Extension of geometry and investigation of deformation on crossed helical gears to increase load capacity and performance

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Abstract: *Crossed helical gear units are used in many applications. They range from actuators and power take-offs to household appliances and functions in automotive engineering and production processes. The design is often based on a material combination of steel-worm and plastic-wheel. Based on the research on high efficient plastic materials, they already replace a large number of steel applications. In order to improve the load capacity and performance of crossed helical gears, they must be understood in detail. This article deals with a new calculation method to design optimized flank geometries and to investigate them with regard to their properties in gear mesh. In addition, the deformation and the load distribution in the gear mesh will be examined in more detail. The observation is made in the normal section. In this section, all relevant influences on the performance of the gearing can be analyzed. The pressure and deformation are verified with the help of an FEM-simulations.*

Keywords*: CROSSED HELICAL GEARS, PLASTIC GEARS, FEM-SIMULATION*

1. Introduction

The multitude of applications of crossed helical gear units requires quickly adaptable and optimally designed gearings. Crossed helical gears are found in actuators and auxiliary drives as well as they are increasingly used in automation processes. The high gear ratio in a small space is a great advantage, like it is in worm gears. The development of highly efficient and resilient plastic materials makes it possible to replace more gear applications with steel wheels by plastic wheels. The material behavior of plastic is different than steel. A number of influencing factors have to be taken into account. This paper describes the approach in a research project in which a new calculation method for designing and calculating the loads on crossed helical gears was developed. To investigate the meshing behavior and load distribution of crossed helical gears, it is necessary to know the exact geometry.

Niemann and Winter [1] calculate the geometry of a crossed helical gear using a counterpart rack. The rack rolls on the two pitch circles of the individual gears. The gears, considered in this article, consists of a worm and a helical gear. For this reason, the terms worm and wheel are used in the following text. Niemann and Winter [1] calculate the meshing, pressures, sliding paths etc. only at the screw point. Boehme [2] has extended the approach. He defines a counterpart rack from two known gears. The rack is a thin counterpart rack in space. Boehme calculates the variables along the entire path of contact. His approach allows changes for axis distance and changes in the axis crossing angle of 90°. VDI 2736-3 [3] provides a simple procedure for the initial evaluation of the load-carrying capacity. In order to be able to take all necessary influences into account, the investigations must be carried out in more detail.

The calculations in this article are also based on the gear geometries derived with the help of a counterpart rack arranged in space. The geometry of the worm and the wheel is generated by applying the law of gearing. Pressures and sliding paths can be calculated in the entire contact path. The distribution of load on the individual teeth has an influence on the running behavior and load-carrying capacity of a gear. The more tooth flanks are in contact, the less load each individual tooth has to bear. The load distribution on the tooth flanks depends on the stiffness of the plastic material and the deformation of the teeth. To verify the calculation, these correlations are analyzed with the help of FEM-simulations.

2. Calculation process

A crossed helical gear consists of two cylindrical gears. The axes of these two gears cross at a shaft angle Σ and have the center distance a.

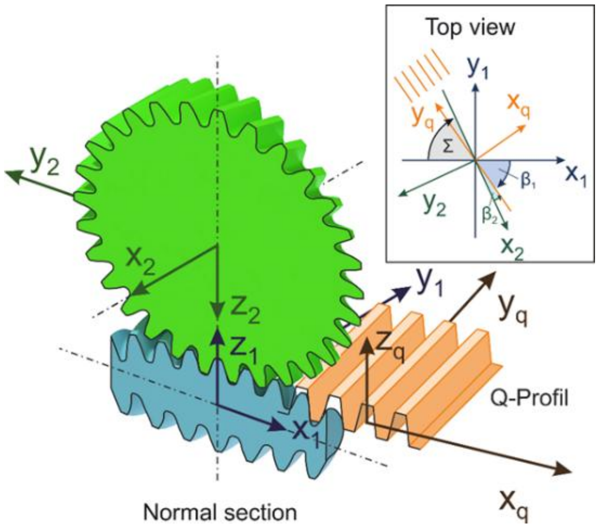

Fig. 1 *Coordinate systems of the crossed helical gear (normal section) and Q-profile*

