

Solution of a generalized system of differential equations of intercoupled energy and mass transfer at short-term phase contact

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Abstract: Modern intensification of materials processing technology leads to an increase in the role of non-stationary interconnected exchange processes compared to stationary unconnected interconnected exchange processes compared to stationary unconnected. This fact is still insufficiently reflected in the field of solving energy and mass transfer problems (EMT) at small Fourier numbers (Fourier numbers $F_{0k} \leq 0.1$) at short-term phase contact (SPC). In this article, a generalized mathematical model of interconnected non-stationary irregular energy and mass transfer mode at short-term contact across a boundary with selective permeability of phases is formalized. In vector-matrix form, a conjugate mixed boundary value problem is solved with excitation in each of the phases of flows of substances absent in the other phase. By analogy with heat exchange and mass exchange, matrices of potential assimilation of phases and a contact matrix are introduced, which allows obtaining a uniform solution for a number of special cases and especially simplified the entry for the vector of interphase flow densities. The mathematical notation of the solutions of the considered parabolic system of partial differential equations of the second order for intensive irreversible processes (Fourier numbers $F_{0k} \leq 0.1$) are written in vector-matrix form and are close to the scalar Higbee theory for mass transfer.

KEY WORDS: SHORT-TERM CONTACT, MATHEMATICAL MODELING, SYSTEMS OF DIFFERENTIAL EQUATIONS, POTENTIALS, PHASES, MODES, VECTOR, MATRIX, HEAT AND MASS TRANSFER, COEFFICIENTS.

1. Introduction

The processes of diffusion-filtration energy and mass transfer through capillary-porous colloidal and colloid-capillary-porous bodies are very complex and closely interconnected [1-4]. In many branches of industry (high-temperature heat exchangers, turbine blades, rocket nozzles, skins of high-speed aircraft, boiler installations, lining of heat and power generating plants, nuclear reactor systems with internal cooling, heat and power grids, etc.) high-intensity and high-temperature technological processes are nonlinear, non-stationary, non-uniform and asymmetric in nature, that is, at sufficiently high thermal, concentration and filtration gradients of potentials, the kinetic coefficients included in the conjugate system of differential equations depend on the parameters of the state of the transfer potentials, which in turn depends on the four-dimensional space of events of independent state parameters. [1-4, 10, 11].

2. Main Body

The generalized mathematical model (GM) of the interconnected non-stationary EMF with variable transfer coefficients and CCF conditions is formulated as a system of equations (SE) written in vector-matrix form [4,5]:

$$C_i \Pi_{it} = (L_i \Pi_{ix})_x, \tau > 0, x \in ((-1)^i \infty, 0), i = 1, 2 \quad (1)$$

$$\Pi_i(x, 0) = \Pi_i[(-1)^i \infty, \tau] = \Pi_{oi}, \quad \forall |x| > 0; \quad (2)$$

$$K_{\Gamma} \Pi_i(0, \tau) = \Pi_{i+1}(0, \tau); \quad (3)$$

$$L_i \Pi_{ix}(0, \tau) = L_i \Pi_{ix}(0, \tau), \quad (4)$$

where Π_{it}, Π_{ix} are the local change in the vector of transfer potentials of dimension n_i , respectively, with respect to time τ and coordinate x in the i -th phase, x is the coordinate along the normal to the plane of contact with zero at the phase boundary, C_i, L_i are the matrices of capacitive and kinetic coefficients, respectively. Moreover, the elements of C_i are defined as the ratio of the stored amount of substance to the corresponding transfer potential and depend on the choice of units of measurement Π_i , just like the elements of L_i . The dimensions of the vectors Π_i are equal to n_i and, generally speaking, do not coincide, for example, in the case of coupled transfer in capillary-porous bodies (phase 2) in contact with a heating metal surface (phase 1), then $n_2 > n_1$ [6,7]. Under the condition $\dim \Pi_i = \dim \Pi_2$ ($n_1 = n_2$), the operations of narrowing the vector or matrix from $R_1 \cup R_2$ to $R_1 \cap R_2$ and expanding by zeros in $R_1 \cup R_2$ are possible, K_{Γ} is the diagonal matrix of

equilibrium constants ($n_1 \times n_2$). In particular, if we are talking about multicomponent diffusion and heat propagation, then the diagonal K_{Γ} consists of Henry constants or coefficients of the linearized representation of the equilibrium compositions of the phases [8] and a unit corresponding to thermal equilibrium when determining the temperature of the phases in the same units of measurement.

The conjugation conditions (3) and (4) mean that in the absence of concentrated resistance at the phase interface $x=0$, the conditions of continuity of flows and transfer potentials apply. Using the self-similarity condition (2) and further omitting the phase index, the matrix equation (1) for the stated limit boundary value problem can be written in the form of a system of ordinary differential equations (SODE) of the form:

$$2tC(t)\Pi_t(t) + [L(t)\Pi_t(t)]_t = \bar{0}, \quad (5)$$

where $t = x/2\sqrt{\tau}$ is the Boltzmann support, $\bar{0}$ is the zero vector. Further denoting the derivative $\Pi_t(t)$ by Z , we obtain an autonomous first-order SODE of dimension $\sum_{i=1}^n n_i$ with respect to $Z(t)$

