Particle-in-cell modeling of non-radiating states of laser-plasma

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Abstract: *In this work we study numerically, by means of Particle-in-Cell simulations, the formation of non-radiating configurations in a plasma, arising from the ionization of atomic gases by intense laser pulse Our simulation show that, after the release of a short terahertz pulse, a nearly non-radiating field and current configuration is formed, even though the charge and current distributions of plasma remain essentially time-dependent. In a plasma target emitting terahertz waves, such a non-radiating state is formed under the action of radiation. Our calculations suggest that formation of a non-radiating configuration occurs in a self-consistent way and can therefore affect the duration of the emitted terahertz pulse.*
KEYWORDS*: PARTICLE-IN-CELL SIMULATIONS, TERAHERTZ RADIATION*

1. Introduction

The generation of time-dependent non-radiative currents in a rarefied plasma is a rather common phenomenon. Current oscillations at a frequency at which the plasma permittivity is zero can be mentioned as a simplest example. Another, more complex example consists in the generation in an inhomogeneous plasma of a current density distribution with a vanishing dipole moment. Strictly speaking, such a configuration can not be called completely non-radiative, but since the radiation occurs because of the higher-order moments, it has a very high frequency, which is much bigger than the plasma frequency. Finally, these could be trivial configurations with an anomalously small dipole moment or cases where the propagation of the wave with a given frequency is prohibited by the dispersion relation.

In this paper we present a tool for a numerical study of time-dependent current configurations with a non vanishing dipole moment that do not emit radiation because of a complete or nearly complete destructive interference in the far-field zone. Such states – the so-called anapole states – have long been known to exist theoretically [1,2] and have been studied by several prominent scientists [3,4], including Ehrenfest, Zeldovich, and Ginzburg, but their practical realization have been realized only recently in metamaterials. For example, a review [5] discusses a small emitter (with a size on the order of a wavelength), where suppression of radiation can not be attributed to the restrictions given by the dispersion relation. In this case, the suppression occurs because the currents and charges in the emitter oscillate in a very specific way.

Our calculations show that in a plasma target *emitting* terahertz (THz) waves, a non-radiating state may be formed under the *back reaction* of the THz *radiation*. In particular, when a THz wave is emitted from a small plasma target (with a size of the order of the wavelength of the THz wave), the back reaction of the radiation on the motion of the charged particles leads to the rapid formation of a plasma configuration that no longer emits radiation. At the same time, it continues to perform rather complex oscillations. Since in this case the formation of a non-radiating configuration occurs in a self-consistent way, it can therefore affect the duration of the THz pulse, making it very short. The structure of the paper is following. First we formulate the problem statement and present the main numerical algorithms we use. Then we discuss our numerical results and finally, report the conclusion.

2. Numerical model and Setup

The initial setup is schematically shown in Figure 1. The rectangular domain $L_{domain} \times \Delta_{domain}$ is partially occupied by a neutral gas. The laser pulse enters the domain from the left. The laser pulse is characterized by the intensity I_0, duration τ_0, focal spot-size r_0, polarization (linear of circular) and spectrum (mono- or bichromatic). The gas cell - by the initial concentration n_0, size of the cell $L_{cell} \times \Delta_{cell}$ and gas type. We assume, that the plasma, formed by the ionization, can be considered as collisionless.

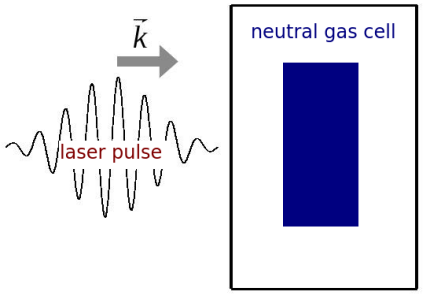


Figure 1. *Initial setup: The rectangular domain is partially occupied by a neutral gas. The laser pulse enters the domain from the left and propagates to the right.*

A self-consistent model to study the behavior of fully or partially ionized gases in the field of an intense laser pulse consists of Vlasov kinetic equations for the distribution functions of plasma ions and electrons and Maxwell's equations for the electromagnetic fields E, B [6].

