased potential of robust stability of spacecraft in the class of three-parametric structurally stable mappings. Since modern control systems are developed and operated in conditions of significant uncertainty, increasing the potential of system's robust stability is one of the key factors that guarantee the system of protection against the generation of deterministic chaos and instability. In this regard, in conditions of great uncertainty, to build a control system with rather wide area of robust stability is proposed a method for constructing Lyapunov functions based on the geometric interpretation of Lyapunov's theorems on asymptotic stability and dynamic systems stability concepts.*

KEYWORDS*: ROBUST STABILITY, LYAPUNOV'S FUNCTION METHOD, THREE-PARAMETRIC STRUCTURALLY STABLE MAPPING.*

1. Introduction

In the modern automatic control theory, one of the key directions is analysis and synthesis of control systems under uncertainty conditions. This is due to various factors such as inaccurate knowledge of mathematical models of technological processes and technical objects, simplifying models, lowering complexity, or neglect of existing nonlinearities and changing control system parameters in an unpredictable manner under the influence of external and internal disturbances. Therefore, the need for creating such automated systems arises, which in time of object parameters change and influence of external disturbances would have been remained not only in steady state but also provide the required quality of operation. The study and synthesis of such systems is carried out in the framework of robust control theory. The idea of robust design is that it is necessary to choose such settings of control parameters so that the influence of external factors on the output characteristics was minimal [1, 2].

Latest studies have identified a great variety of nonlinear system dynamics. One of the fundamental properties of nonlinear systems is generation of deterministic chaos, forming a "strange attractor" in state space. In dynamical systems it is shown as loss of control system stability. Chaotic modes in the system can act sometimes as unwanted, harmful modes, and sometimes as main useful modes of operation. In the first case, when you construct a control system it is required to suppress or avoid the development of undesired scenario of chaotic processes and instabilities, and the second is to ensure the maintenance of the given chaotic motion system. Chaotic systems are a class of models of uncertainty. Robust stability conditions of these systems allow existence of instability regions of controlled system stationary states. Presently conditions for the suppression or exclusion of scenarios with development of chaotic motions process with the help of control is being studied.

Models describing chaotic behavior occur in many fields of science and technology, and in some cases are a more appropriate tool for describing the irregular fluctuations and uncertainty than stochastic, probabilistic model. It suffices to observe that a wide class of chaotic systems is a well — known pseudo-random number generator, appeared long before the introduction into scientific use of the term "chaos". Surprising was revealed the possibility of significant changes in the properties of chaotic systems using a very small variation of its parameters. However, despite the huge number of publications, including several monographs, rigorous results have been accumulated a little, and many issues remain open. Considering a wide range of potential applications, the field represents interest for, both for theorists and engineers on control systems [5].

It should be noted that deterministic chaos is generated in spacecraft orientation and stabilization systems as a result of loss of stability of the existing steady states, i.e. output is determined by the uncertain parameters of a system beyond the boundaries of robust stability. One of the approaches to deterministic chaos control can be expansion of robustness depending on uncertain parameters changes of the system, i.e. the increased potential of robust stability system [6].

The present study represents a new approach to control of deterministic chaos and to the creation of a control system with increased potential of robust stability [7] systems orientation and stabilization of spacecraft, based on the results of the qualitative theory of dynamical systems and catastrophe theory [8, 9], where, in particular, classified and studied elementary structurally-stable mappings, which are limited to and directly determined by the number of parameters. Also used the idea of gradient mode dynamical systems, the potential function of a vector of Lyapunov functions [10, 11, 12]. Also the main results obtained by applying the above methods are covered in this study.

