FATIGUE ANALYSIS APPROACHES FOR VEHICLE COMPONENTS MADE OF RUBBER

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Abstract: Generally, the most frequently used structural materials are metals which have high strength and stiffness. However, there are many cases, when other important properties come to the fore as well as high deformation by elastic behavior, high viscosity namely good damping effect. Vehicle components made of rubber usually exhibit large deformations. One of the most important properties of rubber is the ability to withstand large strains without permanent fractures. This feature makes it ideal for many engineering applications. On the other hand, the task becomes more complicated because of some features of rubber parts. The temperature of rubber increases significantly after deformations. Material properties of rubber change after these above mentioned temperature changes. Thus it is necessary to understand the mechanics underlying the failure process. This paper summarizes the applied equations and the basic physical laws which are responsible for the theoretical background of the strain and temperature changes and the analysis approaches that are available for predicting fatigue life in rubber, especially in vehicle components made of rubber.

Keywords: RUBBER, HIGH DEFORMATIONS, THERMODYNAMICS, FATIGUE ANALYSIS

1. Introduction

Rubber can be classified as a so-called hyperelastic polymer which has a typical geometrical and material nonlinear behavior. It means that the relationship between displacements and internal forces can be described by functions whose order is higher than linear. The geometrical nonlinearity is easy to handle mathematically, however the material nonlinearity is only described approximately [1], [2]. Independently of the experimental investigations which deal with the material behavior of rubber, a number of theoretical works treated rubber as an ideally nonlinear elastic, in particular hyperelastic material. One of the properties of the constitutive equations of hyper-elastic material is that stresses are derived from stored elastic energy function. Hyper-elasticity can be described by particularly convenient constitutive equation given its simplicity and it constitutes the basis for more complex material models such as elastoplasticity, viscoplasticity, and viscoelasticity [3,4,5,6].

Furthermore, the task becomes more complicated because of some features of rubber parts. The temperature of rubber increases significantly. Therefore, the temperature- and displacement fields are coupled, and it means that special solving algorithms are required [7]. So the equations of mechanics and thermodynamics are coupled. As described above, the goals of this paper are the following:

It is necessary to summarize the applied equations and the basic physical laws which are responsible for the theoretical background [8,9]. Clarification of these relationships is essential because the material laws of rubber cannot violate those basic physical laws. It is necessary to extend these relationships like balance of linear momentum and balance of angular momentum, the first and second law of thermodynamics to high deformation of rubber and rubberlike polymers. An algorythm will be presented which allows to calculate strain changes and temperature changes of the rubber part of a vehicle component under certain conditions. The present numerical algorithm is the basis of the further fatigue and lifetime-calculations. After this, it will follow the literature survey which is responsible for predicting fatigue life.

2. Governing equations

2.1 Equilibrium of linear momentum

The differential formulation of the equilibrium of linear momentum in the current configuration is

$$\rho\dot{\vec{v}} = \underline{\underline{\sigma}} \cdot \nabla + \vec{f} \tag{1}$$

where ρ is the mass density, $\vec{v}$ is the velocity, $\underline{\underline{\sigma}}$ is the Cauchy stress, $\vec{f}$ is the volume force.

2.2 Equilibrium of angular momentum

The next equality shows the differential form of the balance of the moments.

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^T \tag{2}$$

2.3 First law of thermodynamics

When deformations repeatedly occur, significant increase in temperature can be observed. The differential form of the first law of thermodynamics is in the current configuration

$$\dot{e}\rho = \left[-\nabla \cdot \vec{q} + h\right] + \underline{\underline{\sigma}} \cdot \cdot \underline{\underline{l}} \tag{3}$$

where e is the internal energy per unit mass, $\vec{q}$ is the heat flux, h is the heat source, $\underline{\underline{l}}$ is the velocity gradient, $\underline{\underline{l}} = \underline{\underline{F}} \cdot \underline{\underline{F}}^{-1}$, $\underline{\underline{l}} = \vec{v} \circ \nabla$.