For further investigations, it is necessary to determine some basic parameters of the gear pair to be designed: the numbers of teeth z_1, z_2, the normal modulus m_n and the centre distance a, the pressure angle in the normal section α_n, the shaft angle Σ and the two helix angles β_1 and β_2. From these parameters, the remaining parameters can be calculated. The reference profile is selected according to standard with DIN 867 [4].

The parameters are defined in individual coordinate systems. The right-hand coordinate system of wheel 1 (system 1) and wheel 2 (system 2) are shown in Fig. 1. For the analysis in the gear meshing, both wheels must be considered in one system. This is done by a global coordinate system X, Y, Z, which corresponds to the system of wheel 1. The counterpart rack has a local coordinate system q, which is also shown in Fig. 1. A fourth order polynomial (equation 1) forms the rack profile. This will be called Q-profile in the further cause of article. The individual coefficients q_0 to q_4 can be chosen arbitrarily. The pitch is considered in the coefficient q_0. q_1 considers the normal pressure angle and influences the gradient of the flank line. q_2, as well as q_3 allow various modifications to the profile for optimum design of the gears. In this article, for the involute form only q_0 and q_1 are used.

$$y_q(u) = q_0 + q_1 \cdot u + q_2 \cdot u^2 + q_3 \cdot u^3 + q_4 \cdot u^4 \quad (1)$$

$$z_q(u) = u \cdot mm \quad (2)$$

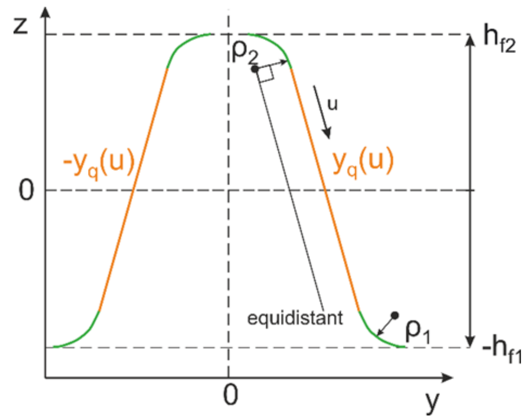

Fig. 2 *Profile of the counterpart rack of an involute flank with q_0 and q_1; the fourth order polynomial from equation 1 is shown in orange*

The parameter u describes the flank course of the Q-profile, which is initially limited by the dedendums h_{f1} and h_{f2}. It is assumed that the tip diameters result from the diameter of the material cylinders during the manufacturing process. For plastic wheels, the type of manufacturing process determines the tip circle. The rounding on the Q-profile are only required to create the tooth root fillets of the two wheels. The transition between the tip fillet of the Q-profile and the flank line is tangential and smooth. Mirroring a flank creates a whole tooth, so the whole counterpart rack can be generated. Fig. 1 shows the counterpart rack, the rack profile is shown in Fig. 2. To describe the complete Q-Profile, tangent and normal vectors at the Q profile have to be determined. These become particularly important when considering the velocities in meshing and the Hertzian theory. For the transformation of the tangent vectors into the transverse sections of the wheels, the helix angles β_1 and β_2 are required. The Q-profile, including the vectors associated with, are transformed in the coordinate systems of wheel 1 or 2.

