INCREASING POTENTIAL OF ROBUST STABILITY OF SYSTEM BY ONE PARAMETER IN A CLASS OF FOUR-PARAMETER STRUCTURALLY STABLE MAPPINGS

УВЕЛИЧЕНИЕ ПОТЕНЦИАЛА РОБАСТНОЙ УСТОЙЧИВОСТИ СИСТЕМЫ ПО ОДНОМУ ПАРАМЕТРУ В КЛАССЕ ЧЕТЫРЕХПАРАМЕТРИЧЕСКИХ СТРУКТУРНО-УСТОЙЧИВЫХ ОТОБРАЖЕНИЙ

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Abstract: *This article is about robust stable control system for objects with uncertain parameters with an approach to the choice of control laws in the class of four parametric structurally stable mappings, which allow increasing the robust stability potential and the quality of the control system. This article presents robust stability of a system with one input and one output in the class of catastrophe functions "parabolic umbilic". To study the robust stability of stationary states of the system, the main provisions of the Lyapunov function method are used.*
KEYWORDS*: ROBUST STABILITY,* PARABOLIC UMBILIC, LYAPUNOV FUNCTION, UNCERTAIN PARAMETERS

Introduction

The problem of robustness is the most important problem in control theory. At present a large number of scientific works devoted to the topic of robustness have been published. One of the fundamental statements that determined the emergence of the theory of robustness is the Kharitonov theorem, first formulated in [1].

A.A. Andronov and L.S. Pontryagin introduced [2] and developed [3] the concept of the roughness of the dynamic system. The initial definition of this concept was of a local nature, those characterized the change in the behavior of trajectories for small changes in the right-hand sides of the equations of motion. Roughness was treated both as a qualitative property (preservation of the topological structure of the phase space) and analytical (uniform continuity of the dependence of the trajectories on the parameter characterizing the indicated small changes). Further significant development was given to studies of non-local roughness, analysis of changes in the behavior of systems with large finite changes in parameters and the synthesis of control laws, which in some cases provide the best "protection" from a large uncertainty in the knowledge of the properties of the object. In this case, the term "robustness" was often used, usually understood as the ability to maintain the stability of the system under conditions of parametric or nonparametric uncertainty in the description of the control object.

Preconditions and means for resolving the problem

An approach is proposed for constructing the Lyapunov function from the gradient in the form of a function vector [4,5], and the antigradient with respect to the state variable of the required Lyapunov function is given by the components of the velocity vector (the right-hand side of the equation of state) of the system. For a system with an increased robust stability potential with one input and one output built in the class of four-parameter structurally stable mappings (parabolic umbilic) [6]. The region of stability is obtained in the form of the simplest inequalities in the indeterminate parameters of the control object and the selectable parameters of the regulator. Investigation of the stability of the system is based on the ideas of the second method of Lyapunov.

Solution of the problem

We consider a stationary closed control system with one input and one output, described by the equation of state

$$\dot{x} = Ax + Bu \tag{1}$$

Where $x(t) \in R^n -$ object state vector; $u(t) \in R^1 -$ Scalar control function; A – Matrix of a control object with uncertain parameters $n \times n$; $B -$ control matrix $m \times 1$. Matrix A и B have the following form

$$A = \begin{Vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & . \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{Vmatrix}, \quad B = \begin{Vmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{Vmatrix}$$

Law of control $u(t)$ is given in a closed loop in the form of four-parameter structurally stable maps (parabolic umbilic) [7]

$$u(x) = \frac{1}{b}\left(-x_2^2 x_1 - x_1^4 - k_1 x_2^2 - k_2 x_1^2 + k_3 x_1 + k_4 x_2\right), \tag{2}$$

The system (1) is given in the expanded form

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = x_3 \\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots, \\ \dfrac{dx_n}{dt} = -x_2^2 x_1 - x_1^4 - k_1 x_2^2 - k_2 x_1^2 - (a_n - k_3)x_1 + \\ + (a_{n-1} - k_4)x_2 -,\dots,- a_2 x_{n-1} - a_1 x_n \end{cases} \tag{3}$$