2.4 Second law of thermodynamics

The behaviour of viscoelastic materials is described by the second law of thermodynamics. The second law of thermodynamics in the current configuration can be written as

$$\dot{\eta}T\rho \geq -\nabla \cdot \vec{q} + \frac{\vec{q} \cdot \nabla T}{T} + h \tag{4}$$

where η is the entropy per unit mass and T is the absolute temperature. It will be practical to change the variable from entropy per unit mass to temperature by applying the Legendre-transformation and by using the Helmholtz-free energy

$$\psi = e - \eta T \tag{5}$$

Substitute the Eqn. (5) into to the Eqn. (3) and subtract Eqn. (3) from Eqn. (4) the following expression will be generated

$$-\left(\dot{\psi} + \eta\dot{T}\right)\rho + \underline{\underline{\sigma}} \cdot \cdot \underline{\underline{l}} - \frac{\vec{q} \cdot \nabla T}{T} - \mathsf{D} \geq 0 \tag{6}$$

which is known as Clausius-Duhem inequality [2].

2.4 Constitutive model

The property of an elastic element is that the total mechanical energy is reversible. The free energy of the body is the function of the strain and temperature. Dissipation comes only from heat conduction.
In order to make the further calculations easier it is necessary to split the Eq. (5) to temperature-dependent and temperature-independent parts. Based on known functions $\tilde{\psi}_0\left(\underline{\underline{C}}\right)$ and $e_0\left(\underline{\underline{C}}\right)$ for the free energy and the internal energy at a given reference temperature and the given heat capacity at a reference temperature, one obtains the following general structure for the thermoelastic free energy from the Eq. (5):

$$\psi\left(\underline{\underline{C}},T\right)=\tilde{\psi}\left(\underline{\underline{C}},\tilde{T}\right)=\frac{T}{T_0}\psi_0\left(\underline{\underline{C}}\right)+(1-\frac{T}{T_0})e_0\left(J\right)+\int_{T_0}^{T}\hat{c}\left(\underline{\underline{C}},\tilde{T}\right)\left(1-\frac{T}{\tilde{T}}\right)d\tilde{T} \quad (7)$$

where $\underline{\underline{C}}$ is the right Cauchy-Green strain tensor [2]. In the following section we are going to investigate the isotrop materials and we are going to apply the Neo-Hookean material law. It means that which is used in free energy depends on the scalar invariant of the right Cauchy-Green strain tensor. The internal energy is zero applying the entropic theory and the heat capacity is constant with good approximation.

2.5 Equation of heat conduction

Starting from the first law of thermodynamics and introducing the internal energy and changing the variable from entropy to temperature, the equation will have the next form:

$$\rho_0 c\dot{T}=\left(\underline{\underline{S}}-\rho_0\frac{\partial\psi}{\partial\underline{\underline{C}}}\right)\cdot\cdot\frac{1}{2}\dot{\underline{\underline{C}}}+\rho_0\frac{\partial^2\psi}{\partial T\partial\underline{\underline{C}}}\cdot\cdot\dot{\underline{\underline{C}}}T-q_0\nabla_0+h_0 \quad (8)$$

where $\left(\underline{\underline{S}}-\rho_0\frac{\partial\psi}{\partial\underline{\underline{C}}}\right)\cdot\cdot\frac{1}{2}\dot{\underline{\underline{C}}}$ is the non-recoverable part of the mechanical power, which is zero in the case of a pure elastic element [1], [2]. In this case the reological model is regarded to be a pure elastic element. So the free energy of the body is characterized by the deformation and temperature. Furthermore, we are assuming that there are not heat sources in the rubber and the temperature field shows homogeneous distribution. Thus, the equation of the heat conduction is the following:

$$c\dot{T}=T\frac{\partial^2\psi}{\partial T\partial\underline{\underline{C}}}\cdot\cdot\dot{\underline{\underline{C}}} \quad (9)$$