$$L(t)Z_t + K(t)Z = \bar{0}, \quad (6)$$

here $K(t) = 2tC(t) + L_t(t)$. In turn, SODE (6) can be represented as

$$Z_t = A(t)Z, \quad (7)$$

where $A(t) = -[K(t)][L(t)]^{-1} = \|a_{ik}(t)\|$ is a continuous matrix function in some (finite or infinite) interval (a,b) of change of the real argument t ($i, k = \overline{1, n}$). The general solution of equation (7) has the form

$$Z = C_0 M_{t_0}^t, \quad (8)$$

where $Z(t_0) = Z_0 = E$ is the identity matrix; $M_{t_0}^t(A)$ is the matricant that determines the normalized solution (7), C_0 is an arbitrary constant matrix with determinant $|C_0| \neq 0$. The matricant and successive approximations Z_j can be found from the recurrence relations

$$M_{t_0}^t(A) = Z_j = Z_{j-1} + \left[\int_{t_0}^t A(t) dt \right]^j \quad j = \overline{1, n}. \quad (9)$$

In the special case when A is a constant matrix

$$C_0 = Z_0 e^{-At_0}, \quad (10)$$

the general solution of equation (7) can be written as an integral matrix

$$Z = Z_0 \exp[A(t - t_0)], \quad (11)$$

where

$$e^{At} = \sum_{k=1}^s Z_k = \sum_{k=1}^s (X_{k1} + X_{k2} + \dots + X_{km_k} t^{m_k-1}) e^{\lambda_k t} \quad (12)$$

Here

$$\psi(\lambda) = (\lambda - \lambda_1)^{m_1} (\lambda - \lambda_2)^{m_2} \dots (\lambda - \lambda_s)^{m_s}$$

$$(\lambda_i \neq \lambda_k \text{ at } i \neq k; i, k = \overline{1, s})$$

- the minimal polynomial of matrix A , and X_{kj} are linearly independent constant matrices that are polynomials in A , since in the right-hand side of formula (12) each term Z_k is a solution to equation (7), which is satisfied by the product $\varphi(A)e^{At}$ for any function $\varphi(\lambda)$, but $Z_k = f(A) = \varphi(A)e^{At}$, if $f(A) = \varphi(A)e^{At}$ and $\varphi(\lambda_k) = 1$, and the remaining $m - 1$ values of the function $\varphi(\lambda)$ on the spectrum of matrix A are equal to zero [6]

Expressions (11), (12) make it possible to specify the structure of the desired solutions in the form of a linear combination of integral matrices. Then, as in [5-7], the solution to the CCF problem with constant transfer coefficients is represented as a linear combination of additional Gaussian probability integrals

$$Z(t) = Z_0 + \operatorname{erfc}(v_j^{-1}t)C, \quad (13)$$

where $\operatorname{erfc}(t/v_j)$ is the additional error integral function, v_j is the square root of the eigenvalue of the matrix A . In (13), the erfc operation is applied to the diagonal elements of the matrix $v^{-1}t$ element-wise:

$$Z(t) = Z_0 + \sum_{i=1}^n c_i \operatorname{erfc}(t/v_i)t_i, \quad (14)$$

where the j -th eigenvector t_j is the solution of the ES $At_j = v_j t_j$. It is easy to see that ES (5) with constant C and L and condition (2) is satisfied identically, and the vectors $C_1 \in R_1$ and $C_2 \in R_2$ of the constants of integration are determined from the boundary conditions (3) and (4). The generalized solution of the boundary value problem of the CCF with a functional dependence of the transfer coefficients on the potentials (the parameters of the state of the system) is reduced by two quadratures to the solution of the following system of nonlinear integral equations:

$$\Pi(t) = C_0 \int_{t_0}^t \exp \left\{ - \int_{t_0}^t [L_t(t) + 2tC(t)] L^{-1}(t) dt \right\} dt. \quad (15)$$

3. Conclusions

A generalized mathematical model of the interconnected non-stationary irregular energy and mass transfer mode during short-term contact across the boundary with selective permeability of phases is formalized.

A conjugate mixed boundary value problem is solved in vector-matrix form when excitation of flows of substances absent in the other phase is carried out in each of the phases.

By analogy with heat and mass exchange, matrices of potential assimilation of phases and a contact matrix are introduced, which allows obtaining a uniform solution for a number of special cases and especially simplified the entry for the density vector of interphase flows.

The mathematical entry of solutions of the considered parabolic system of partial differential equations of the second order for intensive irreversible processes (Fourier criterion $Fok \leq 0.1$) is

written in vector-matrix form and is close to the scalar Higbee theory for mass transfer.

A set of thermophysical, chemical and thermodynamic characteristics together with mathematical modeling allows to reveal the regularities of external and internal heat and mass transfer in the most complete way, to find scientifically based ways of intensification and modular design of similar processes and devices of chemical engineering, biotechnology, industrial power engineering, mechanical engineering and construction industry (heating, cooling and condensation, rectification, evaporation, vacuum and convective drying, drying by pressure reduction, oscillating and combined drying method, drying of thermo- and xerolabile products by pulse pressure and vacuum release, absorption and adsorption, extraction, liquid distillation, crystallization, etc.).

The obtained results serve the development of the general theory of mathematical modeling, are useful in setting up experimental studies on non-stationary heat and mass transfer in the analysis and calculation of exchange processes of chemical engineering, biotechnology and bioengineering, industrial power engineering and design.

4. References

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