The stationary states of the system are determined by solving the equation

$$\begin{gathered} x_{2s} = 0, \quad x_{3s} = 0,\dots, \quad x_{n-1,s} = 0 \\ -x_{2s}^2 x_{1s} - x_{1s}^4 - k_1 x_{2s}^2 - k_2 x_{1s}^2 - (a_n - k_3)x_{1s} + \\ -(a_{n-1} - k_4)x_{2s} -,\dots,- a_1 x_{ns} = 0 \end{gathered} \tag{4}$$

From equation (4) we find the stationary states

$$x_{1s} = 0, \quad x_{2s} = 0,\dots, \quad x_{ns} = 0 \tag{5}$$

Other stationary states will be determined by solving equations

$$-k_1 x_{2s} - a_{n-1} + k_4 = 0, \tag{6}$$

or

$$-x_{1s}^3 - k_2 x_{1s} - (a_n - k_3) = 0, \tag{7}$$

Let us consider the stability of the stationary state (5). To do this, we find the components of the gradient vector from the components of the Lyapunov vector-function [8,9,10].

$$\frac{\partial V_1(x)}{\partial x_1}=0,\ \frac{\partial V_1(x)}{\partial x_2}=-x_2,\ \frac{\partial V_1(x)}{\partial x_3}=0,...,\ \frac{\partial V_1(x)}{\partial x_n}=0$$
$$\frac{\partial V_2(x)}{\partial x_1}=0,\ \frac{\partial V_2(x)}{\partial x_2}=0,\ \frac{\partial V_2(x)}{\partial x_3}=-x_3,...,\ \frac{\partial V_2(x)}{\partial x_n}=0$$
$$\cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots$$
$$\frac{\partial V_{n-1}(x)}{\partial x_1}=0,\ \frac{\partial V_{n-1}(x)}{\partial x_2}=0,\ \frac{\partial V_{n-1}(x)}{\partial x_3}=0,\ \frac{\partial V_{n-1}(x)}{\partial x_n}=-x_n$$
$$\frac{\partial V_n(x)}{\partial x_1}=\frac{1}{2}x_2^2x_1-x_1^4-k_2x_1^2-(a_n-k_3)x_1$$
$$\frac{\partial V_n(x)}{\partial x_2}=\frac{1}{2}x_2^2x_1-k_1x_2^2-(a_{n-1}-k_4)x_2,$$
$$\cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots$$
$$\frac{\partial V_n(x)}{\partial x_n}=a_1x_n$$

From (3) we find the decomposition of the components of the velocity vector into the coordinates of the system

$$\begin{cases}\left(\frac{dx_1}{dt}\right)_{x_1}=0,\left(\frac{dx_1}{dt}\right)_{x_2}=x_2,\left(\frac{dx_1}{dt}\right)_{x_3}=0,\dots,\left(\frac{dx_1}{dt}\right)_{x_n}=0\\ \left(\frac{dx_2}{dt}\right)_{x_1}=0,\left(\frac{dx_2}{dt}\right)_{x_2}=0,\left(\frac{dx_2}{dt}\right)_{x_3}=x_3,\dots,\left(\frac{dx_2}{dt}\right)_{x_n}=0\\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\\ \left(\frac{dx_{n-1}}{dt}\right)_{x_1}=0,\left(\frac{dx_{n-1}}{dt}\right)_{x_2}=0,\left(\frac{dx_{n-1}}{dt}\right)_{x_3}=0,\dots,\left(\frac{dx_{n-1}}{dt}\right)_{x_n}=x_n\\ \left(\frac{dx_n}{dt}\right)_{x_1}=-\frac{1}{2}x_2^2x_1+x_1^4+k_2x_1^2+(a_n-k_3)x_1,\\ \left(\frac{dx_n}{dt}\right)_{x_2}=-\frac{1}{2}x_2^2x_1+k_1x_2^2-(a_{n-1}-k_4)x_2\\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\\ \left(\frac{dx_n}{dt}\right)_n=-a_1x_n\end{cases}$$