We measure the mass and charge of the ions in the electron mass m_e and the elementary charge e, normalize the velocities to the speed of light c, use the values of the main wavelength λ_0 of the laser pulse as the length unit and the amplitude of the dimensionless vector potential $a_0 = eE_0\lambda_0/(2\pi m_e c^2)$ – as the unit for the electromagnetic field amplitude. Note that for a linearly polarized electromagnetic pulse with wavelength $\lambda_0 = 800nm$ the value $a_0 = 1$ of the normalized vector-potential corresponds to the intensity of $I_0 = 2.1 \cdot 10^{18} W/cm^2$.

We solve the Vlasov equations using Particle-in-Cell (PIC) method [7-9] and obtain the solutions of the Maxwell equations using a finite-difference time domain (FDTD) scheme [10,11].

To solve the equations of motion for the charged particles the Boris scheme [12] is used. It is of the second order in time $O(\Delta_t^2)$, time reversible and unconditionally stable [13]. Formally, the scheme is implicit, but its implementation can be done in an explicit way. In [14] it has been proven that the long-term accuracy of the Boris algorithm is ensured because it preserves the phase space volume.

In Cartesian geometry the solution of Maxwell's equations is performed on a uniform rectangular grid $x_i = i \cdot \Delta_x, y_k = k \cdot \Delta_y$, with $\Delta_{x,y}$ being the grid spacings. The values of the components of the electric and magnetic fields are defined on the staggered grids and are shifted relative to each other by a half step size in time. Since the conventional FDTD scheme [10,11] uses central differences to approximate the derivatives of the fields, it has a second order of approximation over all grid steps $O(\Delta_t^2, \Delta_x^2, \Delta_y^2)$ with Δ_t being the time step. The scheme is explicit and conditionally stable if $c\Delta_t(\frac{1}{\Delta_x} + \frac{1}{\Delta_y}) \le 1$. The FDTD scheme ensures an automatic fulfillment of discretized Gauss's law for the magnetic field $\nabla \cdot \vec{B} = 0$ at any time step, provided it is fulfilled at $t = 0$. To garanty an automatic fulfillment of the discrete analog of Gauss's

law for the electric field $\nabla \cdot \vec{E} = 4\pi e(Zn_i - n_e)$, where n_e is the electron concentration and n_i and Z are the ion concentration and ion charge, we use an algorithm of reconstruction of the current density first proposed in [15] and later generalized in [16].

Ionization of the gas cell by the electric field is taken into account using formulas for the tunnel ionization probability $w(E)$. These use the instantaneous value of the electric field strength [17,18], in contrast to the widely used ADK [19] formula, in which the electric field is averaged over the period. Numerically we proceed as follows. At $t = 0$ the computational domain contains a neutral gas, which is modeled by identical particles having a distribution by coordinates according to the initial gas concentration $n_0(x, y)$. At each point of the domain (x_j, y_j), occupied by a neutral atom or an ion in a low charge state the ionization probability $p = 1 - e^{-w(E)\Delta_t}$ is calculated. The refection method [20] is used to decide whether the ionization event takes place or not. If an ionization event takes place, the charge state of the parent atom/ion is increased by one and an immobile "new" electron is born at (x_j, y_j). At the next time step positions and velocities of the "new" electrons are updated and their contributions to the charge and current density are calculated. Since the intensity of laser radiation is not very high, ionization can lead to significant losses of pulse energy. To take into account the depletion of the laser pulse due to ionization losses we follow [21,22] and introduce an "ionization" current $\vec{j}_{ion}$, whose direction coincides with the direction of the electric field $\vec{E}(x_j, y_j)$ at the position (x_j, y_j) of the "parent" ion, and whose density, is such that $(\vec{j}_{ion} \cdot \vec{E}(x_j, y_j)) = W_E$, where W_E is the work of the electric field spent on ionization during the time step Δ_t. The energy conservation in every numerical cell is controlled: if the remaining electric field energy is not sufficient for further ionization, then no further ionization occurs.