The path of contact results from the points of contact with the Q-profile. The observation is made in the transverse section of the worm or the wheel. For each point on the path of contact, there is one point of contact on the rack and one on the flank. The rack moves translationally along the profile reference line. The gear rotates with angular velocity $\omega_{1,2}$ around its own axis of rotation. A profile point P on the rack meets the wheel point S at the pitch point E. The z-coordinate of the point on the rack corresponds to the z-coordinate of the pitch point. For the y-coordinate, the law of gearing is considered. The parameters from equation 3 depend on the variable u, which describes the flank course.

$$y_{e1}(u) = (r_{01} - z_{p1}(u)) \cdot \left(\frac{z'_{p1}(u)}{y'_{p1}(u)}\right) \qquad (3)$$

With the aid of a known point E, a point S on the flank is determined. Both gears mesh with each other, if each mesh with the counterpart rack.

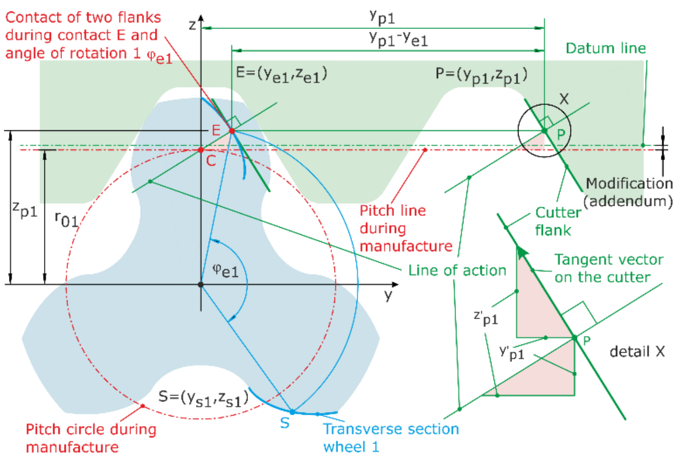

Fig. 3 *Deriving the gear geometry from the counterpart rack and the gear law*

The rack can be used in different angular positions. This enables variable shaft angles according to Boehme [2]. The path of contact is limited by the two tip diameters. The start of meshing A results from the cross point of the path of contact with the tip diameter of gear 2, the end of meshing E results from the cross point with the tip diameter of gear 1.

The path of contact is derived from the two transverse sections. In each transverse section, there exists a pitch point $E_{1,2}$. From these, the pitch point ES of the crossed helical gear is obtained. From the coordinates of the contact points in each contact position, the line of contact is obtained. Graphically, the pitch points are determined by finding the point of intersection of the path of contact with the X, Z plane, or the Y, Z plane. This results in a pitch point in each transverse section. The total contact ratio ε_γ results from the transverse contact ratio ε_α and the overlap ratio ε_β under consideration of the overlap angles φ_α and φ_β and the number of teeth z_1.

$$\varepsilon_\gamma = \frac{\varphi_\alpha + \varphi_\beta}{360°} \cdot z_1 \qquad (4)$$

To ensure that the gears created can be used in practice without problems, they are examined for meshing interference.

The equivalent radii of curvature for arbitrary curves are calculated using Bronstein [5]. To describe the tooth surfaces in three dimensions, the parameters u and v are being used. The parameter v describes the flank in the width direction, the parameter u describes the flank surface in the height direction. A shaft angle of 90° between the mesh lines can cause them to coincide with the principal curvature directions and lead to problems in the numerical evaluation. To prevent this, a transformation of the mesh lines u, v by the angles τ_g and τ_h, to the lines g and h is applied.