2.6 Example

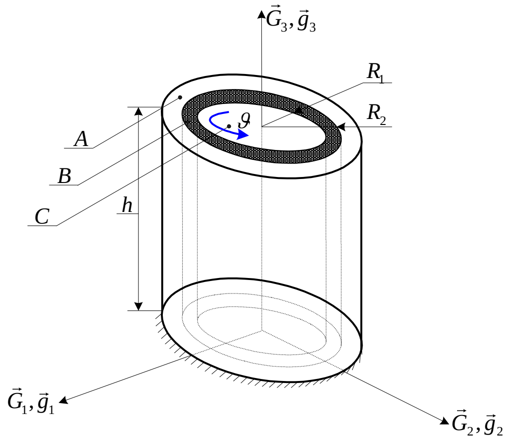

Fig. 1. Mechanical model of a silent block

Let us consider the mechanical model of a silent block, thus the A, B, C axisymmetric bodies (see Fig.2). The A and C bodies are rigid bodies, and B is a deformable one. Regarding the structure of the silent block it consists of two metal elements whose are connected by the rubber which is vulcanized between them. The inside rubber part provides a non-linear elastic connection between the two metal elements in the following way: it transfers loads however filters out the harmful vibrations, i.e. it has damping effect. All three bodies are axysymmetric and their symmetry axes are the same.

The external body (A) is fixed and the internal one is imposed by a given rotation.

Further assumptions:

Planes perpendicular to the symmetric axis will be planes after the deformation. The magnitude of the displacement is linear function of the measured distance from the axes of symmetry. Furthermore, we are assuming that there aren't heat sources in the rubber and the temperature field shows homogeneous distribution, $h_0=0, q_0=0$. Fig.2 presents the temperature change by the effect of vibration frequency 1 Hertz.

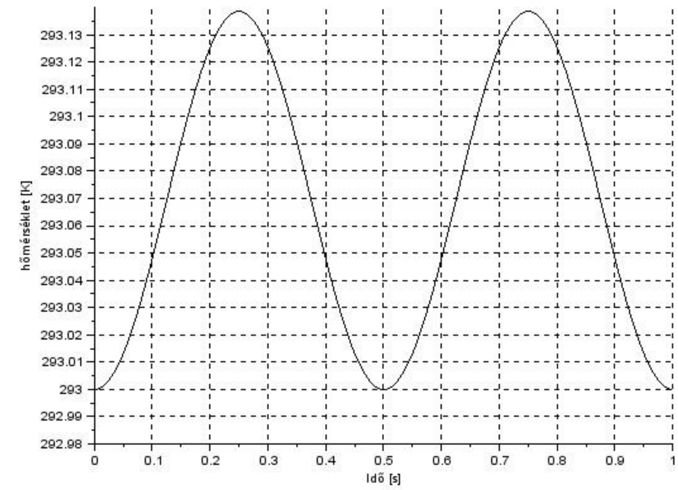

Fig. 2. Temperature change as the function of time

The calculation of temperature changes which is caused by the effect of vibration frequency 1 Hertz is obtained by the using of Eq.(9), i.e. $c\dot{T}=T\frac{\partial^2\psi}{\partial T\partial\underline{\underline{C}}}\cdot\cdot\dot{\underline{\underline{C}}}$. Substitution of the next relation to the Eq.(9), $\varphi=\vartheta\sin\left(\omega t\right)\frac{R_2-R}{R_2-R_1}$ the temperature changes can be calculated. Generation of calculations and the representation of the temperature changes as the function of time were determined by the SCILAB program. This calculation will be the basis of the further fatigue and lifetime calculations. In the next a brief literature survey on fatigue analysis approaches will be presented.