2. Main mathematical model

Let's review nonlinear system of the spacecraft [13]:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = \dfrac{1}{I_x}(I_y - I_z)x_2x_6 + \dfrac{1}{I_x}(-M_{xu} + M_{xf}) \\ \dfrac{dx_3}{dt} = x_4 \\ \dfrac{dx_4}{dt} = \dfrac{1}{I_x}(I_y - I_z)x_4x_6 + \dfrac{1}{I_y}(-M_{yu} + M_{yf}) \\ \dfrac{dx_5}{dt} = x_6 \\ \dfrac{dx_6}{dt} = \dfrac{1}{I_z}(I_x - I_y)x_2x_4 + \dfrac{1}{I_z}(-M_{zu} + M_{zf}) \end{cases} \quad (1)$$

where I_x, I_y, I_z - are main central products of spacecraft inertia relative to the corresponding axes; M_{xu}, M_{yu}, M_{zu} и M_{xf}, M_{yf}, M_{zf} - accordingly, the projection of controlling and destabilizing moments on to the corresponding axes.

The control laws are given in the form of three-parametric structurally stable mappings:

$$\begin{cases} -M_{xu}+M_{xf} = -x_1^3 - x_2^3 - k_{12}x_1x_2 + k_1x_1 + k_2x_2 \\ -M_{yu}+M_{yf} = -x_3^3 - x_4^3 - k_{34}x_3x_4 + k_3x_3 + k_4x_4 \\ -M_{zu}+M_{zf} = -x_5^3 - x_6^3 - k_{56}x_5x_6 + k_5x_5 + k_6x_6 \end{cases} \quad (2)$$

The system (1) considering (2) can be written as:

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = a(\frac{1}{b}-\frac{1}{c})x_2x_6 - ax_1^3 - ax_2^3 - ak_{12}x_1x_2 + ak_1x_1 + ak_2x_2 \\ \frac{dx_3}{dt} = x_4 \\ \frac{dx_4}{dt} = b(\frac{1}{c}-\frac{1}{a})x_4x_6 - bx_3^3 - bx_4^3 - bk_{34}x_3x_4 + bk_3x_3 + bk_4x_4 \\ \frac{dx_5}{dt} = x_6 \\ \frac{dx_6}{dt} = c(\frac{1}{a}-\frac{1}{b})x_2x_4 - cx_5^3 - cx_6^3 - ck_{56}x_5x_6 + ck_5x_5 + ck_6x_6 \end{cases} \quad (3)$$

3. Stationary states of control system

Now we determine stationary states of the system

$$\begin{cases} x_{2S} = 0 \\ a(\frac{1}{b}-\frac{1}{c})x_{2S}x_{6S} - ax_{1S}^3 - a_{2S}^3 - ak_{12}x_{1S}x_{2S} + ak_1x_{1S} + ak_2x_{2S} = 0 \\ x_{4S} = 0 \\ b(\frac{1}{c}-\frac{1}{a})x_{4S}x_{6S} - bx_{3S}^3 - b_{4S}^3 - bk_{34}x_{3S}x_{4S} + bk_3x_{3S} + bk_4x_{4S} = 0 \\ x_{6S} = 0 \\ c(\frac{1}{a}-\frac{1}{b})x_{2S}x_{4S} - cx_{5S}^3 - c_{6S}^3 - ck_{56}x_{5S}x_{6S} + ck_5x_{5S} + ck_6x_{6S} = 0 \end{cases} \quad (4)$$

From equations (4) we find stationary states of the system:

$$x_{1S} = 0,\ x_{2S} = 0,\ x_{3S} = 0,\ x_{4S} = 0,\ x_{5S} = 0,\ x_{6S} = 0 \quad (5)$$

Other stationary states are specified by solution of the equation:

$$-x_{1S}^2 + k_1 = 0,\ x_{2S} = 0,\ -x_{3S}^2 + k_3 = 0, \quad (6)$$
$$x_{4S} = 0,\ -x_{5S}^2 + k_5 = 0,\ x_{6S} = 0$$

The solution of the equations (6) is

$$x_{1S}^{2,3} = \pm\sqrt{k_1},\ x_{2S} = 0,\ x_{3S}^{2,3} = \pm\sqrt{k_3},\ x_{4S} = 0,\ x_{5S}^{2,3} = \pm\sqrt{k_5},\ x_{6S} = 0 \quad (7)$$

Lets research robust stability of stationary states (5) and (7) of the system (3) using Lyapunov's method.