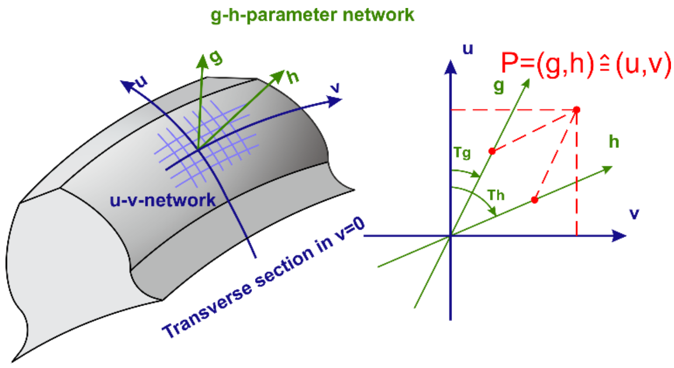

Fig. 4 *Parameter network for describing the tooth surface*

The principal curvatures are calculated using Bronstein [5]. With the corresponding formulas, curvatures on curved surfaces can be determined in general. With the help of an extension [6], the principal curvature directions are determined to match the previously determined principal curvatures. It is possible that a principal curvature radii runs towards infinity. To exclude singularities in calculation the reciprocal of the radii is used. This procedure is also particularly suitable for calculating modified flank shapes. There are no restrictions with regard to geometry. To determine the angle between the two principal curvature directions, both wheel flanks are brought together at the contact points. This is important to find the position and orientation of the two semiaxes of the contact ellipse.

The calculation program analyze the Hertzian pressure along the entire path of contact. The Hertzian theory is carried out according to known methods [7]. Instead of the rolling cylinders, two ellipsoids in contact are considered for a general view. The necessary tooth normal force is determined from the torque balance about the x_1 axis.

Plastic has a different material behavior than steel. The temperature dependence of the E-modulus as well as the stiffness behavior have a significant influence. A material parameter considers the influence of the plastic gear. The load distribution on the tooth flanks in tooth meshing changes due to the load-dependent deformation of the plastic wheel. To calculate the deformations, the steel worm is considered as a rigid element. Plastic is the more stressed and deformed material. The gears will fail on the plastic wheel.

In his work, Pech [7] derived the deformation of a plastic gear based on a bending beam. He uses a non-linear material model and describes the shape of the beam with the help of the tooth tip and height, as well as the normal module of the gear. The total deformation of the plastic gear is determined from the tooth bending and the surface deformation according to Hertz. The following figure shows the equations and correlations of Pech graphically.

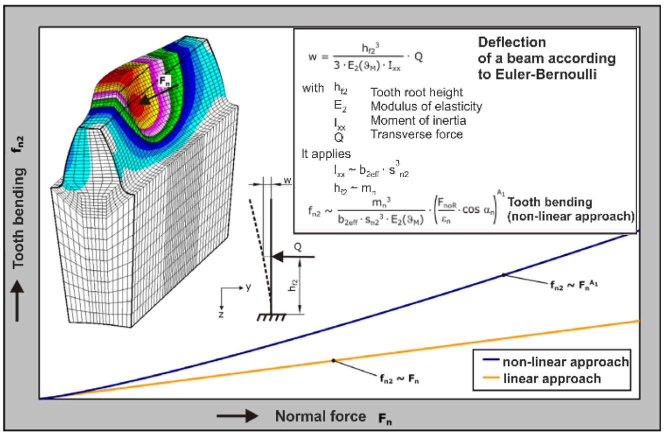

***Fig. 5** Total deformation of a gear flank with the approach of Pech [7]*

The procedure according to Pech is integrated in the new calculation. Now it is possible to determine the total deformations λ_{zus} depending on the load F_n at each meshing position. The parameter $\kappa u \cdot F_n$ describes the load distribution depended on the teeth in contact. At the end, the total deformation is a combination of the bending deformation λ_{pechF} and the Hertz surface deformation δ_{HF}. A curve of tooth deformation in the form of a 3rd order polynomial is assumed. This is consistent with the data according to Pech. The course and the assignment is done with the parameter $f_{R2}(u)^3$.

$$\lambda_{pechF}(\kappa u, u) = f_{R2}(u)^3 \cdot A_0 \cdot \frac{(m_n)^3}{b_{2eff} \cdot (s_{n2})^3 \cdot E_{2\vartheta}} \cdot (\kappa u \cdot F_n \cdot \cos(\alpha_n))^{A_1} \tag{5}$$

$$\lambda_{zus}(\kappa u, u) = \lambda_{pech}(\kappa u, u) + \delta_{HF}(\kappa u, u) \tag{6}$$

This allows further information to be derived from the extended path of contact. The load distribution in the contact area can be determined with the help of the deformation from the plastic gear. The calculation for distributing the load among the meshing tooth flanks in detail is being further refined in current research work.