3. Literature survey: types of fatigue analysis approaches of rubber

The facility of rubber is the resistance finite strains without permanent deformation makes it ideal for engineering applications e.g. vehicle components made of rubber, as can be seen in the first figure. These loading cases impose large static and time-varying strains over a long period. The fatigue failure analysis have two distinct sections. The first section is a time during which cracks nucleate in regions. The second section is a time during nucleated cracks grow to the point of failure. Two further models for predicting fatigue life in rubber have to be considered. One theory occupies with the predicting crack nucleation life, given the history of quantities like stress and strain in the current configuration. The other theory derives from fracture mechanics, focuses on prediction of the growth of each cracks. Some of the information described in this paper has been reviewed previously [10-15]. Another paper summarizes factors that influence the fatigue life of rubber [16]. These factors are the effects of the mechanical loading history, environmental effects, rubber formulation, and effects due to dissipative aspects of the constitutive response of rubber.

3.1 Models for predicting crack nucleation life

Let us consider fatigue crack nucleation life which is defined as the number of cycles required to cause the appearance of a crack or cracks. The first study of this was Wöhler's work in the 1860's [17]. The maximum principal strain and the strain energy density are the two widely used fatigue life parameters for crack nucleation approach in rubber. Strain can be directly determined from displacements, which can be measured in rubber. The strain energy density can be estimated from a hyperelastic strain energy density function, which can be defined in terms of strains. The alternating and mean values of maximal principal strain predict nucleation life. The earliest fatigue studies in rubber occupied with the developing of the description of the number of cycles to failure as a function of starin. Cadwell et al. [18] considered unfilled rubber and investigated minimum engineering strains and strain amplitudes in determined range. They stated that, for constant strain amplitude, the fatigue life of rubber improves with extending minimum strain, up to a high strain level, and the additional minimum strain decreased the life. Generally, in the case of strain crystallized rubbers, by increasing the minimum strain of the strain cycle can significantly elongate the fatigue life.

The strain energy density is the second most important parameter which can be used for prediction of fatigue crack initiation. The energy release rate is proportional to the product of strain energy density in certain conditions and the crack size [19, 20]. Several paper can be mentioned in which researchers investigated strain energy density as a fatigue life parameter in rubber. According to the research of Roberts and Benzies [21], and Roach [22], the equibiaxial tension fatigue life of natural rubber is longer than simple tension fatigue life, in the case of using equal strain energy density. Furthermore, Roach gave the best correlation between simple and equibiaxial tension fatigue data, i.e. Roach proposed –in the case simple tension-that all of the strain energy density is available for flaw growth.

3.2 Models by using crack growth approach

The pre-existing cracks or flaws are in the focus of the crack growth approaches. The two main bases of this type of approach are the works which were publicated by Inglis and later Griffith [23]. Griffith offered a fracture criterion based on energy balance including both the mechanical energy of a cracked body, and the energy associated with the crack surfaces. This approach was further developed by for the case of rubber by Thomas, Lake, Mullins, Lindley and Rivlin. The original application of this approach in the case of rubber was predict to static strength, and later Thomas extended it to the analysis of the growth of the cracks under cyclic loadings in natural rubber. Thomas discovered a square-law context between energy release rate and crack growth rate in the case of unfilled natural rubber. According to Griffith's hypothesis the crack growth is due to the conversion of the stored potential energy to surface energy is in connection with new crack surfaces. He presented that the surface energy associated with the crack faces of a broken glass cane was equal to the elastic energy caused by the fracture. The potential energy (in rubber) released from surrounding material is spent on both reversible and irreversible changes to create the new surfaces. The energy release rate is simply the change in the stored mechanical energy ∂u, per unit change in crack surface area ∂A. This quantity is often called tearing energy T in rubber literature.

$$T = \frac{\partial u}{\partial A}$$

The energy release rate was first applied to the analysis of rubber specimens under static loading, and the above mentioned concept also applied to crack growth under cyclic loading. The experience was that the maximum energy release rate achieved during a cycle determined the crack growth rate, for R=0 cycles [24].

4. Summary

An algorythm was presented which allows to calculate strain changes and temperature changes of the rubber part of a vehicle component under certain conditions. In the future I would like to develop a solving computer program in order to apply it as a thermodynamically consistent description. The present numerical algorithm is the basis of the further fatigue and lifetime-calculations. The literature survey will be used to create the connection with the calculations.

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