4. Research of control system's robust stability

We investigate the robust stability of the system's (3) stationary state (5), developed by Lyapunov functions method [12]. We denote components of vectors gradient through:

$$\frac{\partial V_1(x_1,...,x_6)}{\partial x_1} = 0,\ \frac{\partial V_1(x_1,...,x_6)}{\partial x_2} = -x_2,\ \frac{\partial V_1(x_1,...,x_6)}{\partial x_3} = 0,...,$$
$$\frac{\partial V_1(x_1,...,x_6)}{\partial x_6} = 0$$
$$\frac{\partial V_2(x_1,...,x_6)}{\partial x_1} = ax_1^3 + \frac{1}{2}ak_{12}x_1x_2 - ak_1x_1,$$
$$\frac{\partial V_2(x_1,...,x_6)}{\partial x_2} = ax_2^3 + \frac{1}{2}ak_{12}x_1x_2 - ak_2x_2 - \frac{1}{2}a(\frac{1}{b}-\frac{1}{c})x_2x_6,...,$$
$$\frac{\partial V_2(x_1,...,x_6)}{\partial x_6} = -\frac{1}{2}a(\frac{1}{b}-\frac{1}{c})x_2x_6$$
$$\frac{\partial V_3(x_1,...,x_6)}{\partial x_1} = 0,...,\ \frac{\partial V_3(x_1,...,x_6)}{\partial x_4} = -x_4,\ \frac{\partial V_3(x_1,...,x_6)}{\partial x_5} = 0,$$
$$\frac{\partial V_3(x_1,...,x_6)}{\partial x_6} = 0$$
$$\frac{\partial V_4(x_1,...,x_6)}{\partial x_1} = 0,\ \frac{\partial V_4(x_1,...,x_6)}{\partial x_2} = 0,$$
$$\frac{\partial V_4(x_1,...,x_6)}{\partial x_3} = bx_3^3 + \frac{1}{2}bk_{34}x_3x_4 - bk_3x_3,$$
$$\frac{\partial V_4(x_1,...,x_6)}{\partial x_4} = bx_4^3 + \frac{1}{2}bk_{34}x_3x_4 - bk_4x_4 - \frac{1}{2}b(\frac{1}{c}-\frac{1}{a})x_4x_6,$$
$$\frac{\partial V_4(x_1,...,x_6)}{\partial x_5} = 0,\ \frac{\partial V_4(x_1,...,x_6)}{\partial x_6} = -\frac{1}{2}b(\frac{1}{c}-\frac{1}{a})x_4x_6$$
$$\frac{\partial V_5(x_1,...,x_6)}{\partial x_1} = 0,...,\ \frac{\partial V_5(x_1,...,x_6)}{\partial x_5} = 0,\ \frac{\partial V_5(x_1,...,x_6)}{\partial x_6} = -x_6$$
$$\frac{\partial V_6(x_1,...,x_6)}{\partial x_1} = 0,\ \frac{\partial V_6(x_1,...,x_6)}{\partial x_2} = -\frac{1}{2}c(\frac{1}{a}-\frac{1}{b})x_2x_4,$$
$$\frac{\partial V_6(x_1,...,x_6)}{\partial x_3} = 0,\ \frac{\partial V_6(x_1,...,x_6)}{\partial x_4} = -\frac{1}{2}c(\frac{1}{a}-\frac{1}{b})x_2x_4,$$
$$\frac{\partial V_6(x_1,...,x_6)}{\partial x_5} = cx_5^3 + \frac{1}{2}ck_{56}x_5x_6 - ck_5x_5,$$
$$\frac{\partial V_6(x_1,...,x_6)}{\partial x_6} = cx_6^3 + \frac{1}{2}ck_{56}x_5x_6 - ck_6x_6$$