3. Simulation results

Numerical simulations discussed below have been performed at the Joint Supercomputer Center of the Russian Academy of Sciences and at the Siberian Supercomputer Center of the Siberian Branch of the Russian Academy of Sciences. In order to efficiently use the resources provided by these Centers the numerical code has been parallized. The details of the parallelization strategy can be found in e.g. [23]. Data exchange between processor elements is handled by means of MPI routines, FORTRAN has been chosen as the basic programming language. We performed series of simulations in a wide range of parameters, which are summarized in a Table 1 and Table 2 below. In all the simulations the laser pulse was bichromatic with a wavelength of the second harmonic $\lambda_1 = \lambda_0/2$. The amplitude of the electric field of the second harmonic was $a_1 = 0.2a_0$.

a_0	polarization	λ_0, nm	r_0/λ_0	$\tau_0 c/\lambda_0$
0.02 - 0.05	CP(LP)-LP(CP)	800 - 2000	5-50	10 - 100

Table 1: Basic parameters of the laser pulse used in simulations. Abbreviation CP/LP denotes circular/linear polarization of the laser pulse.

$n_0, 10^{18} cm^{-3}$	Δ_{cell}/λ_0	L_{cell}/λ_0	L_{domain}/λ_0	$\Delta_{domain}/\lambda_0$
1 - 20	4 - 50	5 - 200	100 - 250	100 - 250

Table 2: Basic parameters of the argon gas cell used in simulations.

In Figures 2 and 3, we plot, exemplary, the distribution of the period -averaged electric field (E_y component in Fig. 2 and E_z component in Fig. 3) for the simulation of the interaction of a circularly polarized bichromatic laser pulse with $\lambda_0 = 2\mu m, a_0 = 0.05$ with small argon cell of $\Delta_{cell} = L_{cell} = 5\lambda_0$ of the initial concentration of $n_0 = 9 \times 10^{17} cm^{-3}$. The y- component of the electric field is show for $t = 100\lambda_0/c = 667 fs$ and the z-component of the electric field is shown at three subsequent time moments, t = 30 λ_0/c, 70 λ_0/c and 100 λ_0/c. The generation and propagation of a short terahertz pulse is clearly visible.

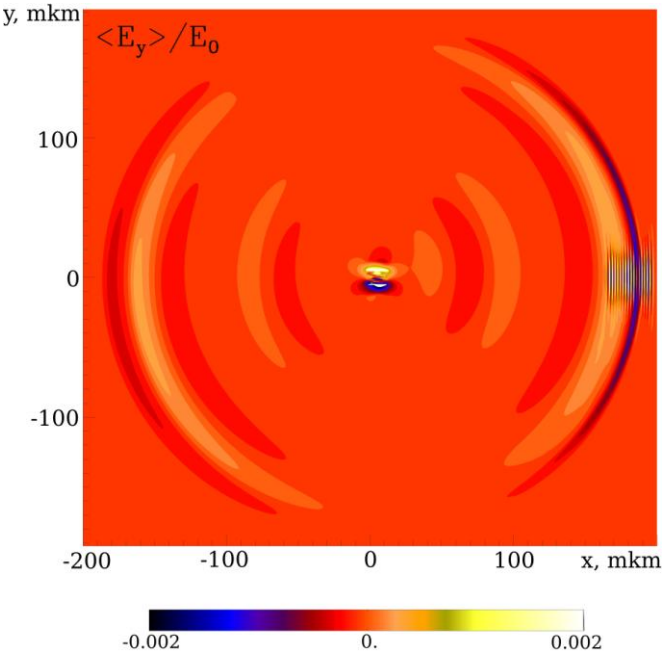


Figure 2. *The distribution of the period averaged y-component of the electric field at* $t = 100\lambda_0/c = 667 fs$.