The velocities and efficiencies at the pitch point are determined analogously to Boehme [2]. It is possible to calculate them at any position of the contact line. With the help of the velocities at the flank, the lubrication of crossed helical gears can also be studied in more detail in subsequent research topics. To be able to calculate the sliding paths (worm and wheel), it is important to know the orientation of the contact ellipse. The orientation is considered relative to a horizontal line through the pitch point. The sliding path is an indication for wear and can be calculated with the contact time and the contact ellipse. It is determined how the flank of a wheel moves as a function of the running parameter u through the contact ellipse. The contact time is determined with the help of the tangential velocity and the path through the contact ellipse.

3. Simulation

The contact points of crossed helical gears of all meshing tooth flanks are located in the normal section plane. Therefore, to analyse tooth deformation, it is useful to determine the 2D tooth contours of both gears in this plane.

Fig. 6 shows a worm and the normal section plane (gray) through the z-axis, which is orthogonal to the tooth flanks. The transverse section geometry (orange) is also shown below.

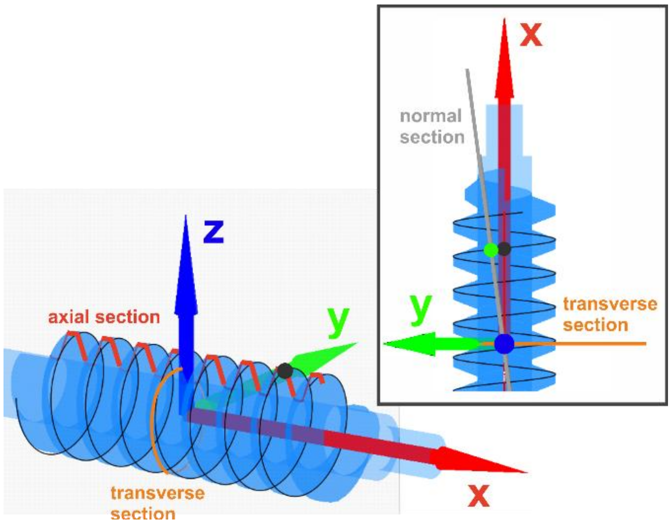

***Fig. 6** Worm with screw helix of an axial section point*

At first, for crossed helical gears with a helix angle of $\beta > 45°$, it is useful to determine the axial section geometry from the transverse section contour. In the further course, this is done for the worm. For this purpose, the helix function for an arbitrary tooth flank point P is used. In Fig. 6, the point P and the respective helix function are shown in black. With the curve parameter t, the helix describes the screwing of a flank point around the x-axis. Here, t_{tot} is the total number of revolutions over the wheel width b. The radius r and the angle φ are calculated in the x-plane ($x = x_P$) of the point P.

$$\text{Helix}(t, x_P, y_P, z_P) = \begin{pmatrix} b \cdot \frac{t}{t_{tot}} + x_P \\ r(y_P, z_P) \cdot \cos(2\pi \cdot t + \varphi(y_P, z_P)) \\ r(y_P, z_P) \cdot \sin(2\pi \cdot t + \varphi(y_P, z_P)) \end{pmatrix} \tag{7}$$

$$r(y_P, z_P) = \sqrt{{y_P}^2 + {z_P}^2} \tag{8}$$

$$\varphi(y_P, z_P) = \arctan2(y_P, z_P) \tag{9}$$

To calculate the axial section contour, the curve parameter t is determined, when the y-component results in 0. In this way for each point in the transverse section, an axial section point is directly defined.