Also, each component of the velocity vector are to be presented in the form of decomposition according to the coordinate axes $x_1, \ldots, x_6$:

$$\left(\frac{dx_1}{dt}\right)_{x_1} = 0,\ \left(\frac{dx_1}{dt}\right)_{x_2} = x_2,...,\ \left(\frac{dx_1}{dt}\right)_{x_6} = 0$$
$$\left(\frac{dx_2}{dt}\right)_{x_1} = -(ax_1^3 + \frac{1}{2}ak_{12}x_1x_2 - ak_1x_1),$$
$$\left(\frac{dx_2}{dt}\right)_{x_2} = -(ax_2^3 + \frac{1}{2}ak_{12}x_1x_2 - ak_2x_2 - \frac{1}{2}a(\frac{1}{b}-\frac{1}{c})x_2x_6),...,$$
$$\left(\frac{dx_2}{dt}\right)_{x_6} = \frac{1}{2}a(\frac{1}{b}-\frac{1}{c})x_2x_6$$
$$\left(\frac{dx_3}{dt}\right)_{x_1} = 0,...,\ \left(\frac{dx_3}{dt}\right)_{x_4} = x_4,\ \left(\frac{dx_3}{dt}\right)_{x_5} = 0,\ \left(\frac{dx_3}{dt}\right)_{x_6} = 0$$
$$\left(\frac{dx_4}{dt}\right)_{x_1} = 0,\ \left(\frac{dx_4}{dt}\right)_{x_2} = 0,\ \left(\frac{dx_4}{dt}\right)_{x_3} = -(bx_3^3 + \frac{1}{2}bk_{34}x_3x_4 - bk_3x_3),$$
$$\left(\frac{dx_4}{dt}\right)_{x_4} = -(bx_4^3 + \frac{1}{2}bk_{34}x_3x_4 - bk_4x_4 - \frac{1}{2}b(\frac{1}{c}-\frac{1}{a})x_4x_6),$$
$$\left(\frac{dx_4}{dt}\right)_{x_5} = 0,\ \left(\frac{dx_4}{dt}\right)_{x_6} = \frac{1}{2}b(\frac{1}{c}-\frac{1}{a})x_4x_6$$
$$\left(\frac{dx_5}{dt}\right)_{x_1} = 0,...,\ \left(\frac{dx_5}{dt}\right)_{x_6} = x_6$$
$$\left(\frac{dx_6}{dt}\right)_{x_1} = 0,\ \left(\frac{dx_6}{dt}\right)_{x_2} = \frac{1}{2}c(\frac{1}{a}-\frac{1}{b})x_2x_4,\ \left(\frac{dx_6}{dt}\right)_{x_3} = 0,$$
$$\left(\frac{dx_6}{dt}\right)_{x_4} = \frac{1}{2}c(\frac{1}{a}-\frac{1}{b})x_2x_4,\ \left(\frac{dx_6}{dt}\right)_{x_5} = -(cx_5^3 + \frac{1}{2}ck_{56}x_5x_6 - ck_5x_5),$$
$$\left(\frac{dx_6}{dt}\right)_{x_6} = -(cx_6^3 + \frac{1}{2}ck_{56}x_5x_6 - ck_6x_6)$$