The total time derivative of the Lyapunov vector-valued function is

$$\frac{dV}{dt}=-x_2^2-x_3^2-,...,-x_n^2-$$
$$-\left(\frac{1}{2}x_2^2x_1-x_1^4-k_2x_1^2-(a_n-k_3)x_1\right)^2-$$
$$-\left(\frac{1}{2}x_2^2x_1-k_1x_2^2-(a_{n-1}-k_4)x_2\right)-\left(a_{n-2}x_3\right)^2-,\ ...,\ -\left(a_1x_n\right)^2 \tag{8}$$

From (8) we get that the total time derivative of the Lyapunov vector-valued function is a sign-negative function, hence a sufficient condition for the asymptotic stability of system (3) with respect to the stationary state (5) is satisfied.

By a gradient from the Lyapunov function we construct Lyapunov functions in the form

$$V(x)=\frac{1}{4}x_2^2x_1^2+\frac{1}{5}x_1^5+\frac{1}{5}x_2^5\frac{1}{3}k_2x_1^3+\frac{1}{2}(a_n-k_3-1)x_1^2+$$
$$+\frac{1}{2}(a_{n-1}-k_4-1)x_2^2+\frac{1}{2}(a_{n-2}-1)x_3^2+,\ ...,\ +\frac{1}{2}(a_1-1)x_n^2 \tag{9}$$

The positive definiteness of the Lyapunov function (9) will be determined by conditions

$$\begin{cases}a_n-k_3-1>0\\ a_{n-1}-k_4-1>0\\ a_{n-2}-1>0\\ \vdots\\ a_1-1>0\end{cases} \tag{10}$$

Thus, the stationary state (5) of system (3) will be asymptotically stable if conditions (10) are satisfied

Let us investigate the stability of the stationary state (7) and write for this equation of state (3) in the deviations with respect to the stationary state (7)

$$\begin{cases}\frac{dx_1}{dt}=x_2\\ \frac{dx_2}{dt}=x_3\\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots,\\ \frac{dx_n}{dt}=-x_2^2x_1-x_1^4-k_1x_2^2-k_2x_1^2-2(a_n-k_3)x_1+\\ +(a_{n-1}-k_4)x_2-,...,-a_2x_{n-1}-a_1x_n\end{cases} \tag{11}$$

We find the gradient vectors from the components of the Lyapunov vector-valued function:

$$\frac{\partial V_1(x)}{\partial x_1}=0,\ \frac{\partial V_1(x)}{\partial x_2}=-x_2,\ \frac{\partial V_1(x)}{\partial x_3}=0,...,\ \frac{\partial V_1(x)}{\partial x_n}=0$$
$$\frac{\partial V_2(x)}{\partial x_1}=0,\ \frac{\partial V_2(x)}{\partial x_2}=0,\ \frac{\partial V_2(x)}{\partial x_3}=-x_3,...,\ \frac{\partial V_2(x)}{\partial x_n}=0$$
$$\cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots$$
$$\frac{\partial V_{n-1}(x)}{\partial x_1}=0,\ \frac{\partial V_{n-1}(x)}{\partial x_2}=0,\ \frac{\partial V_{n-1}(x)}{\partial x_3}=0,$$
$$\frac{\partial V_{n-1}(x)}{\partial x_n}=-x_n$$
$$\frac{\partial V_n(x)}{\partial x_1}=\frac{1}{2}x_2^2x_1-x_1^4-k_2x_1^2-2(a_n-k_3)x_1$$
$$\frac{\partial V_n(x)}{\partial x_2}=\frac{1}{2}x_2^2x_1-k_1x_2^2-2(a_{n-1}-k_4)x_2,$$
$$\cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots$$
$$\frac{\partial V_n(x)}{\partial x_n}=a_1x_n \tag{12}$$