The normal section profile can also be determined with the helix function in a similar way to the axial section profile. For crossed helical gears according to Boehme [2], the normal section plane does not necessarily have to include the z-axis. In this case, the point M in this section (x_M, y_M, z_M) describes the shortest distance orthogonal to the z-axis. By transforming the helix, the normal section is located in the xz-plane. The coordinate transformation can be described mathematically with equation 10.

$$\text{Helix}_n(t, x_P, y_P, z_P) = \begin{pmatrix} \cos(\beta - 90°) & -\sin(\beta - 90°) & 0 \\ \sin(\beta - 90°) & \cos(\beta - 90°) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \left(\text{Helix}(t, x_P, y_P, z_P) - \begin{pmatrix} x_M \\ y_M \\ z_M \end{pmatrix} \right) \tag{10}$$

The contour in the normal section results from the transformed helix and the condition y = 0. The numerical root-finding algorithm provides the curve parameter t of the desired point in the normal section. Fig. 7 shows the axial section point in black. From this point a normal section point results, shown in green. Performing the described procedure for all points (of the axial section profile) results in the green contour in the normal section.

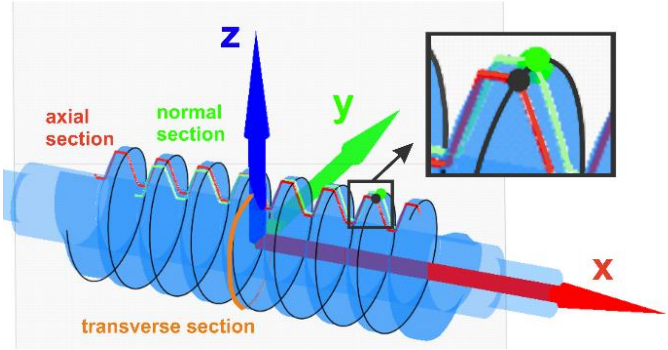

Fig. 7 *Determining the normal section contour from the axial section*

With the tooth stiffness model it is possible to determine the deformation of points on the tooth flank in the normal section. This enables the calculation of the actual overlap and the load distribution. To validate this model, an FE-routine in the normal section geometry of the gears was set up. Nodes are defined on the contour lines. By screwing them along the helix functions, the 3D-mesh of the gears is generated.

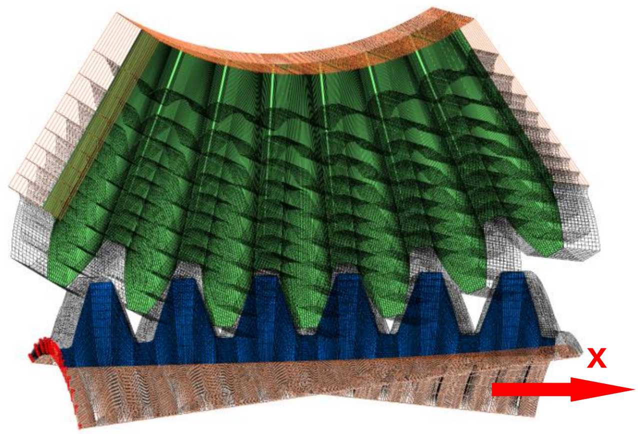

Fig. 8 *FE-mesh generation from the normal section profiles*

By generating the flanks from the normal section, only the relevant parts of the gears (that can come into contact) are meshed. This context is shown in Fig. 8 as well as the mesh that is refined in the area of the tooth contacts. The constraints are drawn in orange. The crossed helical gear is constrained in all degrees of freedom. Only at the worm, axial displacement in the x-direction is allowed. The axial force resulting from the input torque is introduced at the worm (red). The advantage of this FE-model results in the deformations and occurring stresses which are calculated directly at the nodes in the normal section.

