# FORMATION OF PROJECT AND RESEARCH SKILLS OF STUDENTS IN CALCULATION OF LIMITS

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**Abstract.** The paper is devoted to the problem about formation of project and research skills of students of natural sciences direction, using project and research activities on the subject "Mathematical Analysis".
***KEYWORDS:*** *PROJECT-RESEARCH ACTIVITY, COMPETENCE, RADICAL EXPRESSION, RATIONALIZATION, INDETERMINACY.*

## *Introduction*

Today different goals and objectives are qualitatively assigned to the educational system, which are defined by the dictated social order, by a new model of expert and professional. Consequently, it is necessary to form a personality, which is able to organize his professional career in the ever-changing socio-cultural conditions. Therefore, current priorities of higher education are aimed at development of specialist personality, the leading component of which is the possession of an independent creative and research activities [1].

Recently, the majority of researchers are turning to active teaching methods, such as - project and research. This trend is, firstly, is due to the fact that in recent years there is an increasing need of high school and university students to participate in the project and research activities. Therefore, the process of incorporation schoolchildren and students in cognitive activity is activated. In its turn, analysis of the performed works content allows to make a conclusion that in the most cases activity of executors is not entirely self-sufficient as research activity.

According to the heads of project and research works, the most of students are not able to independently propose and substantiate a hypothesis, plan an activity, formulate a goal, search and analyze necessary information, represent results of research, realize reflection, competently write a report. This is due to the fact that not only the students but also their teachers are not trained to project and research activities [2].

For the future bachelors of mathematics the most interest is the formation of project-research competence of students of natural sciences direction while studying mathematical analysis, as this course has broad opportunities to develop students' key subject competencies. Since the mathematical analysis is the main subject of general mathematical training for specialists, then it is real to form project and research competence exactly while teaching mathematical analysis.

The question is how to organize the learning process, where we not just give knowledge about the studied problems to students and build up their skills to work on the project, as well as research skills, but also solve the deeper problem of formation of project and research competence, the presence of which is necessary to continue mathematical education. The necessity of development these skills and competencies lies in the concept of higher education, since the main objectives of higher mathematical education are: development of creative student's abilities, formation of belief systems, value orientations, cognitive, subject and research skills and competences, ensuring students the readiness to continue professional education.

As an illustration, we present the project and research work "Calculation of limits, using rationalize substitutions", performed by students of H.A. Yasawi IKTU (Turkestan, Kazakhstan).

## *Results and discussion*

As it is known, historically, that the study of rationalize substitutions by students is generally carried out in the course of mathematical analysis in the section "Integration of radical expressions", but in the calculation of limits these substitutions do not find their proper application. The main method of integration of these or other classes of differential expressions is to search the substitution $t = w(x)$, which leads the integrand to a rational form and gives the opportunity to present the integral in the final form as a function of $t$.

If the function $w(x)$, which is to be substituted instead of $t$, is expressed in terms of elementary functions, then the integral is represented in the final form as a function of $x$. The method of solution, based on the rationalize substitutions, is called the method of rationalization (see., e.g., [3]).

This method, applied to rationalize various irrational and trigonometric expressions, and hence to solve irrational and trigonometric equations and inequalities, to calculate limits, etc., is a generalization of the well-known idea about the use of rationalize integration substitutions [4], where these substitutions are widely used to rationalize the various classes of differential expressions by I. Newton, L. Euler, P. Chebyshew and other mathematicians.

However, experience has shown that the use of rationalization method in the calculation of sequences and functions limits is very effective. Moreover, as rationalize substitutions they can apply or its analogical substitution, rationalizing integrand, or their minor modifications.

Let us consider some examples for calculating limits, using the rationalize substitutions. We denote rational functions of their arguments by $R(z(x), Z(x), \dots)$ and $R^*(t)$; where in the second function is obtained from the first one as a result of rationalize substitution $t = w(x)$. In view of the manifold rationalize substitutions we consider only some of them and give examples for calculating limits of specific functions.

**I.** Computation of limits, that contain radicals:

$$\lim_{x \to x_0} R\left(x, \sqrt[l]{\frac{ax+b}{cx+d}}\right),$$

where $R$ is a rational function of two arguments, $l > 1-$ natural number, $a, b, c, d -$ constants.

Put

$$t = w(x) = \sqrt[l]{\frac{ax+b}{cx+d}}, \quad t^l = \frac{ax+b}{cx+d},$$

$$x = z(t) = \frac{dt^l - b}{a - ct^l}.$$

The limit transforms to

$$\lim_{t \to t_0} R(z(t), t);$$

Here the limit has a rational form, since $R, z -$ are rational functions.

***Example 1.*** Compute the limit:

$$\lim_{x \to 0} \frac{\sqrt[m]{2x+1} - \sqrt[n]{2x+1}}{x},$$

where $m, n -$ are integers,

***Solution.*** A direct calculation leads to indeterminacy of the type $\left(\frac{0}{0}\right)$. Use the substitution $t = \sqrt[l]{ax+1}$, i.e. $\sqrt[m]{2x+1} = t$, $2x+1 = t^m$, $\sqrt[n]{2x+1} = t$, $2x+1 = t^n$, $x = t^{mn} - 1$.