From (11) we find the decomposition of the components of the velocity vector into coordinates

$$\begin{cases}\left(\frac{dx_1}{dt}\right)_{x_1}=0,\left(\frac{dx_1}{dt}\right)_{x_2}=x_2,\left(\frac{dx_1}{dt}\right)_{x_3}=0,\ldots,\left(\frac{dx_1}{dt}\right)_{x_n}=0\\ \left(\frac{dx_2}{dt}\right)_{x_1}=0,\left(\frac{dx_2}{dt}\right)_{x_2}=0,\left(\frac{dx_2}{dt}\right)_{x_3}=x_3,\ldots,\left(\frac{dx_2}{dt}\right)_{x_n}=0\\ \ldots\\ \left(\frac{dx_{n-1}}{dt}\right)_{x_1}=0,\left(\frac{dx_{n-1}}{dt}\right)_{x_2}=0,\left(\frac{dx_{n-1}}{dt}\right)_{x_3}=0,\ldots,\left(\frac{dx_{n-1}}{dt}\right)_{x_n}=x_n\\ \left(\frac{dx_n}{dt}\right)_{x_1}=-\frac{1}{2}x_2^2x_1+x_1^4+k_2x_1^2+2(a_n-k_3)x_1,\\ \left(\frac{dx_n}{dt}\right)_{x_2}=-\frac{1}{2}x_2^2x_1+k_1x_2^2-2(a_{n-1}-k_4)x_2\\ \ldots\\ \left(\frac{dx_n}{dt}\right)_{n}=-a_1x_n\end{cases}$$

The total time derivative of the Lyapunov vector-valued function is defined as the scalar product

$$\begin{aligned}&\frac{dV}{dt}=-x_2^2-x_3^2-,\ldots,-x_n^2-\\&-\left(\frac{1}{2}x_2^2x_1-x_1^4-k_2x_1^2-2(a_n-k_3)x_1\right)^2-\\&-\left(\frac{1}{2}x_2^2x_1-k_1x_2^2-2(a_{n-1}-k_4)x_2\right)-(a_{n-2}x_3)^2-,\ldots,-(a_1x_n)^2\end{aligned} \quad (13)$$

It is obvious from (13) that a sufficient condition for asymptotic stability will always be satisfied, that is, the total derivative of the Lyapunov vector-valued function is a sign-negative function.

Using the gradient (12), we construct the Lyapunov function

$$\begin{aligned}V(x)=&\frac{1}{4}x_2^2x_1^2+\frac{1}{5}x_1^5+\frac{1}{5}x_2^5\frac{1}{3}k_2x_1^3+\frac{1}{2}(a_n-k_3-1)x_1^2+\\&+\frac{1}{2}(a_{n-1}-k_4-1)x_2^2+\frac{1}{2}(a_{n-2}-1)x_3^2+,\ldots,+\frac{1}{2}(a_1-1)x_n^2\end{aligned}$$

Necessary conditions for the asymptotic stability of the equilibrium state (7), that is, the condition for positive definiteness of the Lyapunov function for the equilibrium state (7), which exists only when $k_3-a_n>0$ that is to say $a_n<k_3<\infty$ will be satisfied if

$$\begin{cases}a_n-k_3-1>0\\a_{n-1}-k_4-1>0\\a_{n-2}-1>0\\ \vdots\\a_1-1>0\end{cases}$$

Results and discussion

Thus, the system (3) due to the introduction into the contour of the control law in the form of four-parameter structurally stable mappings becomes stable over unlimitedly wide limits of the variation of the indeterminate parameter a_n, a_{n-1} and adjustable controller parameters k_3 and k_4. The stationary state of the system (5) exists and it is stable when the parameters a_n, k_3 and a_{n-1}, k_4 in the regions $-\infty<k_3<a_n$ and $-\infty<k_4<a_{n-1}-1$ respectively, and the stationary state (7) exists and is stable when the parameters a_n, k_3 and a_{n-1}, k_4 in the regions $<k_3<a_n$ and $a_n<k_3<\infty$ and $a_{n-1}<k_4<\infty$. For the rest of the object parameters there are restrictions $a_i>1, i\le n-2$.

In real control systems, uncertainty is inevitably present, and the control system must be operative in the presence of uncertainty. Such a control is called robust.

Conclusion

As a result, we see that the control system with increased potential of robust stability in a class of four-parameter structurally stable mappings for objects is stable with any changes of uncertain parameters accordingly excludes the generation of deterministic chaos in dynamic system and guarantees the performance and reliability of control system under conditions of uncertainty.

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