4. Results and discussion

To prove the calculation, a reference gear pair was calculated. Pech [7] and Boehme [2] use the following gear-geometry:

Table 1: *Basic data of the reference gear pair [7] and [2]*

	worm	wheel
Center distance	30 mm	
Normal module	1,25 mm	
Normal pressure angle	20°	
Axis crossing angle	90°	
Helix angle	82,493°	7,507°
Number of teeth	1	40
Tip diameter	12,068 mm	52,932 mm
Root diameter	6,443 mm	47,307 mm
Profile shift factor	0	0
Face width	32 mm	10 mm

In the following table, the calculation results from different methods are compared. Boehme [2] compared his results with those of Pech [7]. VDI 2736-3 [3] is generally used as a basic design guideline.

Table 2: *Comparison of the calculation results to the reference gearing*

	New method	Boehme [2]	VDI 2736-3 [3]
Total contact ratio	1.8367	1.8369	1.8369
Tooth normal force	212.835 N	216.93 N	215.86 N
Hertzian pressure	110.79 N/mm^2	110.83 N/mm^2	110.12 N/mm^2
Efficiency	54.55 %	54.32 %	54.54 %
Sliding velocity	0.758 m/s	0.769 m/s	0.758 m/s
Sliding path wheel 2	35.46 mm	38.81 mm	-
Hertzian deformation	62.35 µm	-	-
Total deformation	109.58 µm	-	-

The parameter show good agreements in the results. VDI 2736-3 calculates the parameter at the screw point. To compare the values at the screw point, the results from the calculation according to Boehme and the new method are also given at this point. However, the values can be determined at any point on the contact line. With the help of the new method, it is also possible to obtain a good result regarding the total deformation with simple equations. This is an important basis for the investigation of the load distribution in the tooth mesh.

5. Conclusion

In this research work, the calculations on crossed helical gears were extended. The load distribution in the multiple engagement was examined depending on the existing load and the stiffness of the plastic material. The tooth deformation was taking into account with the load distribution. The simplifications assumed in the calculation provide good agreement in both approaches. A counterpart rack profile is defined using the fourth-order polynomial. The geometries of the two wheels are derived with one side of the rack in each case. By meshing with the rack, both wheels also mesh with each other. It is possible to consider shaft angles deviating from 90°. In addition to crossed helical gears, the calculation can also be used for spur gears. This new calculation process for crossed helical gears forms the basis for new improved, designed and optimized gears of various application areas. It shows great potential for improvements. The calculation values always assume the highest load on the gear unit. The potential for improvement is far from exhausted and will be further investigated in this ongoing study, also like the study of the load distribution in the tooth engagement in detail. New flank geometries and modifications for crossed helical gears are being developed to improve the gears and make them more efficient.

6. References

1. G. Niemann, H. Winter, *Maschinenelemente Band 1-3,* In: Springer-Verlag [1960]
2. C. Boehme.: *Berechnungsverfahren zur Erweiterung der Anwendungsgrenzen und der Optimierung von Schraubradgetrieben*, Ruhr-Universität Bochum [2020]
3. Verein Deutscher Ingenieure, VDI 2736: *Thermoplastische Zahnräder* Blatt 1-4., Berlin: [Beuth-Verlag 2014/2016]
4. Deutsche Industrie Norm, DIN 867: *Bezugsprofile für Evolventenverzahnungen an Stirnrädern (Zylinderrädern),* Beuth Verlag, Berlin, [1986]
5. Bronstein, I. N., Semendjajew K. A., *Taschenbuch der Mathematik*, B.G. Teuber Verlagsgesellschaft [1991]
6. Massachusetts Institute of Technology 3.4 Principal curvatures Link: http://web.mit.edu/hyperbook/Patrikalakis-Maekawa-Cho/node30.html [13.05.22]
7. M. Pech, *Tragfähigkeit und Zahnverformung von Schraubradgetrieben der Werkstoffpaarung Stahl/Kunststoff*, Bochum [2011]