In this representation, the gradient vector from the Lyapunov vector-functions and from the velocity vector and its projections on the coordinate axis of the full derivative on time from Lyapunov vector-functions can be represented in the form of:

$$\frac{dV(x)}{dt}=\frac{\partial V(x)}{\partial x}\frac{dx}{dt}=\sum_{i=1}^{6}\sum_{j=1}^{6}\frac{\partial V_i(x)}{\partial x_j}\left(\frac{dx_i}{dt}\right)_{x_j}=-x_2^2-$$
$$-\left(ax_1^3+\frac{1}{2}ak_{12}x_1x_2-ak_1x_1\right)^2-\left(ax_2^3+\frac{1}{2}ak_{12}x_1x_2-\right.$$
$$\left.-ak_2x_2-\frac{1}{2}a(\frac{1}{b}-\frac{1}{c})x_2x_6\right)^2-\frac{1}{4}a^2\left(\frac{1}{b}-\frac{1}{c}\right)^2x_2^2x_6^2-x_4^2-$$
$$-\left(bx_3^3+\frac{1}{2}bk_{34}x_3x_4-bk_3x_3\right)^2-\left(bx_4^3+\frac{1}{2}bk_{34}x_3x_4-\right. \tag{8}$$
$$\left.-bk_4x_4-\frac{1}{2}b(\frac{1}{c}-\frac{1}{a})x_4x_6\right)^2-\frac{1}{4}b^2\left(\frac{1}{c}-\frac{1}{a}\right)^2x_4^2x_6^2-$$
$$-x_6^2-\frac{1}{4}c^2\left(\frac{1}{a}-\frac{1}{b}\right)^2x_2^2x_4^2-\frac{1}{4}c^2\left(\frac{1}{a}-\frac{1}{b}\right)^2x_2^2x_4^2-$$
$$-\left(cx_5^3+\frac{1}{2}ck_{56}x_5x_6-ck_5x_5\right)^2-\left(cx_6^3+\frac{1}{2}ck_{56}x_5x_6-ck_6x_6\right)^2$$

The full derivative on time from Lyapunov vector functions (8) in this construction is negative sign function.

For the components of the gradient vector it is possible to construct the Lyapunov function:

$$V_1(x)=-\frac{1}{2}x_2^2,$$
$$V_2(x)=\frac{1}{4}ax_1^4+\frac{1}{4}ak_{12}x_1^2x_2-\frac{1}{2}ak_1x_1^2+\frac{1}{4}ax_2^4+\frac{1}{4}ak_{12}x_1x_2^2-$$
$$-\frac{1}{2}ak_2x_2^2-\frac{1}{4}a\left(\frac{1}{b}-\frac{1}{c}\right)x_2^2x_6-\frac{1}{4}a\left(\frac{1}{b}-\frac{1}{c}\right)x_2x_6^2,$$
$$V_3(x)=-\frac{1}{2}x_4^2,$$
$$V_4(x)=\frac{1}{4}bx_3^4+\frac{1}{4}bk_{34}x_3^2x_4-\frac{1}{2}bk_3x_3^2+\frac{1}{4}bx_4^4+\frac{1}{4}bk_{34}x_3x_4^2-$$
$$-\frac{1}{2}bk_4x_4^2-\frac{1}{4}b\left(\frac{1}{c}-\frac{1}{a}\right)x_4^2x_6-\frac{1}{4}b\left(\frac{1}{c}-\frac{1}{a}\right)x_4x_6^2,$$
$$V_5(x)=-\frac{1}{2}x_6^2,$$
$$V_6(x)=\frac{1}{4}cx_5^4+\frac{1}{4}ck_{56}x_5^2x_6-\frac{1}{2}ck_5x_5^2+\frac{1}{4}cx_6^4+\frac{1}{4}ck_{56}x_5x_6^2-$$
$$-\frac{1}{2}ck_6x_6^2-\frac{1}{4}c\left(\frac{1}{a}-\frac{1}{b}\right)x_2^2x_4-\frac{1}{4}c\left(\frac{1}{a}-\frac{1}{b}\right)x_2x_4^2$$