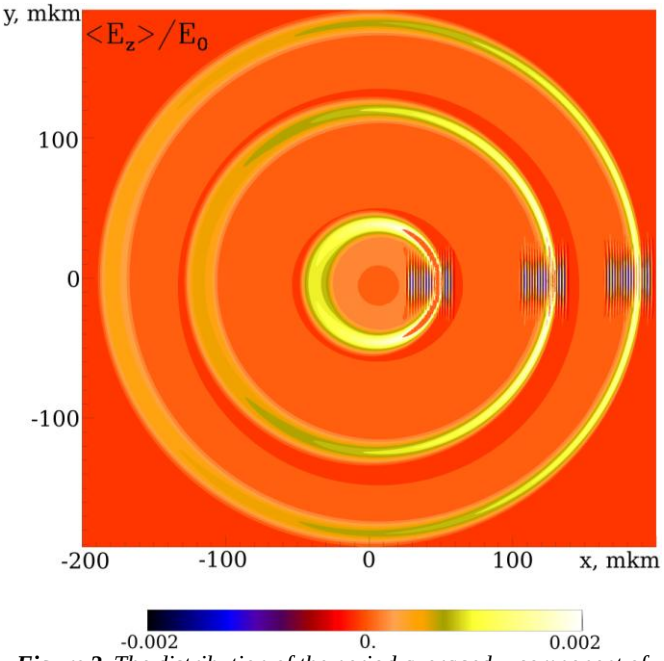
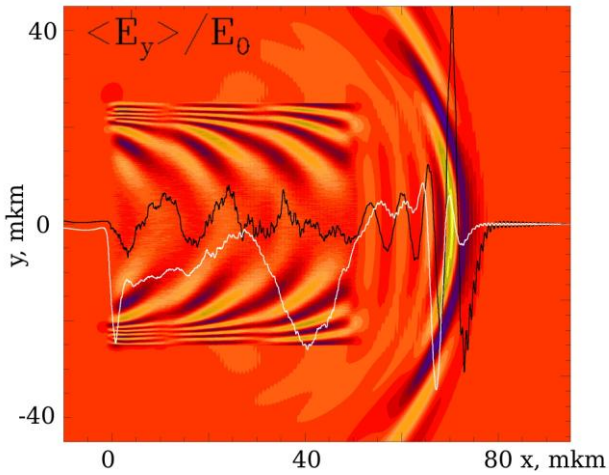


Figure 3. *The distribution of the period averaged y-component of the electric field at* $t = 30,70,100\lambda_0/c$, *corresponding to 200, 467 and 667 fs.*

An example of a non-radiating configuration is plotted in Figure 4. The distribution of the y-component of the electric field at is shown for $t = 120\lambda_0/c = 320 fs$. Here the circularly polarized bichromatic laser pulse with $\lambda_0 = 0.8\mu m, a_0 = 0.05$ interacted with larger argon cell of $\Delta_{cell} = L_{cell} = 62.5\lambda_0$ of the initial concentration of $n_0 = 1.1 \times 10^{19} cm^{-3}$. After a release of a short terahertz pulse, this configuration doesn't emit radiation anymore, although the

distribution of the fields and currents remains essentially time-dependent.



4. Conclusion

In this paper a numerical tool to study dynamical configurations of plasma created through the ionization of an initially neutral gas cell by an intense laser pulse has been presented. The mathematical model consists of Vlasov equations for the distribution functions of the plasma species and Maxwell equations for the electromagnetic field. To solve the Vlasov equations the widely used PIC method has been applied. To self-consistently update the electromagnetic fields the FDTD method has been used. Series of simulations within the wide range of laser pulse and gas cell parameters has been performed to examine the formation of non-radiating configurations in a plasma arising from the ionization of atomic gases by an intense bichromatic laser pulse. The results of the simulations have confirmed that in the interaction of a relatively long but not monochromatic laser pulse with an initially neutral gas cell an asymmetric photoelectron momentum distribution is formed. The resulting strong, time-dependent photoelectron current leads to the emission of a short electromagnetic pulse in the THz domain. After the release of a short THz pulse, a nearly non-radiating plasma configuration is formed, even though the charge and current distributions remain essentially time-dependent. Our calculations suggest that the formation of a non-radiating configuration occurs in a self-consistent way and can therefore affect the duration of the THz pulse, making it potentially very short.

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