Then

$$\lim_{x\to 0}\frac{\sqrt[m]{2x+1}-\sqrt[n]{2x+1}}{x}=\left|\begin{array}{l}t=\sqrt[l]{ax+1},\\ x\to 0,\\ t\to 1\end{array}\right|=$$

$$=\frac{1}{2}\lim_{t\to 1}\frac{t^m-t^n}{t^{mn}-1}=\frac{1}{2}\left(\frac{1}{m}-\frac{1}{n}\right).$$

**II.** Computation of limits, that contain trigonometric functions:

$$\lim_{x\to x_0} R(cosx, sinx).$$

Limits of this kind always can be rationalized by the substitution $t=tg\dfrac{x}{2}\ \ (-\pi<x<\pi).$

$$sinx=\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}=\frac{2t}{1+t^2},$$

Indeed, thus,

$$cosx=\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}=\frac{1-t^2}{1+t^2},\qquad x=2arctgt,$$

$$\lim_{x\to x_0} R(cosx, sinx)=\lim_{t\to t_0} R^*(t),$$

Therefore, limits of the type

$$\lim_{x\to x_0} R(cosx, sinx)$$

*is always calculated in the final form; to express them, except for the functions occurring in the calculation of rational expressions, it is necessary only trigonometric functions.*

Mentioned substitution, which is a universal for limit, contained trigonometric functions, leads sometimes to complex calculations. The goal can be achieved by simple substitutions in the following cases:

а) $t=cosx,\quad t_0=\lim\limits_{x\to x_0} cosx,$ if $R(cosx,-sinx)=R(cosx,sinx);$

б) $t=sinx,\quad t_0=\lim\limits_{x\to x_0} sinx,$ if $R(-cosx,sinx)=R(cosx,sinx);$

в) $t=tgx,\quad t_0=\lim\limits_{x\to x_0} tgx,$ if $R(-cosx,-sinx)=R(cosx,sinx).$

***Example 2.*** Compute the limit:

$$\lim_{x\to\frac{\pi}{6}}\frac{cos^2 3x}{ctg^2 3x+sin6x}.$$

***Solution.*** It is also indeterminacy of the type $\left(\dfrac{0}{0}\right)$. We use the substitution $t=tg3x.$ As $x\to\dfrac{\pi}{6},\ t\to\infty.$ We make appropriate transformations:

$$cos^3 3x=\frac{1}{1+tg^2 3x}=\frac{1}{1+t^2},\quad ctg^2 3x=\frac{1}{tg^2 3x}=\frac{1}{t^2},$$

$$sin6x=\frac{2tg3x}{1+tg^2 3x}=\frac{2t}{1+t^2}.$$

Then

$$\lim_{x\to\frac{\pi}{6}}\frac{cos^2 3x}{ctg^2 3x+sin6x}=\left|\begin{array}{l}t=tg3x,\\ x\to\frac{\pi}{6},\\ t\to\infty\end{array}\right|=$$

$$=\lim_{t\to\infty}\frac{\frac{1}{1+t^2}}{\frac{1}{t^2}+\frac{2t}{1+t^2}}=\lim_{t\to\infty}\frac{\frac{1}{1+t^2}}{\frac{1+t^2+2t^3}{t^2(1+t^2)}}=$$

$$=\lim_{t\to\infty}\frac{t^2}{2t^3+t^2+1}=$$

$$=\lim_{t\to\infty}\frac{\frac{1}{t}}{2+\frac{1}{t}+\frac{1}{t^3}}=\frac{0}{2}=0.$$

***Example 3.*** Compute the limit:

$$\lim_{x\to\frac{\pi}{4}}\frac{sin4x-cos2x}{tg4x}.$$

***Solution.*** It is also indeterminacy of the type $\left(\dfrac{0}{0}\right)$. Use the substitution $t=tgx.$ Due to that $x\to\dfrac{\pi}{4},$ then $t\to 1.$ We make appropriate transformations:

$$sin4x=2sin2xcos2x=\frac{4t}{1+t^2}\cdot\frac{1-t^2}{1+t^2}=$$

$$=\frac{4t(1-t^2)}{(1+t^2)^2},\quad cos2x=\frac{1-t^2}{1+t^2},$$

$$tg4x=\frac{2tg2x}{1-tg^2 2x}=\frac{\frac{tg2x}{1-tg^2x}}{1-\left(\frac{2tgx}{1-tg^2 2x}\right)^2}=$$

$$=\frac{\frac{4t}{1-t^2}}{\frac{(1-t^2)^2-4t^2}{(1-t^2)^2}}=\frac{4t(1-t^2)}{(1-t^2)^2-4t^2}.$$

Then

$$\lim_{x\to\frac{\pi}{4}}\frac{sin4x-cos2x}{tg4x}=\left|\begin{array}{l}t=tgx,\\ x\to\frac{\pi}{4},\\ t\to 1\end{array}\right|=$$

$$=\lim_{t\to 1}\frac{\frac{4t(1-t^2)}{(1+t^2)^2}-\frac{1-t^2}{1+t^2}}{\frac{4t(1-t^2)}{(1-t^2)^2-4t^2}}=$$

$$=\lim_{t\to\infty}\frac{\frac{4t(1-t^2)-(1-t^2)(1+t^2)}{(1+t^2)^2}}{\frac{4t(1-t^2)}{(1-t^2)^2-4t^2}}=$$

$$=\lim_{t\to 1}\frac{\dfrac{(1-t^2)(4t-1-t^2)}{(1+t^2)^2}}{\dfrac{4t(1-t^2)}{(1-t^2)^2-4t^2}}=$$

$$=\lim_{t\to 1}\frac{(4t-1-t^2)\left[(1-t^2)^2-4t^2\right]}{4t(1+t^2)^2}=$$

$$=\frac{(4-1-1)\left[(1-1)^2-4\right]}{4(1+1)^2}=-\frac{1}{2}.$$

Thus, familiarizing students to project and research activities, and development their respective intellectual and spiritual - moral qualities can be a solution of many problems of modern mathematical education, and provide the most natural way out of the designated position.

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