The Lyapunov function derived in vector form, can represent a scalar function of the form of:

$$V(x_1,...,x_6)=\frac{1}{4}ax_1^4+\frac{1}{4}ak_{12}x_1^2x_2-\frac{1}{2}ak_1x_1^2+\frac{1}{4}ax_2^4+$$
$$+\frac{1}{4}ak_{12}x_1x_2^2-\frac{1}{2}(ak_2+1)x_2^2-\frac{1}{4}a\left(\frac{1}{b}-\frac{1}{c}\right)x_2^2x_6-$$
$$-\frac{1}{4}a\left(\frac{1}{b}-\frac{1}{c}\right)x_2x_6^2+\frac{1}{4}bx_3^4+\frac{1}{4}bk_{34}x_3^2x_4-\frac{1}{2}bk_3x_3^2+$$
$$+\frac{1}{4}bx_4^4+\frac{1}{4}bk_{34}x_3x_4^2-\frac{1}{2}(bk_4+1)x_4^2- \tag{9}$$
$$-\frac{1}{4}b\left(\frac{1}{c}-\frac{1}{a}\right)x_4^2x_6-\frac{1}{4}b\left(\frac{1}{c}-\frac{1}{a}\right)x_4x_6^2+$$
$$\frac{1}{4}cx_5^4+\frac{1}{4}ck_{56}x_5^2x_6-\frac{1}{2}ck_5x_5^2+\frac{1}{4}cx_6^4\frac{1}{4}ck_{56}x_5x_6^2-$$
$$-\frac{1}{2}(ck_6+1)x_6^2-\frac{1}{4}c\left(\frac{1}{a}-\frac{1}{b}\right)x_2^2x_4-\frac{1}{4}c\left(\frac{1}{a}-\frac{1}{b}\right)x_2x_4^2$$

By Morse theorem from the catastrophe theory the Lyapunov's function (9) can be represented as a quadratic form [8]:

$$V(x_1,...,x_6)=-\frac{1}{2}ak_1x_1^2-\frac{1}{2}(ak_2+1)x_2^2-\frac{1}{2}bk_3x_3^2-$$
$$-\frac{1}{2}(bk_4+1)x_4^2-\frac{1}{2}ck_5x_5^2-\frac{1}{2}(ck_6+1)x_6^2 \tag{10}$$

Conditions for robust stability of the system (3) stationary state (5) we will obtain by taking into account negative definiteness of the total derivative (8) obtained from the quadratic form (10) in the form of:

$$k_1<0,\ k_2<-\frac{1}{a},\ k_3<0,\ k_4<-\frac{1}{b},\ k_5<0,\ k_6<-\frac{1}{c}; \tag{11}$$

We investigate the stability of the state equation (3) written in deviations relative to the stationary state (7):

$$\begin{cases}\frac{dx_1}{dt}=x_2\\ \frac{dx_2}{dt}=-ax_1^3-ax_2^3-3a\sqrt{k_1}x_1^2-ak_{12}\sqrt{k_1}x_2-\\ -ak_{12}x_1x_2-2ak_1x_1+ak_2x_2\\ \frac{dx_3}{dt}=x_4\\ \frac{dx_4}{dt}=-bx_3^3-bx_4^3-3b\sqrt{k_3}x_3^2-bk_{34}\sqrt{k_3}x_4-\\ -bk_{34}x_3x_4-2bk_3x_3+bk_4x_4\\ \frac{dx_5}{dt}=x_6\\ \frac{dx_6}{dt}=-cx_5^3-cx_6^3-3c\sqrt{k_5}x_5^2-ck_{56}\sqrt{k_5}x_6-\\ -ck_{56}x_5x_6-2ck_5x_5+ck_6x_6\end{cases} \tag{12}$$

We introduce the notation for the components of the vector gradient:

$$\frac{\partial V_1(x_1,...,x_6)}{\partial x_1}=0,\ \frac{\partial V_1(x_1,...,x_6)}{\partial x_2}=-x_2,\ \frac{\partial V_1(x_1,...,x_6)}{\partial x_3}=0,...,$$
$$\frac{\partial V_1(x_1,...,x_6)}{\partial x_6}=0$$
$$\frac{\partial V_2(x_1,...,x_6)}{\partial x_1}==ax_1^3+3a\sqrt{k_1}x_1^2+\frac{1}{2}ak_{12}x_1x_2+2ak_1x_1,$$
$$\frac{\partial V_2(x_1,...,x_6)}{\partial x_2}==ax_2^3+ak_{12}\sqrt{k_1}x_2+\frac{1}{2}ak_{12}x_1x_2-ak_2x_2,...,$$
$$\frac{\partial V_2(x_1,...,x_6)}{\partial x_6}=0,$$
$$\frac{\partial V_3(x_1,...,x_6)}{\partial x_1}=0,...,\ \frac{\partial V_3(x_1,...,x_6)}{\partial x_4}=-x_4,\ \frac{\partial V_3(x_1,...,x_6)}{\partial x_5}=0,$$
$$\frac{\partial V_3(x_1,...,x_6)}{\partial x_6}=0,$$
$$\frac{\partial V_4(x_1,...,x_6)}{\partial x_1}=0,\ \frac{\partial V_4(x_1,...,x_6)}{\partial x_2}=0,$$
$$\frac{\partial V_4(x_1,...,x_6)}{\partial x_3}=bx_3^3+3b\sqrt{k_3}x_3^2+\frac{1}{2}ak_{34}x_3x_4+2bk_3x_3,$$
$$\frac{\partial V_4(x_1,...,x_6)}{\partial x_4}=bx_4^3+bk_{34}\sqrt{k_3}x_4+\frac{1}{2}bk_{34}x_3x_4-bk_4x_4,...,$$
$$\frac{\partial V_4(x_1,...,x_6)}{\partial x_5}=0,\ \frac{\partial V_4(x_1,...,x_6)}{\partial x_6}=0,$$
$$\frac{\partial V_5(x_1,...,x_6)}{\partial x_1}=0,...,\ \frac{\partial V_5(x_1,...,x_6)}{\partial x_5}=0,$$

$$\frac{\partial V_5(x_1,...,x_6)}{\partial x_6} = -x_6,\ \frac{\partial V_6(x_1,...,x_6)}{\partial x_1} = 0,...,$$
$$\frac{\partial V_6(x_1,...,x_6)}{\partial x_5} = cx_5^3 + 3c\sqrt{k_5}x_5^2 + \frac{1}{2}ck_{56}x_5x_6 + 2ck_5x_5,$$
$$\frac{\partial V_6(x_1,...,x_6)}{\partial x_6} = cx_6^3 + ck_{56}\sqrt{k_5}x_6 + \frac{1}{2}ck_{56}x_5x_6 - ck_6x_6.$$

Also, by introducing notation for the projection of velocity vector components on the coordinate axes, we will find the total derivative from Lyapunov vector-function components [10], which can be represented as following:

$$\begin{aligned}\frac{dV(x)}{dt} &= \frac{\partial V(x)}{\partial x}\frac{dx}{dt} = \sum_{i=1}^{6}\sum_{j=1}^{6}\frac{\partial V_i(x)}{\partial x_j}\left(\frac{dx_i}{dt}\right)_{xj} = \\ &= -x_2^2 - \left(ax_1^3 + 3a\sqrt{k_1}x_1^2 + \frac{1}{2}ak_{12}x_1x_2 + 2ak_1x_1\right)^2 - \\ &- \left(ax_2^3 + ak_{12}\sqrt{k_1}x_2 + \frac{1}{2}ak_{12}x_1x_2 - ak_2x_2\right)^2 - \\ &- x_4^2 - \left(bx_3^3 + 3b\sqrt{k_3}x_3^2 + \frac{1}{2}ak_{34}x_3x_4 + 2bk_3x_3\right)^2 - \\ &- \left(bx_4^3 + bk_{34}\sqrt{k_3}x_4 + \frac{1}{2}bk_{34}x_3x_4 - bk_4x_4\right)^2 - \\ &- x_6^2 - \left(cx_5^3 + 3c\sqrt{k_5}x_5^2 + \frac{1}{2}ck_{56}x_5x_6 + 2ck_5x_5\right)^2 - \\ &- \left(cx_6^3 + ck_{56}\sqrt{k_5}x_6 + \frac{1}{2}ck_{56}x_5x_6 - ck_6x_6\right)^2\end{aligned} \tag{13}$$

The total derivative (13) is negative sign function. We construct the Lyapunov's function in scalar form according to the gradient components of the Lyapunov vector-functions by taking into account (13)

$$\begin{aligned}V(x) &= \frac{1}{4}ax_1^4 + a\sqrt{k_1}x_1^3 + \frac{1}{4}ak_{12}x_1^2x_2 + ak_1x_1^2 + \frac{1}{4}ax_2^4 + \frac{1}{4}ak_{12}x_1x_2^2 - \\ &- \frac{1}{2}(ak_2 - ak_{12}\sqrt{k_1} + 1)x_2^2 + \frac{1}{4}bx_3^4 + b\sqrt{k_3}x_3^3 + \frac{1}{4}bk_{34}x_3^2x_4 + \\ &+ bk_3x_3^2 + \frac{1}{4}bx_4^4 + \frac{1}{4}bk_{34}x_3x_4^2 - \frac{1}{2}(bk_4 - bk_{34}\sqrt{k_3} + 1)x_4^2 + \frac{1}{4}cx_5^4 + \\ &+ c\sqrt{k_5}x_5^3 + \frac{1}{4}ck_{56}x_5^2x_6 + ck_5x_5^2 + \\ &+ \frac{1}{4}cx_6^4 + \frac{1}{4}ck_{56}x_5x_6^2 - \frac{1}{2}(ck_6 - ck_{56}\sqrt{k_5} + 1)x_6^2\end{aligned} \tag{14}$$

Using the theorem of Morse function (14) can be represented in the quadratic form:

$$\begin{aligned}V(x) \approx ak_1x_1^2 - \frac{1}{2}(ak_2 - ak_{12}\sqrt{k_1} + 1)x_2^2 + k_3x_3^2 - \frac{1}{2}(bk_4 - bk_{34}\sqrt{k_3} + 1)x_4^2 + \\ + ck_5x_5^2 - \frac{1}{2}(ck_6 - ck_{56}\sqrt{k_5} + 1)x_6^2\end{aligned}$$

Conditions for robust stability of the system (3) stationary state (7), we will obtain considering negative definiteness of the total derivative of the Lyapunov vector-functions (14) from the quadratic form (16) as:

$$k_1 > 0,\ k_2 < -\frac{1}{a} + k_{12}\sqrt{k_1},\ k_3 > 0,$$
$$k_4 < -\frac{1}{b} + k_{34}\sqrt{k_3},\ k_5 > 0,\ k_6 < -\frac{1}{c} + k_{56}\sqrt{k_5}; \tag{15}$$

Thus, the spacecraft control system built in the class of three-parametric structurally stable mappings will be stable in a wide range of changes of uncertain parameters and guarantees from falling into deterministic chaos mode. Steady state (5) exists and is sustainable when changing the indeterminate parameters of spacecraft in the field (11), and stationary state (7) appears if you lose stability state (5) and they simultaneously do not exist. Stationary state of spacecraft (7) is stable when running system of inequalities (15).

5. Conclusion

This work built robust stable nonlinear control system in the class of three-parametric structurally stable mappings, allowing and maximizing the potential of robust stability. Study of robust stability of the system is based on a geometric interpretation of Lyapunov theorem on asymptotic stability. Given example shows the effectiveness of nonlinear control laws.

6. *References*

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